Notes on the Nature of the Forces which give rise to the Earthquake Motions.

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§1. Introduction.

The existence of a certain regularity in the distribution of the senses of the first movements of an earthquake was first discovered by Prof. Shida. He found that the region disturbed is divided into four parts by the two lines intersecting at the epicentre, in the opposite two of which the senses of the initial motions are outward, and in the other two they are inward. Then, Dr. S. T. Nakamura[1] of this observatory proceeded to make a research in the same line and gave numerous examples. He also detected that, besides the case mentioned above, there is another one in which the disturbed region is composed of two parts, separated by a line, on one side of which the outward initial shocks dominate and on the other side the inward ones. It will be of some interest to seismologists if we may get any information about the forces generating an earthquake from the observed facts just described. The possibility of it has been suggested by G. W. Walker in his book "Modern Seismology." The object of this note is to make a discussion about some simple ideal cases from the dynamical standpoint.

As for the elastic properties the interior of the earth is not homogeneous, as shown by Wiechert and others, and the heterogeneity of the exterior parts may be inferred from the complexity of its geological structure. These conditions greatly affect the propagations of seismic waves. So we must take the heteorogeneity of the earth into consideration in such research as here attempted. But, as our object is to give a rough sketch of the effects of various forces acting at the sources, we assume that the earth is a homogeneous and isotropic body. Galitzin's success in determining the seismic centre from the seismograms taken at a single station and the facts mentioned above seems to justify this supposition for a tentative purpose. The dispersion, absorption, and scattering of the seismic waves are sure to modify the results in some degrees, but these are also neglected on the same account in the present investigation. For the consideration of the cases of near earthquakes the earth's surface may be taken as a plane: the investigations of the modifications due

1) S. T. Nakamura, On the Direction of the First Movement of the Earthquake. Jour. Met. Soc. paJan, 41st Year, No. 2, pp. 1—10. Also, This Bulletin, Vol. 1, No. 1, p. 43 & Vol. 1, No. 2, p. 69.

to the free surface are postponed for the future with those of some of the complications described above.

§2. Effects of a Single Force acting at a Point in an Infinite Elastic Solid.

Suppose a homogeneous, isotropic, elastic medium extended infinitely; and let a single periodic force Re^{ipt} act in the positive direction of X-axis at a point which is taken as the origin. Then the displacement $\mathfrak{d}$ at the point, x, y, z, is given by[1]

$$\mathfrak{d}=\nabla\phi+rot\mathfrak{P}, \quad \text{.........(1)}$$

where

$$\left.\begin{aligned}\phi&=-\frac{R}{4\pi p^2\rho}\frac{\partial}{\partial x}\frac{e^{ip\left(t-\frac{r}{a}\right)}}{r}\\ \mathfrak{P}_x&=0\\ \mathfrak{P}_y&=\frac{R}{4\pi p^2\rho}\frac{\partial}{\partial z}\frac{e^{ip\left(t-\frac{r}{b}\right)}}{r}\\ \mathfrak{P}_z&=-\frac{R}{4\pi p^2\rho}\frac{\partial}{\partial y}\frac{e^{ip\left(t-\frac{r}{b}\right)}}{r}\end{aligned}\right\}, \quad \text{.........(2)}$$

x, y, z, the Cartesian coordinates of a point,
$r=\sqrt{x^2+y^2+z^2}$, the radial distance from the orgin,
t, the time,
p, 2π times frequency,
ρ, the density of the medium,
a, the velocity of the condensational wave, and
b, the velocity of the distortional wave.

a and b are expressed by means of Lame's constants λ, μ and the density ρ as

$$a=\sqrt{\frac{\lambda+2\mu}{\rho}},$$

$$b=\sqrt{\frac{\mu}{\rho}}.$$

$\frac{e^{ip\left(t-\frac{r}{a}\right)}}{r}$ and $\frac{e^{ip\left(t-\frac{r}{b}\right)}}{r}$ satisfy the wave equations $\frac{\partial^2 f}{\partial t^2}=a^2\nabla^2 f$ and $\frac{\partial^2 f}{\partial t^2}=b^2\nabla^2 f$, respectively and so, (1) may be written in the form

1) Stokes, On the Dynamical Theory of Diffractions, Math. and Phys. Papers. Vol. II. p. 276. Love, Mathematical Theory of Elasticity, 4th. Ed., p. 309. Lamb, Phil. Trans. Roy. Soc. London. A. Vol. 203. 1904.

$$\left.\begin{aligned}\mathfrak{d}_x&=\frac{\partial\phi}{\partial x}+\frac{\partial^2\chi}{\partial x^2}+\frac{p^2}{b^2}\chi\\ \mathfrak{d}_y&=\frac{\partial\phi}{\partial y}+\frac{\partial^2\chi}{\partial x\partial y}\\ \mathfrak{d}_z&=\frac{\partial\phi}{\partial z}+\frac{\partial^2\chi}{\partial x\partial z}\end{aligned}\right\}\quad\cdots\cdots\cdots\cdots(3)$$

where

$$\chi=\frac{R}{4\pi p^2\rho}\,\frac{e^{ip\left(t-\frac{r}{b}\right)}}{r}.$$

If we put

$$\phi'=\frac{R}{4\pi p^2\rho}\,\frac{\partial}{\partial x}\,\frac{e^{ip\left(t-\frac{r}{b}\right)}}{r},\quad\cdots\cdots\cdots\cdots(4)$$

then $\mathfrak{d}$ is given by

$$\mathfrak{d}=\nabla(\phi+\phi')+\mathfrak{B},\quad\cdots\cdots\cdots\cdots(5)$$

where

$$\mathfrak{B}_x=\frac{R}{4\pi p^2\rho}\,\frac{p^2}{b^2}\,\frac{e^{ip\left(t-\frac{r}{b}\right)}}{r},\quad \mathfrak{B}_y=\mathfrak{B}_z=0.$$

When the force is directed in the direction, whose direction cosines are l, m, n, the displacement is given by an extension of the above results; i. e.,

$$\mathfrak{d}=\nabla(\phi+\phi')+\mathfrak{B},$$

where

$$\left.\begin{aligned}\phi&=-\frac{R}{4\pi p^2\rho}\left(l\frac{\partial}{\partial x}+m\frac{\partial}{\partial y}+n\frac{\partial}{\partial z}\right)\frac{e^{ip\left(t-\frac{r}{a}\right)}}{r}\\ \phi'&=\frac{R}{4\pi p^2\rho}\left(l\frac{\partial}{\partial x}+m\frac{\partial}{\partial y}+n\frac{\partial}{\partial z}\right)\frac{e^{ip\left(t-\frac{r}{b}\right)}}{r}\\ \mathfrak{B}&=\frac{R}{4\pi p^2\rho}\,\frac{p^2}{b^2}(\mathfrak{i}l+\mathfrak{j}m+\mathfrak{k}n)\frac{e^{ip\left(t-\frac{r}{b}\right)}}{r}\end{aligned}\right\}\quad\cdots\cdots\cdots\cdots(6)$$

Here $\mathfrak{i}, \mathfrak{j}, \mathfrak{k}$ denote unit vectors in the positive x-, y-, z- directions respectively.

The movements of the styles of a seismometer are due to the acceleration $\ddot{\mathfrak{d}}$ of the earth's crust; therefore we take the latter for the following discussion. $\ddot{\mathfrak{d}}$ is given by

$$\ddot{\mathfrak{d}}=\nabla(\ddot{\phi}+\ddot{\phi}')+\ddot{\mathfrak{B}},\quad\cdots\cdots\cdots\cdots(7)$$

where

$$\left.\begin{aligned}\ddot{\phi}&=\frac{R}{4\pi\rho}\left(l\frac{\partial}{\partial x}+m\frac{\partial}{\partial y}+n\frac{\partial}{\partial z}\right)\frac{e^{ip\left(t-\frac{r}{a}\right)}}{r}\\ \ddot{\phi}'&=-\frac{R}{4\pi\rho}\left(l\frac{\partial}{\partial x}+m\frac{\partial}{\partial y}+n\frac{\partial}{\partial z}\right)\frac{e^{ip\left(t-\frac{r}{b}\right)}}{r}\\ \ddot{\mathfrak{B}}&=-\frac{R}{4\pi\rho}\,\frac{1}{b^2}(\mathfrak{i}l+\mathfrak{j}m+\mathfrak{k}n)\,p^2\,\frac{e^{ip\left(t-\frac{r}{b}\right)}}{r}\end{aligned}\right\}\quad\cdots\cdots\cdots\cdots(8)$$

When the force changes with the time as $Rf(t)$ instead of Re^{ipt}, we use Fourier's theorem,

$$f(t)=\frac{1}{\pi}\mathfrak{R}\int_0^\infty e^{ipt}dp\int_{-\infty}^{\infty}f(\lambda)e^{-ip\lambda}d\lambda,$$

and have

$$\ddot{\mathfrak{d}}=\nabla(\phi+\phi')+\mathfrak{B}.\quad\cdots\cdots\cdots\cdots(9)$$

The ϕ, ϕ', $\mathfrak{B}$ of the last expression are different from those already given and as follows,[1]

$$\left.\begin{aligned}\phi&=\frac{R}{4\pi\rho}\left(l\frac{\partial}{\partial x}+m\frac{\partial}{\partial y}+n\frac{\partial}{\partial z}\right)\frac{f\left(t-\frac{r}{a}\right)}{r}\\ \phi'&=-\frac{R}{4\pi\rho}\left(l\frac{\partial}{\partial x}+m\frac{\partial}{\partial y}+n\frac{\partial}{\partial z}\right)\frac{f\left(t-\frac{r}{b}\right)}{r}\\ \mathfrak{B}&=\frac{R}{4\pi\rho}\,\frac{1}{b^2}(\mathfrak{i}l+\mathfrak{j}m+\mathfrak{k}n)\,\frac{f''\left(t-\frac{r}{b}\right)}{r}\end{aligned}\right\}\quad\cdots\cdots\cdots\cdots(10)$$

From these equations, it will be seen that $\ddot{\mathfrak{d}}$ is divided into two parts, i. e., $\ddot{\mathfrak{d}}_a=\nabla\phi$ and $\ddot{\mathfrak{d}}_b=\nabla\phi'+\mathfrak{B}$. The former represents the disturbances which propagate spherically with the velocity a and corresponds to the first preliminary tremors of an earthquakes, and the latter is the same with the velocity b and is appropriate to the secondary preliminary tremors.

