SURFACE WAVE THEORY

This section discusses the surface-wave partial derivatives required for the inversion.

1. Eigenvalue Problem

Love Waves

Following Keilis-Borok *et al.* (1989), the equation for the SH-wave eigenfunction, $V^{(3)}$, in a cylindrical coordinate system is

$$\frac{d}{dz}\begin{bmatrix} V^{(3)} \\ p_{\phi z} \end{bmatrix} = \begin{bmatrix} 0 & 1/\mu \\ \mu k^2 - \rho\omega^2 & 0 \end{bmatrix}\begin{bmatrix} V^{(3)} \\ p_{\phi z} \end{bmatrix} \quad (1)$$

where k is the wavenumber, ω is the angular frequency, μ is the rigidity and ρ is density. The boundary conditions for surface waves are that $p_{\phi z}(z=0)=0$, and $V^{(3)}(z) \rightarrow 0$, as $z \rightarrow \infty$.

A discrete solution to the boundary conditions satisfies the equation

$$L = \omega^2 I_0 - k^2 I_1 - I_2 \quad (2)$$

$$= \omega^2 \int_0^\infty \rho \left(V^{(3)}\right)^2 dz - k^2 \int_0^\infty \mu \left(V^{(3)}\right)^2 dz - \int_0^\infty \mu \left(\frac{dV^{(3)}}{dz}\right)^2 dz$$

$$= 0$$

where L is the Lagrangian. The group velocity is determined from the relation

$$U = \frac{d\omega}{dk} = \frac{kI_1}{\omega I_0} \quad (3)$$

If the medium is split into constant velocity-density layers, of thickness d_m, density ρ_m, and shear-wave velocity β_m, and the layer is bounded by $z = z_m - d_m$ and $z = z_m$, then the partials of the phase velocity with shear velocity for fixed frequency and density, and with density for fixed frequency and shear-wave velocity are

$$\left(\frac{\partial c}{\partial \beta}\right)_m = \frac{\beta_m \rho_m}{UI_0} \int_{z=z_m-d_m}^{z_m} \left[\left(V^{(3)}\right)^2 + \left(\frac{1}{k}\frac{dV^{(3)}}{dz}\right)^2\right] dz \quad (4)$$

and

$$\left(\frac{\partial c}{\partial \rho}\right)_m = \frac{\beta_m}{2\rho_m}\left(\frac{\partial c}{\partial \beta}\right)_m - \frac{c^2}{2UI_0} \int_{z=z_m-d_m}^{z_m} \left(V^{(3)}\right)^2 dz \quad (5)$$

If a layer boundary depth is perturbed by an amount h, while keeping the material properties and the frequencies constant, the variation in phase velocity is determined from

$$\frac{\partial c}{\partial h} = \frac{c^3}{2\omega^2 I_1}\left\{\omega^2 (V^{(3)})^2[\rho]_\pm - k^2 (V^{(3)})^2[\mu]_\pm + [\mu\left(\frac{dV^{(3)}}{dz}\right)^2]_\pm\right\} \tag{6}$$

where the symbol $[\,]_\pm$ indicates the computation of the jump in the parameter across the interface, e.g.,

$$[\,\chi\,]_\pm = \chi(h+0) - \chi(h-0). \tag{7}$$

Rayleigh Waves

In a cylindrical coordinate system, the equation of motion governing the eigenfunctions is

$$\frac{d}{dz}\begin{bmatrix} V^{(1)} \\ V^{(2)} \\ p_{zz} \\ p_{rz} \end{bmatrix} = \begin{bmatrix} 0 & k\lambda/(\lambda+2\mu) & 1/(\lambda+2\mu) & 0 \\ -k & 0 & 0 & 1/\mu \\ -\rho\omega^2 & 0 & 0 & k \\ 0 & -\rho\omega^2 + 4k^2\mu(\lambda+\mu)/(\lambda+2\mu) & -k\lambda/(\lambda+2\mu) & 0 \end{bmatrix}\begin{bmatrix} V^{(1)} \\ V^{(2)} \\ p_{zz} \\ p_{rz} \end{bmatrix} \tag{8}$$

where the vertical component eigenfunction is $V^{(1)}$, and the radial component eigenfunction is $V^{(2)}$. The ellipticity at the free surface is $\varepsilon = V^{(2)}(0)/V^{(1)}(0)$. The boundary conditions for surface waves are that $p_{rz}(z=0)=0$, $p_{zz}(z=0)=0$, $V^{(1)}(z)\rightarrow 0$, as $z\rightarrow\infty$. and $V^{(2)}(z)\rightarrow 0$, as $z\rightarrow\infty$.

