A MATHEMATICAL ANALYSIS OF NUMERICAL INTEGRATION AND ITS APPLICATION TO ELECTROMAGNETIC SEISMOGRAMS

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INTRODUCTION

The problem originally chosen for my doctoral research was an analysis of the S-wave motion of natural earthquakes. Three methods of approach were tried, but for one reason or another failed to solve the problem. These efforts are discussed in the following pages. The last of these attempts led to the more specific problem which appears as the title.

Before going into a discussion of the research, it might be well to consider the background of the original and final problems. As will be seen later, it was necessary to solve the more particular problem before attempting to solve the original one.

As early as 1827 Poisson[47] recognized that there were two types of body waves, longitudinal and transversal, possible in an isotropic elastic solid. Stokes[52] recognized the irrotational and equivoluminal character of these two types of waves and also showed that an elastic solid may be put into a state of isochronous vibration.

According to Milne[42] "the earliest writer who had the idea that an earthquake was a pulse-like motion propagated through solid ground appears to have been Francisci [sic] Travagini . . . in 1679." However, he states that Thomas Young (p. 43) seems to have been the first to have a true conception of earthquake motion and its manner of propagation, and that Gay Lussac moulded this same idea into a more definite form.

Ever since 1832 instruments have been designed and built to record the earthquake motion. The early division of the records thus obtained was into "Preliminary Tremors" and "Large Waves". In 1894, von Rebeur-Paschwitz[53] obtained the first complete record of a distant earthquake with his seismograph and he suggested the possibility of identifying the body waves required by the theory of elasticity on the seismogram.

However, R. D. Oldham[46] was the first to clearly establish in a complete record, the existence of two distinct phases in the Preliminary Tremors: the first consisting mainly of condensational and the second of transversal vibrations.

It would seem that von dem Borne[4] was the first to use the present designation "P" and "S" to identify these phases; however, he attributes his nomenclature to Wiechert.

In the years following the publication of Oldham's work, many investigators attempted to draw up Travel-Time Tables for these (and other) waves; with the increased sensitivity of the later instruments and an improved time control, these tables have been revised or refined by Jeffreys[24], Macelwane[38], and others. Special charts[5] by Brunner and tables[15] by Gutenberg and Richter[14] have been constructed for use in the case of deep-focus earthquakes.

The first mathematical application of the theory of elasticity to seismological problems was given by Knott[27*] although Knott recognized the quasi-elastic character of the earth and hence the limitations of his applications.

Even before the invention of the seismograph, efforts were made to deduce the nature of the ground motion during an earthquake both from

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*In a less complete form, this work was first published in the Transactions of the Seis. Soc. of Japan by Knott in 1899.

the effects of the earthquake within the epicentral region and from the seismoscopes.

Even Oldham tried to determine the motion of the earth particle during each of the three phases which he identified on the seismograms of the Assam, India, Earthquake of 1897.

However, H. Arnold[2] was the first to attempt to integrate a seismogram and thereby obtain an earth-displacement curve. During the course of this investigation, Arnold found that in spite of all he could do the axis of his "first integral" curve or "velocity" curve had a definite slope with respect to his originally chosen horizontal time-axis and that the axis of the "second integral" curve or "displacement" curve was parabolic. Because of this, Arnold abandoned the work.

The complete mathematical expression of the earth displacement in terms of the derivative, the trace amplitudes, and the integrals of a seismogram for an electromagnetic seismograph was first published by Prince Galitzin[11]:

$$x = x_0 + \frac{1}{2A_1k}\Big\{\dot{y} + ay + b\int_0^t ydt + c\int_0^t dt\int_0^t ydt + d\int_0^t dt\int_0^t dt\int_0^t ydt - \left[A + Bt + Ct^2 + Dt^3 + Et^4\right]\Big\}.$$

Galitzin also tried to analyze the S-wave motion[11,12] and thought that he observed polarization, the plane of which could be determined in terms of the azimuths if the place of observation, the azimuth of the epicenter, and the angle of incidence of the ray. The derivation of Galitzin's formula was strictly mathematical, based on an analogy from optics; however, he presupposed that he was dealing with a pure transverse wave. In fact, Neumann developed a method[43] for constructing "displacement diagrams" because of his hypothesis as to the nature of the S-waves which seems to be closely related to that of Galitzin.