First we consider $\ddot{\mathfrak{d}}_a=\nabla\phi$.

Now

$$\begin{aligned}\phi&=\frac{R}{4\pi\rho}\left(l\frac{\partial}{\partial x}+m\frac{\partial}{\partial y}+n\frac{\partial}{\partial z}\right)\frac{f\left(t-\frac{r}{a}\right)}{r}.\\ &=-\frac{R}{4\pi\rho}(lx+my+nz)\left\{\frac{f'\left(t-\frac{r}{a}\right)}{ar^2}+\frac{f\left(t-\frac{r}{a}\right)}{r^3}\right\}\end{aligned}$$

1) Here and in the following, it is assumed that $f(t)$ and a number of its successive derivatives are continuous and may be expressed by Fourier's integrals.

Suppose

$$f(t)=0, \qquad \text{for} \quad t<0,$$
$$=t^n, \quad n>0, \qquad \text{for} \quad 0\leqq t\leqq \varepsilon$$

and for $t>\varepsilon$, it may have any form satisfying the conditions of Fourier's theorem. Then, the ratio of the two terms in the last parenthesis of the last expression for ϕ is,

$$\frac{n\left(t-\frac{r}{a}\right)^{n-1}}{ar^2}\Bigg/\frac{\left(t-\frac{r}{a}\right)^n}{r^3}=\frac{nr}{a\left(t-\frac{r}{a}\right)}.$$

Hence for $0<t-\frac{r}{a}<\varepsilon$, we have

$$\frac{nr}{a\left(t-\frac{r}{a}\right)}>\frac{nr}{a\varepsilon}.$$

Thus, if $\frac{nr}{a\varepsilon}\gg 1$, or, as n is finite if $0<t-\frac{r}{a}<\varepsilon\ll\frac{r}{a}$, the second term may be neglected compared with the first. We have

$$\phi=-\frac{R}{4\pi\rho}(lx+my+nz)\frac{f'\left(t-\frac{r}{a}\right)}{ar^2}, \quad \text{for} \quad 0\leqq t-\frac{r}{a}\leqq\varepsilon.$$

For the object of investigating the first movement of the first preliminary tremors only, this is sufficient. The duration of the movements to be taken must be small compared with the time used by the disturbance to arrive at the station. Thus neglecting the infinitesimal terms of higher orders, we have

$$\left.\begin{aligned}
\ddot{\mathfrak{d}}_{ax}&=\frac{R}{4\pi\rho}(lx+my+nz)x\frac{f''\left(t-\frac{r}{a}\right)}{a^2r^3}\\
\ddot{\mathfrak{d}}_{ay}&=\frac{R}{4\pi\rho}(lx+my+nz)y\frac{f''\left(t-\frac{r}{a}\right)}{a^2r^3}\\
\ddot{\mathfrak{d}}_{az}&=\frac{R}{4\pi\rho}(lx+my+nz)z\frac{f''\left(t-\frac{r}{a}\right)}{a^2r^3}
\end{aligned}\right\}, \text{ for } 0\leqq t-\frac{r}{a}\leqq\varepsilon, \quad \ldots\ldots(11)$$

From these equations, it is seen that

$$\ddot{\mathfrak{d}}_{ax}:\ddot{\mathfrak{d}}_{ay}:\ddot{\mathfrak{d}}_{az}=x:y:z$$

or, the $\ddot{\mathfrak{d}}_a$ is in the radial direction.

By similar processes, we have

$$\left.\begin{aligned}
\ddot{\mathfrak{d}}_{bx}&=-\frac{R}{4\pi\rho}\{(lx+my+nz)x-lr^2\}\frac{f''\left(t-\frac{r}{b}\right)}{l^2r^3}\\
\ddot{\mathfrak{d}}_{by}&=-\frac{R}{4\pi\rho}\{(lx+my+nz)y-mr^2\}\frac{f''\left(t-\frac{r}{b}\right)}{l^2r^3}\\
\ddot{\mathfrak{d}}_{bz}&=-\frac{R}{4\pi\rho}\{(lx+my+nz)z-nr^2\}\frac{f''\left(t-\frac{r}{b}\right)}{b^2r^3}
\end{aligned}\right\}, \text{ for } 0\leqq t-\frac{r}{b}\leqq\varepsilon, \quad \ldots\ldots(12)$$

Further,

$$\ddot{\mathfrak{d}}_{bx}:\ddot{\mathfrak{d}}_{by}:\ddot{\mathfrak{d}}_{bz}=\{(lx+my+nz)x-lr^2\}:\{(lx+my+nz)y-mr^2\}:\{lx+my+nz)z-nr^2\}$$

Multiplying each member of the right-handed side of the last proportionality by x, y, z respectively, we have

$$\{(lx+my+nz)x-lr^2\}x+\{(lx+my+nz)y-mr^2\}y+\{(lx+my+nz)z-nr^2\}z=0$$

By this, $\mathfrak{d}_b$ is shown to be tangential.

These results may hold good also for the case $f(t)=e^{ipt}$, if $\frac{pr}{a}$ or $\frac{pr}{b}\gg 1$, i. e., if the distance be very great compared with the wave-lengths.

§3. The Cases of the Complex Forces.

(i) Double forces. Let the force $Rf(t)$ act at the extremity of a small line element $\frac{\delta s}{2}$ from the origin, whose direction cosines are (λ, μ, ν), then the acceleration at the point x, y, z is given by

$$\ddot{\mathfrak{d}}-\frac{\partial\ddot{\mathfrak{d}}}{\partial x}\lambda\frac{\delta s}{2}-\frac{\partial\ddot{\mathfrak{d}}}{\partial y}\mu\frac{\delta s}{2}-\frac{\partial\ddot{\mathfrak{d}}}{\partial z}\nu\frac{\delta s}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(13)$$

where $\mathfrak{d}$ is given by (9).

The effect of a similar force of the opposite sense acting at the end of $\frac{\delta s}{2}$ directed to $(-\lambda, -\mu, -\nu)$, is given by

$$-\left(\ddot{\mathfrak{d}}+\frac{\partial\ddot{\mathfrak{d}}}{\partial x}\lambda\frac{\delta s}{2}+\frac{\partial\ddot{\mathfrak{d}}}{\partial y}\mu\frac{\delta s}{2}+\frac{\partial\ddot{\mathfrak{d}}}{\partial z}\nu\frac{\delta s}{2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots(14)$$

The resultant effect of (13) and (14) is

$$-\delta s\left(\lambda\frac{\partial}{\partial x}+\mu\frac{\partial}{\partial y}+\nu\frac{\partial}{\partial z}\right)\ddot{\mathfrak{d}}.$$

Now, we denote this by $\dot{\mathfrak{d}}$,

$$\mathfrak{d}=\nabla(\phi_1+\phi_1')+\mathfrak{B}_1, \qquad \text{.........(15)}$$

where

$$\left.\begin{aligned}
\phi_1&=-\frac{R\delta s}{4\pi\rho}\left(\lambda\frac{\partial}{\partial x}+\mu\frac{\partial}{\partial y}+\nu\frac{\partial}{\partial z}\right)\left(l\frac{\partial}{\partial x}+m\frac{\partial}{\partial y}+n\frac{\partial}{\partial z}\right)\frac{f\left(t-\frac{r}{a}\right)}{r}\\
\phi_1'&=\frac{R\delta s}{4\pi\rho}\left(\lambda\frac{\partial}{\partial x}+\mu\frac{\partial}{\partial y}+\nu\frac{\partial}{\partial z}\right)\left(l\frac{\partial}{\partial x}+m\frac{\partial}{\partial y}+n\frac{\partial}{\partial z}\right)\frac{f\left(t-\frac{r}{b}\right)}{r}\\
\mathfrak{B}_1&=-\frac{R\delta s}{4\pi\rho}\,\frac{1}{b^2}\left(\lambda\frac{\partial}{\partial x}+\mu\frac{\partial}{\partial y}+\nu\frac{\partial}{\partial z}\right)(\mathfrak{i}l+\mathfrak{j}m+\mathfrak{k}n)\frac{f''\left(t-\frac{r}{b}\right)}{r}
\end{aligned}\right\}\text{...(16)}$$

From this $\mathfrak{d}_a$ is known to be

$$\mathfrak{d}_a=-\frac{R\delta s}{4\pi\rho}\nabla\left(\lambda\frac{\partial}{\partial x}+\mu\frac{\partial}{\partial y}+\nu\frac{\partial}{\partial z}\right)\left(l\frac{\partial}{\partial x}+m\frac{\partial}{\partial y}+n\frac{\partial}{\partial z}\right)\frac{f\left(t-\frac{r}{a}\right)}{r}.$$

In the right-hand side of this equation (λ, μ, ν) and (l, m, n) enter equally; it is concluded that we cannot distinguish between the two directions from the distribution of $\ddot{\mathfrak{d}}_a$ only.