A discrete solution to the boundary conditions satisfies the equation

$$L = \omega^2 I_0 - k^2 I_1 - 2kI_2 - I_3 = 0 \tag{9}$$

where L is the Lagrangian, and

$$I_0 = \int_0^\infty \rho\left[(V^{(1)})^2 + (V^{(2)})^2\right]dz \tag{10}$$

$$I_1 = \int_0^\infty \left[\mu(V^{(1)})^2 + (\lambda+2\mu)(V^{(2)})^2\right]dz \tag{11}$$

$$I_2 = \int_0^\infty \left[\mu V^{(1)}\frac{dV^{(2)}}{dz} - \lambda V^{(2)}\frac{dV^{(1)}}{dz}\right]dz \tag{12}$$

$$I_3 = \int_0^\infty \left[(\lambda + 2\mu) \left(\frac{dV^{(1)}}{dz} \right)^2 + (\mu) \left(\frac{dV^{(2)}}{dz} \right)^2 \right] dz \tag{13}$$

The group velocity is determined from the relation

$$U = \frac{d\omega}{dk} = (kI_1 + I_2) / \omega I_0 \tag{14}$$

If the medium is split into constant velocity-density layers, of thickness d_m, density ρ_m, compressional-wave velocity α_m, and shear-wave velocity β_m, and the layer is bounded by $z = z_m - d_m$ and $z = z_m$, then the partials of the phase velocity with shear velocity for fixed frequency, compressional velocity and density, with compresisonal velocity for fixed frequency, shear velocity and density, and with density for fixed frequency and compresional and shear velocity are

$$\left(\frac{\partial c}{\partial \alpha} \right)_m = \left(\frac{\alpha_m \rho_m}{UI_0} \right) \int_{z=z_m-d_m}^{z_m} \left[V^{(2)} - \frac{1}{k} \frac{dV^{(1)}}{dz} \right]^2 dz \tag{15}$$

$$\left(\frac{\partial c}{\partial \beta} \right)_m = \left(\frac{\beta_m \rho_m}{UI_0} \right) \int_{z=z_m-d_m}^{z_m} \left[\left(V^{(1)} + \frac{1}{k} \frac{dV^{(2)}}{dz} \right)^2 + \frac{4}{k} V^{(2)} \frac{dV^{(1)}}{dz} \right] dz \tag{16}$$

$$\left(\frac{\partial c}{\partial \rho} \right)_m = \frac{1}{2\rho} \left[\alpha \left(\frac{\partial c}{\partial \alpha} \right)_m + \beta \left(\frac{\partial c}{\partial \beta} \right)_m \right] - \frac{c^2}{2UI_0} \int_{z=z_m-d_m}^{z_m} \left[(V^{(1)})^2 + (V^{(2)})^2 \right] dz \tag{17}$$

If a layer boundary depth is perturbed by an amount h, while keeping the material properties and the frequencies constant, the variation in phase velocity is determined from

$$\left(\frac{\partial c}{\partial h} \right) = \frac{c^3}{2\omega(\omega I_1 + cI_2)} \; . \tag{18}$$

$$\left\{ \omega^2 [\rho((V^{(2)})^2 + (V^{(1)})^2]_\pm - k^2 [(\mu v^1)^2]_\pm - k^2 [(\lambda + 2\mu)(V^{(2)})^2]_\pm \right.$$

$$\left. - [(\lambda + 2\mu) \left(\frac{dV^{(1)}}{dz} \right)]_\pm \, [\mu \left(\frac{dV^{(2)}}{dz} \right)]_\pm \right\}$$

where the symbol $[\,]_\pm$ indicates the computation of the jump in the parameter across the interface, e.g.,

$$[\, \chi \,]_\pm = \chi(h+0) - \chi(h-0). \tag{19}$$

Equation (18) differs slightly from that in Keilis-Borok (1989) in the grouping of the square brackets, in order to implement the correct partials for a water-solid layer boundary, for which the radial eigenfunction is not continuous.