Klotz[25], in 1920, applied the Henrici Analyzer to seismograms, but was only able to conclude that the S-wave motion was somewhat complicated.

Neumann examined the nature of the S-wave motion[43] by constructing the "displacement diagrams" mentioned above and found that

> . . . three elementary activities appear in the majority of cases in the following order: (1) a slight activity with a horizontal component apparently normal to the line of propagation, (2) a major activity following the rule expressed in the foregoing equation,

i.e., $A_{So} = A_{SE} + A_{ES} - A_{Ss}$

where A_{So} = the azimuth of the S-wave at the origin

A_{SE} = the azimuth from station to epicenter

A_{Ss} = the azimuth of the S-wave impulse at any observatory

A_{ES} = the azimuth from epicenter to station.

> . . . and, (3) a major activity with horizontal motion in the line of propagation, probably with PS group. . . .
>
> Before any S-waves emerge, there is nearly always a minor activity which may be a prolongation of the compressional wave activity. . . .

The early applications of the mathematical theory of elasticity assumed that the two types of strain were simultaneously released and hence that the two types of bodily waves started from a common origin simultaneously and that the two phases (one of each type) recorded at a given station had traveled along the same path with different velocities. This would require that the travel-time curves for both P and S pass through the origin of the time-distance curves.

Jeffreys[22] made a statistical study of the travel-times for the Jersey Earthquake and and found that the S-curve had a positive time intercept with respect to the P-curve. His more recent[23] works seem to Jeffreys to confirm this.

Byerly[8] observed a similar phenomena in the case of the California Earthquakes. In addition to the "Curtsey"[6] with which he found the S motion usually to begin, Byerly observed a phase having a travel-time curve parallel to, but preceding by about 20 seconds, that of the true S. This phase he designated as "F" for "False S". The motion seemed to Byerly to be longitudinal in character.

By drawing "displacement diagrams", after Neumann[43] Byerly[9] noted for the S-wave proper that

> although the first crest or two of S shows that the horizontal component is nearly transverse to the path, the motion quickly changes to SV, that is, the horizontal component becomes parallel with the line joining the epicenter and station.

Jeffreys[20, 21] has given the name SV to that component of a pure shear wave which vibrates in a plane parallel to the plane of incidence and the name SH to that component which vibrates perpendicularly to it.

Using Galitzin's[11] Equation, mentioned above Sharpe[50] had some of the records of the earthquake of November 13, 1932 analyzed on the MIT differential analyzer and thereby obtained displacement curves for the P-motion.

Neumann[40,44] has devised an ingenious method for the numerical integration of seismograms obtained from mechanical-optical seismographs. In his particular work, he could consider his instruments to be accelerometers. Neumann's outstanding contributions are: (1) that Arnold's results are precisely what one should expect; (2) that by introducing determinable constants of integration, a displacement curve may be obtained for which the axis, or zero line, is parallel to, or coincident with the originally chosen one.

Adkins[1] working under Byerly, studied the records of the Alaskan Earthquake of June 22, 1937. A portion of this study was also devoted to the S-wave motion. In this work, Adkins drew displacement diagrams after Neumann and Byerly. From these, he was able to verify the existence of a longitudinal type wave about 20 seconds ahead of the true S; i.e., the False S or "P" of Byerly. However, Adkins found from his diagrams that the remainder of the S motion could be considered as composed of two parts which he called S_1 and S_2 where S_1 is neither pure SH nor SV -- thereby disagreeing with Neumann and Byerly -- and S_2 is approximately pure SH.