In the case of $f(t)$ having the form given in §2, we have

$$\mathfrak{d}_a=\nabla\phi_1$$

or

$$\left.\begin{aligned}
\mathfrak{d}_{ax}&=\frac{R\delta s}{4\pi\rho}(\lambda x+\mu y+\nu z)(lx+my+nz)x\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}\\
\mathfrak{d}_{ay}&=\frac{R\delta s}{4\pi\rho}(\lambda x+\mu y+\nu z)(lx+my+nz)y\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}\\
\mathfrak{d}_{az}&=\frac{R\delta s}{4\pi\rho}(\lambda x+\mu y+\nu z)(lx+my+nz)z\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}
\end{aligned}\right\},\ \text{for } 0\leqq t-\frac{r}{a}\leqq\varepsilon, \quad \text{...(17)}$$

and

$$\mathfrak{d}_b=\nabla\phi_1'+\mathfrak{B}_1$$

or,

$$\left.\begin{aligned}
\mathfrak{d}_{bx}&=-\frac{R\delta s}{4\pi\rho}(\lambda x+\mu y+\nu z)\{(lx+my+nz)x-lr^2\}\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}\\
\mathfrak{d}_{by}&=-\frac{R\delta s}{4\pi\rho}(\lambda x+\mu y+\nu z)\{(lx+my+nz)y-mr^2\}\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}\\
\mathfrak{d}_{bz}&=-\frac{R\delta s}{4\pi\rho}(\lambda x+\mu y+\nu z)\{(lx+my+nz)z-nr^2\}\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}
\end{aligned}\right\},\ \text{for } 0\leqq t-\frac{r}{b}\leqq\varepsilon. \quad \text{...(18)}$$

That $\mathfrak{d}_a$ is radial and $\mathfrak{d}_b$ is tangential, is obvious.

(ii) Quadruple forces. Consider the resultant effect of the following four forces $Rf(t)$, acting at the end of the line elements from the origin, directed respectively,

	Force in the direction,	$\frac{\delta s}{2}$ directed to
1)	(l, m, n),	(λ, μ, ν),
2)	(l, m, n),	$(-\lambda, -\mu, -\nu)$,
3)	$(-l, -m, -n)$,	(λ', μ', ν'),
4)	$(-l, -m, -n)$,	$(-\lambda', -\mu', -\nu')$.

The effects of these forces are expressed respectively by

$$\left.\begin{aligned}
&1)\quad \mathfrak{d}-\frac{\delta s}{2}\left(\lambda\frac{\partial}{\partial x}+\mu\frac{\partial}{\partial y}+\nu\frac{\partial}{\partial z}\right)\mathfrak{d}+\frac{1}{2}\left(\frac{\delta s}{2}\right)^2\left(\lambda\frac{\partial}{\partial y}+\mu\frac{\partial}{\partial y}+\nu\frac{\partial}{\partial z}\right)^2\mathfrak{d}\\
&2)\quad \mathfrak{d}+\frac{\delta s}{2}\left(\lambda\frac{\partial}{\partial x}+\mu\frac{\partial}{\partial y}+\nu\frac{\partial}{\partial z}\right)\mathfrak{d}+\frac{1}{2}\left(\frac{\delta s}{2}\right)^2\left(\lambda\frac{\partial}{\partial x}+\mu\frac{\partial}{\partial y}+\nu\frac{\partial}{\partial z}\right)^2\mathfrak{d}\\
&3)\quad -\left\{\mathfrak{d}-\frac{\delta s}{2}\left(\lambda'\frac{\partial}{\partial x}+\mu'\frac{\partial}{\partial y}+\nu'\frac{\partial}{\partial z}\right)\mathfrak{d}+\frac{1}{2}\left(\frac{\delta s}{2}\right)^2\left(\lambda'\frac{\partial}{\partial x}+\mu'\frac{\partial}{\partial y}+\nu'\frac{\partial}{\partial z}\right)^2\mathfrak{d}\right\}\\
&4)\quad -\left\{\mathfrak{d}+\frac{\delta s}{2}\left(\lambda'\frac{\partial}{\partial x}+\mu'\frac{\partial}{\partial y}+\nu'\frac{\partial}{\partial z}\right)\mathfrak{d}+\frac{1}{2}\left(\frac{\delta s}{2}\right)^2\left(\lambda'\frac{\partial}{\partial x}+\mu'\frac{\partial}{\partial y}+\nu'\frac{\partial}{\partial z}\right)^2\mathfrak{d}\right\}
\end{aligned}\right\}.$$

The resultant effect is given by

$$\left(\frac{\delta s}{2}\right)^2\left\{\left(\lambda\frac{\partial}{\partial x}+\mu\frac{\partial}{\partial y}+\nu\frac{\partial}{\partial z}\right)^2-\left(\lambda'\frac{\partial}{\partial x}+\mu'\frac{\partial}{\partial y}+\nu'\frac{\partial}{\partial z}\right)^2\right\}\mathfrak{d}$$

Now, this will be denoted by $\mathfrak{d}$.

$$\mathfrak{d}=\nabla(\phi_2+\phi_2')+\mathfrak{B}_2, \qquad \text{.........(19)}$$

where

$$\left.\begin{aligned}
\phi_2 &= \frac{R}{4\pi\rho}\left(\frac{\delta s}{2}\right)^2\left\{\left(\lambda\frac{\partial}{\partial x}+\mu\frac{\partial}{\partial y}+\nu\frac{\partial}{\partial z}\right)^2-\left(\lambda'\frac{\partial}{\partial x}+\mu'\frac{\partial}{\partial y}+\nu'\frac{\partial}{\partial z}\right)^2\right\}\\
&\qquad\times\left(l\frac{\partial}{\partial x}+m\frac{\partial}{\partial y}+n\frac{\partial}{\partial z}\right)\frac{f\left(t-\frac{r}{a}\right)}{r}\\
\phi_2' &= -\frac{R}{4\pi\rho}\left(\frac{\delta s}{2}\right)^2\left\{\left(\lambda\frac{\partial}{\partial x}+\mu\frac{\partial}{\partial y}+\nu\frac{\partial}{\partial z}\right)^2-\left(\lambda'\frac{\partial}{\partial x}+\mu'\frac{\partial}{\partial y}+\nu'\frac{\partial}{\partial z}\right)^2\right\}\\
&\qquad\times\left(l\frac{\partial}{\partial x}+m\frac{\partial}{\partial y}+n\frac{\partial}{\partial z}\right)\frac{f\left(t-\frac{r}{b}\right)}{r}\\
\mathfrak{T}_2 &= \frac{R}{4\pi\rho}\left(\frac{\delta s}{2}\right)^2\frac{1}{b^2}\left\{\left(\lambda\frac{\partial}{\partial x}+\mu\frac{\partial}{\partial y}+\nu\frac{\partial}{\partial z}\right)^2-\left(\lambda'\frac{\partial}{\partial x}+\mu'\frac{\partial}{\partial y}+\nu'\frac{\partial}{\partial z}\right)^2\right\}\\
&\qquad\times(\mathrm{i}l+\mathrm{j}m+\mathrm{k}n)\frac{f''\left(t-\frac{r}{b}\right)}{r}
\end{aligned}\right\}.\quad(20)$$

From this, we have

$$\mathfrak{S}_a=\frac{R}{4\pi\rho}\left(\frac{\delta s}{2}\right)^2\nabla\left\{\left(\lambda\frac{\partial}{\partial x}+\mu\frac{\partial}{\partial y}+\nu\frac{\partial}{\partial z}\right)^2-\left(\lambda'\frac{\partial}{\partial x}+\mu'\frac{\partial}{\partial y}+\nu'\frac{\partial}{\partial z}\right)^2\right\}$$

$$\times\left(l\frac{\partial}{\partial x}+m\frac{\partial}{\partial y}+n\frac{\partial}{\partial z}\right)\frac{f\left(t-\frac{r}{a}\right)}{r}$$

$$=\frac{R}{4\pi\rho}\left(\frac{\delta s}{2}\right)^2\nabla\left\{(\lambda+\lambda')\frac{\partial}{\partial x}+(\mu+\mu')\frac{\partial}{\partial y}+(\nu+\nu')\frac{\partial}{\partial z}\right\}$$

$$\times\left\{(\lambda-\lambda')\frac{\partial}{\partial x}+(\mu-\mu')\frac{\partial}{\partial y}+(\nu-\nu')\frac{\partial}{\partial z}\right\}\left(l\frac{\partial}{\partial x}+m\frac{\partial}{\partial y}+n\frac{\partial}{\partial z}\right)\frac{f\left(t-\frac{r}{a}\right)}{r}.$$

$(\lambda+\lambda', \mu+\mu', \nu+\nu')$ and $(\lambda-\lambda', \mu-\mu', \nu-\nu')$ are both proportional to the direction cosines of the bisectors of the angle between (λ, μ, ν) and (λ', μ', ν'). These two directions and the direction of forces have the equal relations to $\mathfrak{S}_a$, and so it can be concluded that the detection of the direction of forces from the distribution of the initial movements is impossible.