2. Numerical Partial Derivative Computation and Causality

The subroutines used in the programs *surf96* and *joint96* are essentially the same as those in PROGRAMS.330/VOLIII/src of **Computer Programs in Seismology.** Since the inversion programs requires partial derivatives with respect to layer parameters, a detailed discussion of the relations used is presented.

The program **srfdis96** computes the phase velocity values for a given mode, wave type and frequency. When group velocities are computed, the phase velocities are also computed at the two periods $(1 \pm h)T$ rather than at the single period T. The parameter h, tpically 0.005, is that given on LINE 1 of the control file *sobs.d.* or *jobs.d.* Denote the phase velocity output of **srfdis96** c_o.

The programs **srfdrr96** and **srfdrl96** are essentially the same as **sregn96** and **slegn96** except that the front end of the program rearranges the output for use by the inversion program. Note that the programs **sregn96** and **slegn96** provide the phase velocity partial derivatives but not the group velocity partial derivatives. In addition, although these two programs do provide the causal phase velocity and anelastic attenuation coefficient when the medium is described by a causal Q, the partial derivatives are only for infinite Q.

Given the partial derivatives of phase velocity with respect to layer shear velocity, $\frac{\partial c_o}{\partial \beta}$, compressional velocity, $\frac{\partial c_o}{\partial \alpha}$, and group velocity, U_o, the expressions for group velocity are derived by following Rodi *et al* (1975).

By definition

$$U = \frac{c}{1 + \frac{T}{c}\frac{dc}{dT}} = f(c, dc/dT, \omega, m)$$

where m is a model parameter. By applying the chain rule of differentiation,

$$\frac{\partial U}{\partial m} = \frac{\partial U}{\partial c}\frac{\partial c}{\partial m} + \frac{\partial U}{\partial \left(\frac{dc}{dT}\right)}\frac{\partial}{\partial m}\left(\frac{dc}{dT}\right) = \frac{\partial U}{\partial c}\frac{\partial c}{\partial m} + \frac{\partial U}{\partial \left(\frac{dc}{dT}\right)}\frac{\partial}{\partial T}\frac{\partial c}{\partial m}$$

where the interchange of the order of partial differentiation is permitted since the $c(\omega, m)$ is a continuous function. After some simple algebra, one obtains the followiong expression:

$$\frac{\partial U}{\partial m} = \frac{U}{c}\left(2 - \frac{U}{c}\right)\frac{\partial c}{\partial m} + \frac{U^2}{c^2}\,\omega\,\frac{\partial}{\partial \omega}\frac{\partial c}{\partial m} \tag{20}$$

$$= \frac{U}{c}\left(2 - \frac{U}{c}\right)\frac{\partial c}{\partial m} - \frac{U^2}{c^2}\,T\,\frac{\partial}{\partial T}\frac{\partial c}{\partial m}$$

The required partial with respect to period of the phase velocity partial is *numerically* computed using the relation

$$\frac{\partial}{\partial T}\,\frac{\partial c_o}{\partial v} = T\left(\frac{(\frac{\partial c_o}{\partial v})_{T+hT} - (\frac{\partial c_o}{\partial v})_{T-hT}}{2hT} \right)$$

In these expressions, the parameter m can take on the values $\alpha,\ \beta$, layer thickness or inverse Q. and the subscript o represents the value in the parameter in the purely elastic model.