For the form of $f(t)$ given in §2, we have

$$\mathfrak{S}_a=\nabla\phi_2$$

or,

$$\left.\begin{aligned}
\mathfrak{S}_{ax}&=\frac{R}{4\pi\rho}\left(\frac{\delta z}{2}\right)^2\{(\lambda x+\mu y+\nu z)^2-(\lambda' x+\mu' y+\nu' z)^2\}(lx+my+nz)x\frac{f^{(\mathrm{iv})}\left(t-\frac{r}{a}\right)}{a^4r^5}\\
\mathfrak{S}_{ay}&=\frac{R}{4\pi\rho}\left(\frac{\delta s}{2}\right)^2\{(\lambda x+\mu y+\nu z)^2-(\lambda' x+\mu' y+\nu' z)^2\}(lx+my+nz)y\frac{f^{(\mathrm{iv})}\left(t-\frac{r}{a}\right)}{a^4r^5}\\
\mathfrak{S}_{az}&=\frac{R}{4\pi\rho}\left(\frac{\delta s}{2}\right)^2\{(\lambda x+\mu y+\nu z)^2-(\lambda' x+\mu' y+\nu' z)^2\}(lx+my+nz)z\frac{f^{(\mathrm{iv})}\left(t-\frac{r}{a}\right)}{a^4r^5}
\end{aligned}\right\},(21)$$

and

$$\mathfrak{S}_b=\nabla\phi'+\mathfrak{T}_2$$

or,

$$\left.\begin{aligned}
\mathfrak{S}_{bx}&=-\frac{R}{4\pi\rho}\left(\frac{\delta s}{2}\right)^2\{(\lambda x+\mu y+\nu z)^2-(\lambda' x+\mu' y+\nu' z)^2\}\\
&\qquad\times\{(lx+my+nz)x-lr^2\}\frac{f^{(\mathrm{iv})}\left(t-\frac{r}{b}\right)}{b^4r^5}\\
\mathfrak{S}_{by}&=-\frac{R}{4\pi\rho}\left(\frac{\delta s}{2}\right)^2\{(\lambda x+\mu y+\nu z)^2-(\lambda' x+\mu' y+\nu' z)^2\}\\
&\qquad\times\{(lx+my+nz)y-mr^2\}\frac{f^{(\mathrm{iv})}\left(t-\frac{r}{b}\right)}{b^4r^5}\\
\mathfrak{S}_{bz}&=-\frac{R}{4\pi\rho}\left(\frac{\delta s}{2}\right)^2\{(\lambda x+\mu y+\nu z)^2-(\lambda' x+\mu' y+\nu' z)^2\}\\
&\qquad\times\{(lx+my+nz)z-nr^2\}\frac{f^{(\mathrm{iv})}\left(t-\frac{r}{b}\right)}{b^4r^5}
\end{aligned}\right\},\quad(22)$$

The former is radial and the latter is tangential.

(iii) Two double forces. In the results of (i), put $\lambda=l$, $\mu=m$, $\nu=n$, we have

$$\left.\begin{aligned}
\mathfrak{S}_{ax}&=\frac{R\delta s}{4\pi\rho}(lx+my+nz)^2x\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}\\
\mathfrak{S}_{ay}&=\frac{R\delta s}{4\pi\rho}(lx+my+nz)^2y\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}\\
\mathfrak{S}_{az}&=\frac{R\delta s}{4\pi\rho}(lx+my+nz)^2z\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}
\end{aligned}\right\}$$

and

$$\left.\begin{aligned}
\mathfrak{F}_{bx}&=-\frac{R\delta s}{4\pi\rho}(lx+my+nz)\{(lx+my+nz)x-lr^2\}\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}\\
\mathfrak{F}_{by}&=-\frac{R\delta s}{4\pi\rho}(lx+my+nz)\{(lx+my+nz)y-mr^2\}\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}\\
\mathfrak{F}_{bz}&=-\frac{R\delta s}{4\pi\rho}(lx+my+nz)\{(lx+my+nz)y-nr^2\}\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}
\end{aligned}\right\}.$$

If we put $l=-\lambda=l'$, $m=-\mu=m'$, $n=-\nu=n'$, we have

$$\left.\begin{aligned}
\mathfrak{F}_{ax}&=-\frac{R\delta s}{4\pi\rho}(l'x+m'y+n'z)^2x\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}\\
\mathfrak{F}_{ay}&=-\frac{R\delta s}{4\pi\rho}(l'x+m'y+n'z)^2y\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}\\
\mathfrak{F}_{az}&=-\frac{R\delta s}{4\pi\rho}(l'x+m'y+n'z)^2z\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}
\end{aligned}\right\}$$

and

$$\left.\begin{aligned}
\mathfrak{F}_{bx}&=\frac{R\delta s}{4\pi\rho}(l'x+m'y+n'z)\{(l'x+m'y+n'z)x-l'r^2\}\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}\\
\mathfrak{F}_{by}&=\frac{R\delta s}{4\pi\rho}(l'x+m'y+n'z)\{(l'x+m'y+n'z)y-m'r^2\}\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}\\
\mathfrak{F}_{bz}&=\frac{R\delta s}{4\pi\rho}(l'x+m'y+n'z)\{(l'x+m'y+n'z)z-n'r^2\}\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}
\end{aligned}\right\}.$$

In the former case, the double forces are directed outwards, and in the latter case, they are inward. The resultants of the two are given by

$$\left.\begin{aligned}
\mathfrak{F}_{ax}&=\frac{R\delta s}{4\pi\rho}\{(lx+my+nz)^2-(l'x+m'y+n'z)^2\}x\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}\\
\mathfrak{F}_{ay}&=\frac{R\delta s}{4\pi\rho}\{(lx+my+nz)^2-(l'x+m'y+n'z)^2\}y\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}\\
\mathfrak{F}_{az}&=\frac{R\delta s}{4\pi\rho}\{(lx+my+nz)^2-(l'x+m'y+n'z)^2\}z\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}
\end{aligned}\right\}, \quad \text{......(23)}$$

and

$$\left.\begin{aligned}
\mathfrak{F}_{bx}=-\frac{R\delta s}{4\pi\rho}\Big[&(lx+my+nz)\{(lx+my+nz)x-lr^2\}\\
&-(l'x+m'y+n'z)\{(l'x+m'y+n'z)x-l'r^2\}\Big]\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}\\
\mathfrak{F}_{by}=-\frac{R\delta s}{4\pi\rho}\Big[&(lx+my+nz)\{(lx+my+nz)y-mr^2\}\\
&-(l'x+m'y+n'z)\{(l'x+m'y+n'z)y-m'r^2\}\Big]\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}\\
\mathfrak{F}_{bz}=-\frac{R\delta s}{4\pi\rho}\Big[&(lx+my+nz)\{(lx+my+nz)z-nr^2\}\\
&-(l'x+m'y+n'z)\{(l'x+m'y+n'z)z-n'r^2\}\Big]\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}
\end{aligned}\right\}. \quad (24)$$

From (23), it is seen that, in this case, (l, m, n) and (l', m', n') can be determined by the observation of the distribution of $\mathfrak{F}_a$ alone.

§4. Distributions on the Plane $z=f$: Special Cases.

Now we will consider the distributions of $\mathfrak{F}_a$ and $\mathfrak{F}_b$ on the plane $z=f$, which is taken to be the earth's surface. f is the depth of the seismic centre. $\mathfrak{F}_a$ and $\mathfrak{F}_b$, which were given in the above, are accelerations at a point in an infinite solid. The actual accelerations which are obtained from the seismographic data are those at the free surface; and so must be different from those given above. The modifications are complicated and will not be given in this note. For the case that $f(t)$ is a simple harmonic function, G. W. Walker[1] has calculated them. His results may be used in some cases, but not in all cases. They are different according as the angle between the direction of the primary wave and that of the surface; and as for

1) G. W. Walker, Phil. Trans. Roy. Soc. London, A, Vol. 218, pp. 373—393, 1919.

magnitudes, we have to multiply them by a factor between 0 and 2 according as that angle. They also depend on the angle between the acceleration and the surface. The directions are changed on this account. For the present, all these are not taken into considerations. Besides the directions and magnitudes of $\mathfrak{z}_a$ and $\mathfrak{z}_b$, the ratios of their magnitudes are given, which have more possibility to be compared with the observational results at the present stage of investigation.

(i) Let a single force act at the origin in the positive direction of x-axis, then the results may be given by putting $l=1$, $m=n=o$, in the equations (11) and (12). xy-plane is of course horizontal. We have

$$\left.\begin{aligned}
\mathfrak{z}_{ax} &= \frac{R}{4\pi\rho}\,\frac{f''\left(t-\frac{r}{a}\right)}{a^2r^3}x^2\\
\mathfrak{z}_{ay} &= \frac{R}{4\pi\rho}\,\frac{f''\left(t-\frac{r}{a}\right)}{a^2r^3}xy\\
\mathfrak{z}_{az} &= \frac{R}{4\pi\rho}\,\frac{f''\left(t-\frac{r}{a}\right)}{a^2r^3}xz
\end{aligned}\right\}$$

and

$$\left.\begin{aligned}
\mathfrak{z}_{bx} &= -\frac{R}{4\pi\rho}\,\frac{f'\left(t-\frac{r}{b}\right)}{b^2r^3}(x^2-r^2)\\
\mathfrak{z}_{by} &= -\frac{R}{4\pi\rho}\,\frac{f'\left(t-\frac{r}{b}\right)}{b^2r^3}xy\\
\mathfrak{z}_{bz} &= -\frac{R}{4\pi\rho}\,\frac{f''\left(t-\frac{r}{b}\right)}{b^2r^3}xz
\end{aligned}\right\}$$

By using cylindrical coordinates $x=\varpi\cos\varphi$, $y=\varpi\cos\varphi$, $z=z$, and by putting $z=f$, the horizontal accelerations are given by

$$\left.\begin{aligned}
\mathfrak{z}_{a\varpi} &= \frac{R}{4\pi\rho}\,\frac{f''\left(t-\frac{r}{a}\right)}{a^2r^3}\varpi^2\cos\varphi\\
\mathfrak{z}_{a\varphi} &= 0
\end{aligned}\right\}, \quad \ldots\ldots\ldots\ldots\ldots\ldots(25)$$

and

$$\left.\begin{aligned}
\mathfrak{z}_{b\varpi} &= \frac{R}{4\pi\rho}\,\frac{f''\left(t-\frac{r}{b}\right)}{b^2r^3}f^2\cos\varphi\\
\mathfrak{z}_{b\varphi} &= -\frac{R}{4\pi\rho}\,\frac{f''\left(t-\frac{r}{b}\right)}{b^2r}\sin\varphi
\end{aligned}\right\}\ldots\ldots\ldots\ldots\ldots(26)$$

The direction of the force is $\varphi=o$, φ being the angle between the azimuths of the force and the radius vector from the epicentre which is the point $x=y=o$ in the plane $z=f$. $\mathfrak{z}_a$ is radial and outward when the direction of the latter is near to the former and inward when it is near to the azimuth opposite to the force. It vanishes in a direction perpendicular to the force. The same holds good of the radial component of $\mathfrak{z}_b$. The tangential component of $\mathfrak{z}_b$ is clockwise, when the radius vector from the epicentre is on the left-handed side of the direction of force, and counterclockwise, when it is on the other side. In general, the tangential component is more important of $\mathfrak{z}_b$ than the radial one ; and it may be said that in the distribution of $\mathfrak{z}_b$ there is a tendency of rotation through the smaller angle towards the sense of the force. (Fig. 1.)

The vertical accelerations are given by

$$\mathfrak{z}_{az} = \frac{R}{4\pi\rho}\,\frac{f''\left(t-\frac{r}{a}\right)}{a^2r^3}f\varpi\cos\varphi, \quad \ldots\ldots\ldots\ldots\ldots\ldots(27)$$

$$\mathfrak{z}_{bz} = -\frac{R}{4\pi\rho}\,\frac{f''\left(t-\frac{r}{b}\right)}{b^2r^3}f\varpi\cos\varphi. \quad \ldots\ldots\ldots\ldots\ldots\ldots(28)$$

$\mathfrak{z}_{az}$ is upward for the radius vector near to the sense of the force and downward for that near to the opposite direction. For $\mathfrak{z}_{bz}$, the relation is reversed.

The magnitudes of the horizontal components are

$$|\mathfrak{z}_{ah}| = \frac{R}{4\pi\rho}\,\frac{f''\left(t-\frac{r}{a}\right)}{a^2r^3}\varpi^2|\cos\varphi|.$$

$$|\mathfrak{z}_{bh}| = \frac{R}{4\pi\rho}\,\frac{f''\left(t-\frac{r}{b}\right)}{b^2r^3}\sqrt{\varpi^2(\varpi^2+2f^2)\sin^2\varphi+f^4}.$$

The comparison of the two, that is, $\mathfrak{z}_{ah}$ at $t-\frac{r}{a}$ and $\mathfrak{z}_{bh}$ at $t-\frac{r}{b}$, is given by taking the ratio,

$$\frac{|\mathfrak{F}_{bh}|}{|\mathfrak{F}_{ah}|}=\frac{a^2}{b^2}\frac{\sqrt{\varpi^2(\varpi^2+2f^2)\sin^2\varphi+f^4}}{\varpi^2|\cos\varphi|} \qquad \text{..........................(29)}$$

This ratio is minimum in the radial direction parallel to the force and increases as we approach to the direction at right angles to it.

The ratio of the vertical components is simply

$$\frac{|\mathfrak{F}_{bz}|}{|\mathfrak{F}_{az}|}=\frac{a^2}{b^2} \qquad \text{..........................(30)}$$

(ii) The single force is directed upwards, that is, to the positive sense of the z-axis. In this case we have only to put $l=m=0$, $n=1$ in (11) and (12).

We have

$$\left.\begin{aligned}
\mathfrak{F}_{ax}&=\frac{R}{4\pi\rho}\frac{f''\left(t-\frac{r}{a}\right)}{a^2r^3}zx\\
\mathfrak{F}_{ay}&=\frac{R}{4\pi\rho}\frac{f''\left(t-\frac{r}{a}\right)}{a^2r^3}zy\\
\mathfrak{F}_{az}&=\frac{R}{4\pi\rho}\frac{f''\left(t-\frac{r}{a}\right)}{a^2r^3}z^2
\end{aligned}\right\}$$

and

$$\left.\begin{aligned}
\mathfrak{F}_{bx}&=-\frac{R}{4\pi\rho}\frac{f''\left(t-\frac{r}{b}\right)}{b^2r^3}zx\\
\mathfrak{F}_{by}&=-\frac{R}{4\pi\rho}\frac{f''\left(t-\frac{r}{b}\right)}{b^2r^3}zy\\
\mathfrak{F}_{bz}&=-\frac{R}{4\pi\rho}\frac{f''\left(t-\frac{r}{b}\right)}{b^2r^3}(z^2-r^2)
\end{aligned}\right\}$$

On $z=f$, the horizontal components are, in cylindrical coordinates, as follows,

$$\left.\begin{aligned}
\mathfrak{F}_{a\varpi}&=\frac{R}{4\pi\rho}\frac{f''\left(t-\frac{r}{a}\right)}{a^2r^3}f\varpi\\
\mathfrak{F}_{a\varphi}&=0
\end{aligned}\right\} \qquad \text{..........................(31)}$$

$$\left.\begin{aligned}
\mathfrak{F}_{b\varpi}&=-\frac{R}{4\pi\rho}\frac{f''\left(t-\frac{r}{b}\right)}{b^2r^3}f\varpi\\
\mathfrak{F}_{b\varphi}&=0
\end{aligned}\right\} \qquad \text{..........................(32)}$$

Both are radial and do not depend on the azimuthal angle φ, but their senses are opposite.

The vertical components are

$$\mathfrak{F}_{az}=\frac{R}{4\pi\rho}\frac{f''\left(t-\frac{r}{a}\right)}{a^2r^3}f^2, \qquad \text{..........................(33)}$$

$$\mathfrak{F}_{bz}=\frac{R}{4\pi\rho}\frac{f''\left(t-\frac{r}{b}\right)}{b^2r^3}\varpi^2. \qquad \text{..........................(34)}$$

These also do not depend on the azimuth. They are always upward.

The ratios of $\mathfrak{F}_b$ and $\mathfrak{F}_a$ in horizontal and vertical components are

$$\frac{|\mathfrak{F}_{b\varpi}|}{|\mathfrak{F}_{a\varpi}|}=\frac{a^2}{b^2}, \qquad \text{..........................(35)}$$

and

$$\frac{|\mathfrak{F}_{bz}|}{|\mathfrak{F}_{az}|}=\frac{a^2}{b^2}\frac{\varpi^2}{f^2}. \qquad \text{..........................(36)}$$

Both are independent of the azimuth.

For the case of the downward force, we have only to change the sign where it is necessary.

(iii) Here we investigate the distribution of accelerations produced by the two equal horizontal forces of the opposite senses acting at the two points in a straight line parallel to the directions of the forces. The moment of the two forces is zero. Taking one of the directions of the forces for the positive direction of x-axis, we have to put $l=\lambda=1$, $m=n=\mu=\nu=0$ in the equations (17) and (18). The two forces are directed for the outward senses. We have

$$\left.\begin{aligned}
\mathfrak{F}_{ax}&=\frac{R\delta s}{4\pi\rho}\frac{f''\left(t-\frac{r}{a}\right)}{a^2r^4}x^3\\
\mathfrak{F}_{ay}&=\frac{R\delta s}{4\pi\rho}\frac{f''\left(t-\frac{r}{a}\right)}{a^2r^4}x^2y
\end{aligned}\right\},$$

$$\mathfrak{z}_{az} = \frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^2r^4}x^2z$$

$$\left.\begin{aligned}
\mathfrak{z}_{bx} &= -\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^2r^4}(x^2-r^2)r\\
\mathfrak{z}_{by} &= -\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^2r^4}x^2y\\
\mathfrak{z}_{bz} &= -\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^2r^4}x^2z
\end{aligned}\right\}$$

In cylindrical coordinates, the horizontal components on the plane $z=f$ are

$$\left.\begin{aligned}
\mathfrak{z}_{a\varpi} &= \frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^2r^4}\varpi^3\cos^2\varphi\\
\mathfrak{z}_{a\varphi} &= 0
\end{aligned}\right\}, \quad \ldots\ldots\ldots\ldots(37)$$

and

$$\left.\begin{aligned}
\mathfrak{z}_{b\varpi} &= \frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^2r^4}f^2\varpi\cos^2\varphi\\
\mathfrak{z}_{b\varphi} &= -\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^2r^2}\varpi\sin\varphi\cos\varphi
\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(38)$$

$\mathfrak{z}_a$ is always outward. It is maximum on the radius vector parallel to the forces and vanishes in the directions perpendicular to it. The same holds good of the radial component of $\mathfrak{z}_b$. The tangential component has its maximum value in the direction inclined 45° to that of the forces and vanishes in the directions both parallel and perpendicular to the forces. (Fig. 3.)

The vertical components are

$$\mathfrak{z}_{az} = \frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^2r^4}f\varpi^2\cos^2\varphi, \quad \ldots\ldots\ldots\ldots(39)$$

$$\mathfrak{z}_{bz} = -\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^2r^4}f\varpi^2\cos^2\varphi \quad \ldots\ldots\ldots\ldots(40)$$

Both have maximum magnitudes in the directions of the forces, and vanish in the directions at right angles to them.