If a causal Q is introduced, with a Futterman causality tied to a reference angular frequency ω_r, the elastic parameters will equal the anelastic values only when the angular frequency ω equals ω_r. This Q operator causes the medium velocity to be complex, e.g.,

$$v_{causal} = v\left[1 + \frac{1}{\pi Q_v}\ln\left(\frac{\omega}{\omega_r}\right) + i\,\frac{1}{2Q_v}\right]$$

The causal phase velocity c is given by applying the first terms of a Taylor series expansion:

$$c_{complex} = c_o + \frac{\partial c_o}{\partial \alpha}\,\frac{\partial \alpha_{causal}}{\partial Q_\alpha^{-1}}\,Q_\alpha^{-1} + \frac{\partial c_o}{\partial \beta}\,\frac{\partial \beta_{causal}}{\partial Q_\beta^{-1}}\,Q_\beta^{-1}$$

from which we obtain, assuming that the perturbation is small,

$$c = c_o + \frac{1}{\pi}\ln(\frac{\omega}{\omega_r})\sum\left(\frac{\partial c_o}{\partial \beta}\,\beta Q_\beta^{-1} + \frac{\partial c_o}{\partial \alpha}\,\alpha Q_\alpha^{-1}\right) \tag{21}$$

and the value of the spatial anelastic attenuation factor

$$\gamma = \frac{\omega}{2c_o^2}\sum\left(\frac{\partial c_o}{\partial \beta}\,\beta Q_\beta^{-1} + \frac{\partial c_o}{\partial \alpha}\,\alpha Q_\alpha^{-1}\right) \tag{22}$$

In both (21) and (22) the summation extends over all all layers and the halfspace of the model. To obtain this we expanded the propagation term $e^{-i\omega r/c_{complex}}$ to form $e^{-i\omega r/c_o}\ e^{-\gamma r}$

These values are typically output by the programs **sregn96** and **slegn96** if causality is required. However, those programs do not output the causal partial derivatives or group velocities since these are not required for synthetic seismogram construction. The correct causal relations for these parameters follow. A c or U without the subscript o represents the causal value. Note that a partial with respect to Q_β^{-1} may involve a partial of c_o with respect to α. To first order,

$$\frac{\partial c}{\partial v} = \frac{\partial c_o}{\partial v}\left(1 + \frac{1}{\pi Q_v}\ln(\frac{\omega}{\omega_r})\right) \tag{23}$$

$$\frac{\partial c}{\partial Q_v^{-1}} = \frac{1}{\pi}\ln(\frac{\omega}{\omega_r})\left(\frac{\partial c_o}{\partial v}\,v\right) \tag{24}$$

$$\frac{\partial \gamma}{\partial Q_v^{-1}} = \frac{\omega}{2c_o^2}\,\frac{\partial c_o}{\partial v}\,v \tag{25}$$

for $v = \alpha$ or $v = \beta$. An expression for $\frac{\partial \gamma}{\partial v}$ cannot be obtained by simple first orper perturbation theory, since the change in γ# depends on changes in $\frac{\partial c}{\partial v}$, which are second order effects.

To obtain the expression for the causal group velocity, we express

$$U = U_o + \Delta U = U_o + \frac{\partial U}{\partial Q_\alpha^{-1}}\,\Delta Q_\alpha^{-1} + \frac{\partial U}{\partial Q_\beta^{-1}}\,\Delta Q_\beta^{-1}$$

and use (20). From this we obtain

$$U = U_o\left(1 + (2 - \frac{U_o}{c_o})\left(\frac{c - c_o}{c_o}\right) + \frac{2\gamma U_o}{\pi\omega}\right) \tag{26}$$

where it is assumed that the higher order terms in $\left(\frac{c - c_o}{c_o}\right)$ are negligible.