The magnitudes of the horizontal components are

$$|\mathfrak{z}_{ah}| = \frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^2r^4}\varpi^3\cos^2\varphi,$$

$$|\mathfrak{z}_{bh}| = \frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^2r^4}\sqrt{(\varpi^2+2f^2)\varpi^2\sin^2\varphi+f^4}.\varpi|\cos\varphi|.$$

The ratio of the transverse and longitudinal disturbances is as follows. For the horizontal components we have

$$\frac{|\mathfrak{z}_{bh}|}{|\mathfrak{z}_{ah}|} = \frac{a^2}{b^2}\,\frac{\sqrt{(\varpi^2+2f^2)\varpi^2\sin^2\varphi+f^4}}{\varpi^2|\cos\varphi|} \quad \ldots\ldots\ldots\ldots(41)$$

This differs only by the constant factor $\frac{a}{b}$ from the same for the case of a single horziontal force. For the vertical components we have

$$\frac{|\mathfrak{z}_{bz}|}{|\mathfrak{z}_{az}|} = \frac{a^2}{b^2} \quad \ldots\ldots\ldots\ldots(42)$$

When the forces are directed inward, we have only to change signs in the suitable places.

(iv) When the directions of the forces and the lines of action are turned through a right angle, we have to put $\varphi-\frac{\pi}{2}$ for φ in (37), (38), (39) and (40). We have

$$\left.\begin{aligned}
\mathfrak{z}_{a\varpi} &= \frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^2r^4}\varpi^3\sin^2\varphi\\
\mathfrak{z}_{a\varphi} &= 0
\end{aligned}\right\}$$

$$\left.\begin{aligned}
\mathfrak{z}_{b\varpi} &= \frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^2r^4}f^2\varpi\sin^2\varphi\\
\mathfrak{z}_{b\varphi} &= \frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^2r^2}\varpi\sin\varphi\cos\varphi
\end{aligned}\right\}$$

$$\mathfrak{z}_{az} = \frac{R\partial s}{4\pi\rho}\,\frac{f''\left(t-\frac{r}{a}\right)}{a^2r^4}f\varpi^2\sin^2\varphi,$$

$$\delta_{bz} = -\frac{R\delta s}{4\pi\rho}\,\frac{f''\left(t-\frac{r}{b}\right)}{b^3 r^4} f\varpi^2\sin^2\varphi.$$

Superposing these on the results of (iii), we have the effects of the equal forces acting outwardly in four directions.

$$\left.\begin{aligned} \delta_{a\varpi} &= \frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3 r^4}\varpi^3 \\ \delta_{a\varphi} &= 0 \end{aligned}\right\}, \quad \text{..............................(43)}$$

$$\left.\begin{aligned} \delta_{b\varpi} &= \frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3 r^4} f^2\varpi \\ \delta_{b\varphi} &= 0 \end{aligned}\right\}, \quad \text{..............................(44)}$$

$$\delta_{az} = \frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3 r^4} f\varpi^2, \quad \text{..............................(45)}$$

$$\delta_{bz} = -\frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3 r^4} f\varpi^2. \quad \text{..............................(46)}$$

These do not depend on the azimuth. With a change of constant factors, it may be considered as representing the effects of horizontal forces acting equally in all azimuths simultaneously. For both cases the ratios are

$$\frac{|\delta_{bh}|}{|\delta_{ah}|} = \frac{a^3}{b^3}\,\frac{f^2}{\varpi^2}, \quad \text{..............................(47)}$$

$$\frac{|\delta_{bz}|}{|\delta_{az}|} = \frac{a^3}{b^3} \quad \text{..............................(48)}$$

(v) Changing the signs of the right-hand side of the first six equations of (iv) and superposing on the results of (iii), we have.

$$\left.\begin{aligned} \delta_{a\varpi} &= \frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3 r^4}\varpi^3\cos 2\varphi \\ \delta_{a\varphi} &= 0 \end{aligned}\right\}. \quad \text{..............................(49)}$$

$$\left.\begin{aligned} \delta_{b\varpi} &= \frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3 r^4} f^2\varpi\cos 2\varphi \\ \delta_{b\varphi} &= -\frac{R\delta s}{4\pi\rho}\,\frac{f''\left(t-\frac{r}{b}\right)}{b^3 r^2}\varpi\sin 2\varphi \end{aligned}\right\}, \quad \text{..............................(50)}$$

$$\delta_{az} = \frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3 r^4} f\varpi^2\cos 2\varphi, \quad \text{..............................(51)}$$

$$\delta_{bz} = -\frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3 r^4} f\varpi^2\cos 2\varphi. \quad \text{..............................(52)}$$

These represent the accelerations due to two double forces at right angles, one acting outwardly and the other inwardly. The δ_a has the outward sense on the radial direction near to that of the outward forces and inward sense in the direction near to the inward forces, and it vanishes in the middle directions. The radial component of the δ_b has the same distribution. The distribution of the tangential component has the tendency of rotation from the inward forces to the outward. (Fig. 4.)

The magnitudes of the horizontal components are

$$|\delta_{ah}| = \frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3 r^4}\varpi^3|\cos 2\varphi|,$$

$$|\delta_{bh}| = \frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3 r^4}\sqrt{(\varpi^4 + 2f^2)\varpi^2\sin^2 2\varphi + f^4}\,\varpi.$$

The ratio is

$$\frac{|\delta_{bh}|}{|\delta_{ah}|} = \frac{a^3}{b^3}\,\frac{\sqrt{(\varpi^4+2f^2)\varpi^2\sin^2 2\varphi + f^4}}{\varpi^2|\cos 2\varphi|} \quad \text{..............................(53)}$$

This ratio is minimum in the directions of the forces and becomes very large in the middle directions between the two double forces.

The ratio of the vertical components is

$$\frac{|\delta_{bz}|}{|\delta_{az}|} = \frac{a^3}{b^3} \quad \text{..............................(54)}$$

These results may also be deduced from (iii) of §3.

(vi) Put $l=m=\lambda=\mu=0, n=\nu=1$ in the equations (17) and (18), we have

$$\left.\begin{aligned}
\mathfrak{F}_{ax}&=\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}xz^2\\
\mathfrak{F}_{ay}&=\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}yz^2\\
\mathfrak{F}_{az}&=\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}z^3
\end{aligned}\right\}$$

and

$$\left.\begin{aligned}
\mathfrak{F}_{bx}&=-\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}xz^2\\
\mathfrak{F}_{by}&=-\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}yz^2\\
\mathfrak{F}_{bz}&=-\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}(z^2-r^2)z
\end{aligned}\right\}.$$

In cylindrical coordinates, these are, in the plane $z=f$,

$$\left.\begin{aligned}
\mathfrak{F}_{a\varpi}&=\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}f^2\varpi\\
\mathfrak{F}_{a\varphi}&=0
\end{aligned}\right\},$$

$$\left.\begin{aligned}
\mathfrak{F}_{b\varpi}&=-\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}f^2\varpi\\
\mathfrak{F}_{b\varphi}&=0
\end{aligned}\right\},$$

$$\mathfrak{F}_{az}=\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}f^3,$$

$$\mathfrak{F}_{bz}=\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}f\varpi^2.$$

Superposing this on the results of (iv), we have

$$\left.\begin{aligned}
\mathfrak{F}_{a\varpi}&=\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^2}\varpi\\
\mathfrak{F}_{a\varphi}&=0\\
\mathfrak{F}_{az}&=\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^2}f
\end{aligned}\right\}, \quad\ldots\ldots\ldots(55)$$

$$\mathfrak{F}_{b\varpi}=\mathfrak{F}_{b\varphi}=\mathfrak{F}_{bz}=0, \quad\ldots\ldots\ldots(56)$$

or,

$$|\mathfrak{F}_a|=\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r},$$

$$|\mathfrak{F}_b|=0.$$

This result does not depend on the azimuth. With a suitable change of signs, it may represent the effects of forces acting equally in all directions simultaneously. The distorsional wave is not produced in this case.

(vii) Put $l=1$, $m=n=0$ and $\lambda=0$, $\mu=1$, $\nu=0$ in the equations (17) and (18), we have

$$\left.\begin{aligned}
\mathfrak{F}_{ax}&=\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}x^2y\\
\mathfrak{F}_{ay}&=\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}xy^2\\
\mathfrak{F}_{az}&=\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}xyz
\end{aligned}\right\},$$

$$\left.\begin{aligned}
\mathfrak{F}_{bx}&=-\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}(x^2-r^2)y\\
\mathfrak{F}_{by}&=-\frac{R\partial s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}xy^2
\end{aligned}\right\}.$$

$$\mathfrak{F}_{bz} = -\frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}\varpi yz$$

This represents the effect of two horizontal forces of the opposite senses, that is, a couple acting on the two points on a horizontal straight line at right angle to the forces. The moment of the couple has a clockwise sense. On the plane $z=f$, we have in cylindrical coordinates,

$$\left.\begin{aligned}\mathfrak{F}_{a\varpi} &= \frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}\varpi^3\sin\varphi\cos\varphi\\ \mathfrak{F}_{a\varphi} &= 0\end{aligned}\right\}, \quad\text{.....................(57)}$$

$$\left.\begin{aligned}\mathfrak{F}_{b\varpi} &= \frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}f^2\varpi\sin\varphi\cos\varphi\\ \mathfrak{F}_{b\varphi} &= -\frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^2}\varpi\sin^2\varphi\end{aligned}\right\} \quad\text{.....................(58)}$$

The $\mathfrak{F}_s$ is outward on the fore half to the left-hand side or on the back half to the right-hand side of one of the two forces. It is inward on the remaining part. It vanishes in the directions of the forces and those perpendicular to them. The same holds good of the radial component of the $\mathfrak{F}$. The tangential component has always the same sense as the moment of the couple. It vanishes on the directions of the forces. (Fig. 5.)

The vertical components are

$$\mathfrak{F}_{az} = \frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}f\varpi^2\sin\varphi\cos\varphi, \quad\text{.......................(59)}$$

$$\mathfrak{F}_{bz} = -\frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}f\varpi^2\sin\varphi\cos\varphi. \quad\text{...... ....(60)}$$

The magnitudes of the horizontal components are

$$|\mathfrak{F}_{ah}| = \frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}\varpi^3|\sin\varphi\cos\varphi|,$$

$$|\mathfrak{F}_{bh}| = \frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}\sqrt{(\varpi^2+2f^2)\varpi^2\sin^2\varphi+f^4}\,.\,\varpi|\sin\varphi|.$$

$|\mathfrak{F}_{bh}|$ becomes zero on the radial directions parallel to the forces. The ratio is

$$\frac{|\mathfrak{F}_{bh}|}{|\mathfrak{F}_{ah}|} = \frac{a^3}{b^3}\,\frac{\sqrt{(\varpi^2+2f^2)\varpi^2\sin^2\varphi+f^4}}{\varpi^2|\cos\varphi|}. \quad\text{.....................(61)}$$

This differs from that in the case of a single horizontal force only by the factor $\frac{a}{b}$. It is minimum in the directions of the forces and becomes great as we approach to the perpendicular directions.

The ratio of the vertical components is

$$\frac{|\mathfrak{F}_{bz}|}{|\mathfrak{F}_{az}|} = \frac{a^3}{b^3}. \quad\text{...(62)}$$

(viii) Put $\varphi-\frac{\pi}{2}$ for φ in (57), (58), (59), (60), we have

$$\left.\begin{aligned}\mathfrak{F}_{a\varpi} &= -\frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}\varpi^3\sin\varphi\cos\varphi\\ \mathfrak{F}_{a\varphi} &= 0\end{aligned}\right\},$$

$$\left.\begin{aligned}\mathfrak{F}_{b\varpi} &= -\frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}f^2\varpi\sin\varphi\cos\varphi\\ \mathfrak{F}_{b\varphi} &= -\frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^2}\varpi\cos^2\varphi\end{aligned}\right\},$$

$$\mathfrak{F}_{az} = -\frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}f\varpi^2\sin\varphi\cos\varphi,$$

$$\mathfrak{F}_{bz} = \frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}f\varpi^2\sin\varphi\cos\varphi.$$

Superposing this on the results of (vii), we have

$$\mathfrak{F}_{a\varpi} = \mathfrak{F}_{a\varphi} = \mathfrak{F}_{az} = 0. \quad\text{..(63)}$$

$$\left.\begin{aligned}
\mathfrak{F}_{b\varpi} &= 0,\\
\mathfrak{F}_{b\varphi} &= -\frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^2}\varpi\\
\mathfrak{F}_{bz} &= 0
\end{aligned}\right\} \qquad \text{.........................(64)}$$

These are the results of two couples of the same sense whose forces are at right angle to each other. They do not depend on the azimuth. With a suitable change of factors they may represent the effects of a curling force at the source. There exists only a distorsional disturbance, and not a condensational.

(ix) Put $l=m=0$, $n=1$ and $\lambda=1$, $\mu=\nu=0$ in the equations (17) and (18), we have

$$\left.\begin{aligned}
\mathfrak{F}_{ax} &= \frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}x^2z\\
\mathfrak{F}_{ay} &= \frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}xyz\\
\mathfrak{F}_{az} &= \frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}xz^2
\end{aligned}\right\},$$

and

$$\left.\begin{aligned}
\mathfrak{F}_{bx} &= -\frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}x^2z\\
\mathfrak{F}_{by} &= -\frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}xyz\\
\mathfrak{F}_{bz} &= -\frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}(z^2-r^2)xz
\end{aligned}\right\}$$

Here is considered the disturbance due to the two vertical forces forming a couple, whose moment is horizontal. The components of the accelerations in the plane $z=f$, are, in cylindrical coordinates,

$$\left.\begin{aligned}
\mathfrak{F}_{a\varpi} &= \frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}f\varpi^2\cos\varphi\\
\mathfrak{F}_{a\varphi} &= 0
\end{aligned}\right\}, \qquad \text{.........................(65)}$$

$$\left.\begin{aligned}
\mathfrak{F}_{b\varpi} &= -\frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}f\varpi^2\cos\varphi\\
\mathfrak{F}_{b\varphi} &= 0
\end{aligned}\right\}, \qquad \text{.....................(66)}$$

$$\mathfrak{F}_{az} = \frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{a}\right)}{a^3r^4}f^2\varpi\cos\varphi. \qquad \text{.............................(67)}$$

$$\mathfrak{F}_{bz} = \frac{R\delta s}{4\pi\rho}\,\frac{f'''\left(t-\frac{r}{b}\right)}{b^3r^4}\varpi^3\cos\varphi. \qquad \text{.............................(68)}$$

The $\mathfrak{F}_a$ is outward on the side of the upward force and inward on the opposite side. It vanishes in a straight line in the middle. The $\mathfrak{F}_b$ is also radial, but the sense is opposite. It is inward on the side of the upward force and outward on the other side. (Fig. 2.) The vertical components are upward on the side of the upward force and downward on the other side.

The ratio of the horizontal components is

$$\frac{|\mathfrak{F}_{bh}|}{|\mathfrak{F}_{ah}|} = \frac{a^3}{b^3}. \qquad \text{...(69)}$$

That of the vertical components is

$$\frac{|\mathfrak{F}_{bz}|}{|\mathfrak{F}_{az}|} = \frac{a^3}{b^3}\,\frac{\varpi^2}{f^2} \qquad \text{...(70)}$$

Both do not depend on the azimuth.

(x) Put $l=m=0$, $n=1$, $\lambda=1$, $\mu=\nu=0$, and $\mu'=1$, $\lambda'=\nu'=0$, in the equations (21) and (22), we have

$$\left.\begin{aligned}
\mathfrak{F}_{ax} &= \frac{R}{4\pi\rho}\left(\frac{\delta s}{2}\right)^2\frac{f^{(iv)}\left(t-\frac{r}{a}\right)}{a^4r^5}(x^2-y^2)xz\\
\mathfrak{F}_{ay} &= \frac{R}{4\pi\rho}\left(\frac{\delta s}{2}\right)^2\frac{f^{(iv)}\left(t-\frac{r}{a}\right)}{a^4r^5}(x^2-y^2)yz\\
\mathfrak{F}_{az} &= \frac{R}{4\pi\rho}\left(\frac{\delta s}{2}\right)^2\frac{f^{(iv)}\left(t-\frac{r}{a}\right)}{a^4r^5}(x^2-y^2)z^2
\end{aligned}\right\},$$

and

$$\left.\begin{aligned}
\mathfrak{F}_{hx} &= -\frac{R}{4\pi\rho}\left(\frac{\partial s}{2}\right)^2 \frac{f^{(iv)}\left(t-\frac{r}{b}\right)}{b^4r^5}(x^2-y^2)xz \\
\mathfrak{F}_{hy} &= -\frac{R}{4\pi\rho}\left(\frac{\partial s}{2}\right)^2 \frac{f^{(iv)}\left(t-\frac{r}{b}\right)}{b^4r^5}(x^2-y^2)yz \\
\mathfrak{F}_{hz} &= -\frac{R}{4\pi\rho}\left(\frac{\partial s}{2}\right)^2 \frac{f^{(iv)}\left(t-\frac{r}{b}\right)}{b^4r^5}(x^2-y^2)(z^2-r^2)
\end{aligned}\right\}.$$

These represent the accelerations produced by the four equal vertical forces acting on the four corners of a square, the two at the opposite corners being upward and the other two downward. On the plane $z=f$, the components are given by

$$\left.\begin{aligned}
\mathfrak{F}_{a\varpi} &= \frac{R}{4\pi\rho}\left(\frac{\partial s}{2}\right)^2 \frac{f^{(iv)}\left(t-\frac{r}{a}\right)}{a^4r^5}f\varpi^3\cos 2\varphi \\
\mathfrak{F}_{a\varphi} &= 0
\end{aligned}\right\}\text{..................(71)}$$

$$\left.\begin{aligned}
\mathfrak{F}_{b\varpi} &= -\frac{R}{4\pi\rho}\left(\frac{\partial s}{2}\right)^2 \frac{f^{(iv)}\left(t-\frac{r}{b}\right)}{b^4r^5}f\varpi^3\cos 2\varphi \\
\mathfrak{F}_{b\varphi} &= 0
\end{aligned}\right\}\text{..................(72)}$$

$$\mathfrak{F}_{az} = \frac{R}{4\pi\rho}\left(\frac{\partial s}{2}\right)^2 \frac{f^{(iv)}\left(t-\frac{r}{a}\right)}{a^4r^5}f'\varpi^2\cos 2\varphi,\text{..................(73)}$$

$$\mathfrak{F}_{bz} = \frac{R}{4\pi\rho}\left(\frac{\partial s}{2}\right)^2 \frac{f^{(iv)}\left(t-\frac{r}{b}\right)}{b^4r^5}\varpi^4\cos 2\varphi,\text{ ....(74)}$$

The $\mathfrak{F}_a$ is outward on the radial direction nearer to the directions of the diagonal combining the upward forces and inward on the remaining radial direction. It vanishes in the directions middle between the two diagonals. The same relation holds good of $\mathfrak{F}_b$, with the exchange of the outward and inward senses. Both are radial. (Fig 6.) The vertical components are upward where the horizontal component of $\mathfrak{F}_a$ is outward, and they are downward where the latter is inward. The ratios are

$$\frac{|\mathfrak{F}_{bh}|}{|\mathfrak{F}_{ah}|}=\frac{a^4}{b^4}\text{ ....(75)}$$

and

$$\frac{|\mathfrak{F}_{bz}|}{|\mathfrak{F}_{az}|}=\frac{a^4}{b^4}\frac{\varpi^2}{f^2}\text{ ....(76)}$$

Both are independent of the azimuth.