The partial derivatives of the causal group velocity

$$\frac{\partial U}{\partial v} = \frac{\partial U_o}{\partial v}\left(\frac{U}{U_o} - \frac{U_o}{c_o}\left(\frac{c - c_o}{c_o}\right) + \frac{2\gamma U_o}{\pi\omega}\right) \tag{27}$$

$$+ \frac{\partial c_o}{\partial v}\,\frac{U_o}{c_o}\left(- 2\,\frac{2\gamma U_o}{\pi\omega} + \frac{U_o}{c_o}\,\frac{1}{\pi Q_v}\right.$$

$$\left. + (2 - \frac{U_o}{c_o})[\frac{1}{\pi Q_v}\ln(\frac{\omega}{\omega_r}) - \left(\frac{c - c_o}{c_o}\right)] + \frac{U_o}{c_o}\left(\frac{c - c_o}{c_o}\right)\right)$$

$$\frac{\partial U}{\partial Q_v^{-1}} = \frac{U_o}{c_o}(2 - \frac{U_o}{c_o})\,\frac{\partial c}{\partial Q_v^{-1}} \tag{28}$$

$$+ \frac{1}{\pi}\,\frac{U_o^2}{c_o^2}\,\frac{\partial c_o}{\partial v}\,v$$

An iterative linear inversion is performed because of the non-linear nature of the problem. At any stage there is a current model which is used to predict the observations and also a lack of fit. For simplicity, the compresional-wave velocity α is not directly inverted, rather it is tied to the shear-wave velocity after determining the chane in the shear-wave velocity by one of the *assumptions* made when running **surf96** or **joint96**. In addition, the ratio $\frac{Q_\beta}{Q_\alpha}$ is fixed and the expression use in the inversion programs would look like

$$\frac{\partial c}{\partial Q_v^{-1}} = \frac{1}{\pi} \ln(\frac{\omega}{\omega_r}) \left(\frac{\partial c_o}{\partial \beta} \beta + \frac{\partial c_o}{\partial \alpha} \alpha \frac{Q_\beta}{Q_\alpha} \right)$$

rather than the single term in (24).

We may thus express the difference between observed and predicted values in a linear model of changes in shear velocity model and changes in the inverse Q model. Since the effect of compressional-wave Q may not be negligible, the compresional-wave Q is related to the shear-wave Q by a ratio Q_α / Q_β for the layers. Given these assumptions, the inversion can take on two characters, non-causal and causal. To keep the notation general, the partial derivatives with respect to layer velocity can either be causal or non-causal.

The differences in observed and predicted phase velocities are modeled as

$$c_{obs} - c_{pred} = \frac{\partial c}{\partial \beta_1} \Delta\beta_1 + \cdots + \frac{\partial c}{\partial \beta_n} \Delta\beta_n \tag{29}$$

$$+ \frac{\partial c}{\partial Q_{\beta_1}^{-1}} \Delta Q_{\beta_1}^{-1} + \cdots + \frac{\partial c}{\partial Q_{\beta_n}^{-1}} \Delta Q_{\beta_n}^{-1}$$

The difference between observed and predicted group velocities is given by

$$U_{obs} - U_{pred} = \frac{\partial U}{\partial \beta_1} \Delta\beta_1 + \cdots + \frac{\partial U}{\partial \beta_n} \Delta\beta_n \tag{30}$$

$$+ \frac{\partial U}{\partial Q_{\beta_1}^{-1}} \Delta Q_{\beta_1}^{-1} + \cdots + \frac{\partial U}{\partial Q_{\beta_n}^{-1}} \Delta Q_{\beta_n}^{-1}$$

The difference between observed and gamma values predicted by the current model is

$$\gamma_{obs} - \gamma_{pred} = \tag{31}$$

$$\frac{\partial \gamma}{\partial Q_{\beta_1}^{-1}} \Delta Q_{\beta_1}^{-1} + \cdots + \frac{\partial \gamma}{\partial Q_{\beta_n}^{-1}} \Delta Q_{\beta_n}^{-1}$$

The inversion programs give the user the choice of an (1) non-causal inversion, (2) a causal uncoupled solution or (3) a causal-coupled solution. The meaning of this is easily given in the following table:

TERM	Non-Causal	Causal Uncoupled	Causal Coupled
Phase Vel	c_o	(2)	(2)
Group Vel	U_o	(27)	(27)
$\frac{\partial c}{\partial \beta}$	$\frac{\partial c_o}{\partial \beta}$	(23)	(23)
$\frac{\partial U}{\partial \beta}$	$\frac{\partial U_o}{\partial \beta}$	(28)	(28)
$\frac{\partial c}{\partial Q_v^{-1}}$	0	0	(24)
$\frac{\partial U}{\partial Q_v^{-1}}$	0	0	(28)
$\frac{\partial \gamma}{\partial Q_v^{-1}}$	(7)	(7)	(25)

Zero entries in the table, indicate that the respective partials are set to zero. Otherwise, the partials are given by the function or indicated equation.