§5 Comparisons of the Various Cases.

We will now compare the various cases which give a similar distribution of $\mathfrak{F}_{a\varpi}$.

(a) In the cases (ii), (iii), (iv), and (vi) of the last section, the $\mathfrak{F}_{a\varpi}$'s are all outward. In (ii), (iv), and (vi), their magnitudes are distributed symmetrically about the point $x-y=0$, which is epicentre. In (iii), the magnitude changes from one azimuth to another and is not symmetrical about the epicentre; it vanishes in two certain senses. As to the $\mathfrak{F}_{bh}$'s there are differences. In (ii) and (iv), they are radial and symmetrical about the epicentre. But in the former it is inward and in the latter it is outward. In (iii), it is not radial in general and not symmetrical. Its radial component is always outward and its tangential component vanishes on four perpendicular directions and changes its sense from one quadrant to the neighbouring. In (vi), $\mathfrak{F}_{b}$ does not exist. The ratio $\frac{|\mathfrak{F}_{bh}|}{|\mathfrak{F}_{ah}|}$ is also various. In (ii), it is constant throughout the surface. In (iii), it is smallest on the direction of the force and becomes very great near the perpendicular direction. In (iv), it becomes great as we approach the epicentre.

(b) In the cases (i) and (ix), the $\mathfrak{F}_{a\varpi}$'s are different on the two parts of the surface divided by a straight line through the epicentre. It is outward in the one side and inward in the other side; and it vanishes in the very line. As for the distributions of $\mathfrak{F}_{bh}$, there is a large discrepancy. In (i), it is not radial in general. Its radial component has the same sense as $\mathfrak{F}_{a\varpi}$. It is marked through its tangential component. In (ix), $\mathfrak{F}_{bh}$ is radial on all azimuths and has the opposite sense as $\mathfrak{F}_{a\varpi}$. The ratio $\frac{|\mathfrak{F}_{bh}|}{|\mathfrak{F}_{ah}|}$ is also different. It is constant in (ix), but varies as the azimuth in (i). (Figs. 1 & 2.)

(c) In the cases (v), (vii), and (x), $\mathfrak{F}_{a\varpi}$ vanishes on the two perpendicular lines through the epicentre; they divide the whole surface in four quadrants, of which one of two opposite pairs contain the outward sense and the other two the inward. By means of $\mathfrak{F}_{bh}$, we can discriminate the three cases. In (v) and (vii), it is not radial in general, but in (x), it is so all over. In the latter case the radial component of $\mathfrak{F}_{bh}$ is always opposite to $\mathfrak{F}_{a\varpi}$ and in the former cases, it has the same sense. By distribution of the tangential components the former two cases can be discriminated. In the case (vii), it has the same sense everywhere and it vanishes in the directions of the forces, that is, on one of the vanishing lines of $\delta_{a\varpi}$. In the case (v), the whole surface is divided into four quadrants, whose boundaries are the four directions of the forces, that is, the two bisecting lines of the angles between the two vanishing lines of $\delta_{a\varpi}$, the sense of the tangential component is reversed as we pass from one quadrant to the next. The distribution of the ratio $\frac{|\delta_{bh}|}{|\delta_{ah}|}$ also gives the means of determining the nature of forces. In the case (v), it is minimum in the middle directions of the adjacent two of the four directions, in which $\delta_{a\varpi}$ changes its sense; and it become very great near the latter four directions. In the case (vii), it is minimum on one of two opposite pairs of the four radius vectors in which $\delta_{a\varpi}$ vanishes; and it becomes very great as we approach to the other pair perpendicular to those just mentioned. In the case (x), it is constant all over and does not depend on the azimuth. (Figs. 4, 5 & 6.)

§6. Concluding Remarks.

In all the cases described above, the source was considered as a point or a number of consecutive points. This condition is not realized in the actual cases: in the latter, the sources may have an extension of many kilometers. But, it is not probable that the disturbance begins at once all over such an extensive region; the fact may be that, at first, it occurs at some point or in a small part of the extensive source and then the disturbances are induced in the other parts. The above results must be applied to the initial disturbance only; or it may be applied to each shock of the induced disturbances separately, if we can identify them on the seismograms. The arrival of the first condensational disturbance is distinguished without difficulty on the seismograms; but that of the corresponding distorsional one is not easy to identify, because it is possible that the following shocks of condensational nature arrive before the beginning of the distorsional disturbances. In this case, it will be very convenient if we can determine whether of the registered displacements corresponds to the initial: and in such determinations, the ratio $\frac{|\delta_{bh}|}{|\delta_{ah}|}$ above calculated will be of some service when the distribution of the first movements is already known. In the determination of the position of seismic source, it is very important to take the times of the arrival of the corresponding waves of condensation and distorsion; hitherto we have no means of such identification. As regards the induced disturbances, the same must hold good, but the discrimination will be very difficult. In connexion with this problem it will be very important to compare the details of the seismograms of the same earthquake taken at different stations.

Fig. 1.

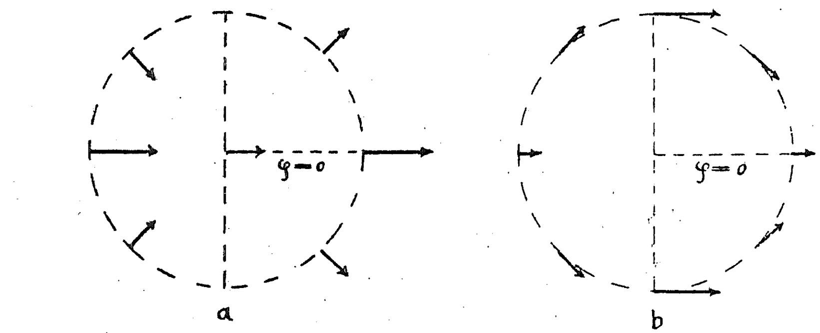

Fig. 2.

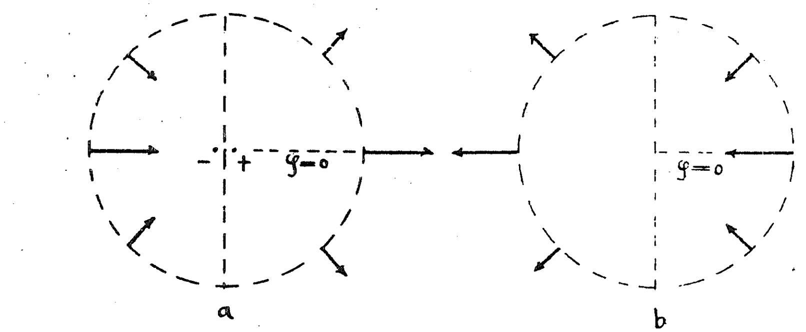

Fig. 3.

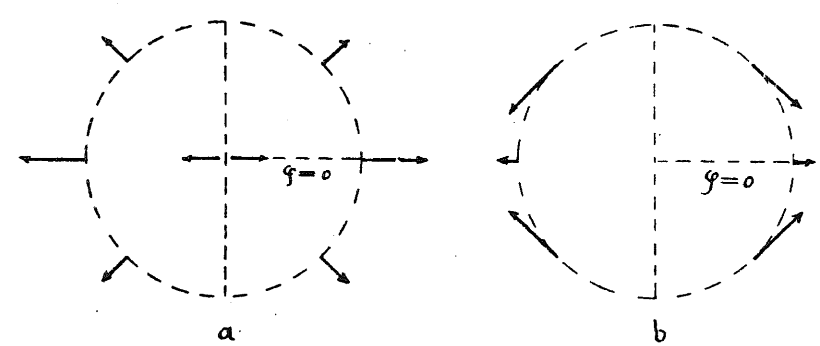

Fig. 4.

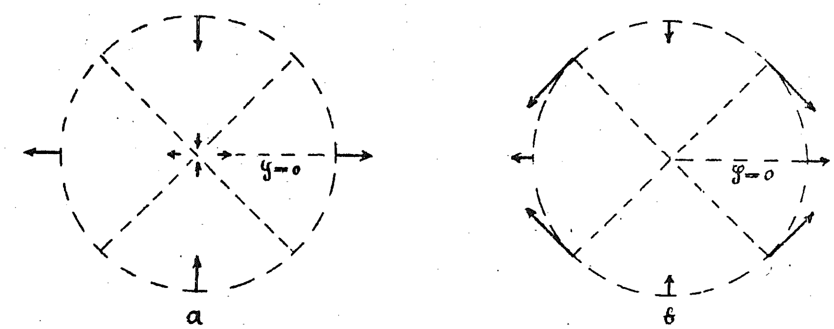

Fig. 5.

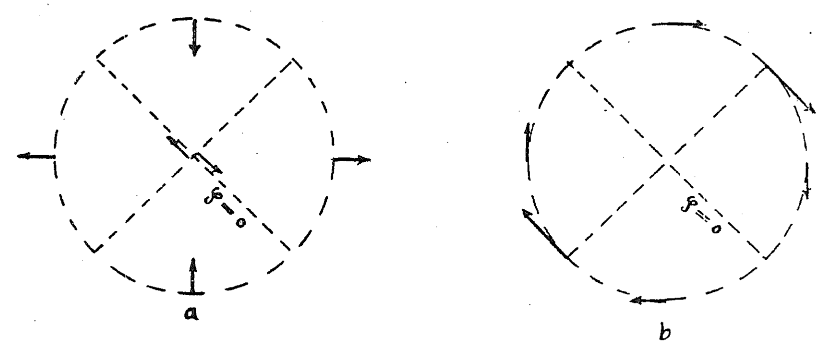

Fig. 6.