3. Sphericity Corrections

Sphericity corrections are based on the work of Schwab and Knopoff (1972). The concept is to convert the spherical earth model into a flat earth model, compute the dispersion, and then adjust the dispersion from the flat earth model to make the spherical earth dispersion. The sphericity correction for Love waves is exact, but that for the Rayleigh waves are approximate, both because of the nature of the problem and also because the effects of gravitation are ignored.

In the discussion that follows, the subscript s represents the spherical earth parameter, and the subscript f represents the value used in the flat earth computations. If **obs.d** file indicates the use of a spherical earth model, then the program **srfdis** performs an earth flattening approximation, and computes the dispersion for the equivalent flat earth model.

Love Waves

Let r be the radial distance from the center of the sphere, and let the surface be given by $r = a$. Also let z be the depth from the free surface in the equivalent flat earth model. The transformation used is

$$z = a \ln(a / r) \tag{32}$$

Given a spherical layer bounded by r_i and r_{i-1} , with $r_{i-1} > r_i$, The thickness of the spherical i'th layer is is

$$(h_i)_s = r_{i-1} - r_i \tag{33}$$

and the thickness of the transformed flat layer model is

$$(h_i)_f = a \ln(a / r_i) - a \ln(a / r_{i-1}) \tag{34}$$

The transformed shear-wave velocity and density in the equivalent flat layer model are given by

$$\overline{(\beta_i)_f} = (\beta_i)_s \frac{2a}{r_i + r_{i-1}} \tag{35}$$

$$\overline{(\rho_i)_f} = (\rho_i)_s \left(\frac{2a}{r_i + r_{i-1}} \right)^{-5} \tag{36}$$

The Love wave equation is from Schwab and Knopoff (1972). Given this flat earth model, flat earth phase, c_f, and group, U_f, are computed, as well as the partial derivatives with respect to velocity, density, and layer thickness. The program **srfdrl96** computes the corresponding spherical model values through the relations:

$$c_s(\omega) = c_f \left[1 + (3c_f / 2a\omega)^2 \right]^{-1/2} \tag{37}$$

$$U_s(\omega) = U_f \left[1 + (3c_f / 2a\omega)^2 \right]^{1/2} \tag{38}$$

To obtain the partials, the chain rule of differentiation is used:

$$\frac{\partial c_s}{\partial p_s} = \frac{\partial c_s}{\partial c_f} \frac{\partial c_f}{\partial p_f} \frac{\partial p_f}{\partial p_s}$$

where p is β or ρ. The resulting expressions are

$$\left(\frac{\partial c_s}{\partial \beta_s} \right)_i = \left(1 + (3c_f / 2a\omega)^2 \right)^{-3/2} \left(\frac{\partial c_f}{\partial \beta_f} \right)_i \frac{2a}{r_i + r_{i-1}} \tag{39}$$

$$\left(\frac{\partial U_s}{\partial \beta_s} \right)_i = \left\{ \left(1 + (3c_f / 2a\omega)^2 \right)^{1/2} \left(\frac{\partial U_f}{\partial \beta_f} \right)_i \right. \tag{40}$$

$$\left. + U_f c_f \left(\frac{\partial c_f}{\partial \beta_f} \right)_i (3 / 2a\omega)^2 \left(1 + (3c_f / 2a\omega)^2 \right)^{-1/2} \right\} \frac{2a}{r_i + r_{i-1}}$$

$$\left(\frac{\partial c_s}{\partial h_s} \right)_i = \left(1 + (3c_f / 2a\omega)^2 \right)^{-3/2} \left(\frac{\partial c_f}{\partial h_f} \right)_i \frac{a}{r_i} \tag{41}$$

$$\left(\frac{\partial U_s}{\partial h_s}\right)_i = \left\{\left(1+(3c_f/2a\omega)^2\right)^{1/2}\left(\frac{\partial U_f}{\partial h_f}\right)_i \right. \tag{42}$$

$$\left. + U_f c_f \left(\frac{\partial c_f}{\partial h_f}\right)_i (3/2a\omega)^2 \left(1+(3c_f/2a\omega)^2\right)^{-1/2}\right\} \frac{a}{r_i}$$

Rayleigh Waves

Let r be the radial distance from the center of the sphere, and let the surface be given by $r = a$. Also let z be the depth from the free surface in the equivalent flat earth model. The transformation used is

$$z = a \ln(a/r) \tag{43}$$

The thickness of the spherical layer bounded by $r_{i-1} > r_i$, is is

$$(h_i)_s = r_{i-1} - r_i \tag{44}$$

and the thickness of the transformed flat layer model is

$$(d_i)_f = a \ln(a/r_i) - a \ln(a/r_{i-1}) \tag{45}$$

The mean compressional- and shear-wave velocities and density in the transformed flat layer model are given by

$$\overline{(\alpha_i)_f} = (\alpha_i)_s \frac{2a}{r_i + r_{i-1}} \tag{46}$$

$$\overline{(\beta_i)_f} = (\beta_i)_s \frac{2a}{r_i + r_{i-1}} \tag{47}$$

$$\overline{(\rho_i)_f} = (\rho_i)_s \left(\frac{2a}{r_i + r_{i-1}}\right)^{-2.275} \tag{48}$$

The exponent for the Rayleigh wave was determined empirically, e.g., *http://www.eas.slu.edu/eqc/eqc_cps/TUTORIAL/SPHERICITY/index.html*. Given this flat earth model, flat earth phase, c_f, and group, U_f, are computed, as well as the partial derivatives with respect to velocity, density, and layer thickness. The program **srfdrl** computes the corresponding spherical model values through the relations:

$$c_s(\omega) = c_f\left[1+(c_f/2a\omega)^2\right]^{-1/2} \tag{49}$$

$$U_s(\omega) = U_f\left[1+(c_f/2a\omega)^2\right]^{1/2} \tag{50}$$

To obtain the partials, the chain rule of differentiation is used:

$$\frac{\partial c_s}{\partial p_s} = \frac{\partial c_s}{\partial c_f} \frac{\partial c_f}{\partial p_f} \frac{\partial p_f}{\partial p_s}$$

where p is β or ρ.

$$\left(\frac{\partial c_s}{\partial \beta_s}\right)_i = \left(1 + (c_f / 2a\omega)^2\right)^{-3/2} \left(\frac{\partial c_f}{\partial \beta_f}\right)_i \frac{2a}{r_i + r_{i-1}} \tag{51}$$

$$\left(\frac{\partial U_s}{\partial \beta_s}\right)_i = \left\{\left(1 + (c_f / 2a\omega)^2\right)^{1/2} \left(\frac{\partial U_f}{\partial \beta_f}\right)_i \right. \tag{52}$$

$$\left. + U_f c_f \left(\frac{\partial c_f}{\partial \beta_f}\right)_i (1/2a\omega)^2 \left(1 + (c_f / 2a\omega)^2\right)^{-1/2} \right\} \frac{2a}{r_i + r_{i-1}}$$

$$\left(\frac{\partial c_s}{\partial h_s}\right)_i = \left(1 + (c_f / 2a\omega)^2\right)^{-3/2} \left(\frac{\partial c_f}{\partial h_f}\right)_i \frac{a}{r_i} \tag{53}$$

$$\left(\frac{\partial U_s}{\partial h_s}\right)_i = \left\{\left(1 + (c_f / 2a\omega)^2\right)^{1/2} \left(\frac{\partial U_f}{\partial h_f}\right)_i \right. \tag{54}$$

$$\left. + U_f c_f \left(\frac{\partial c_f}{\partial h_f}\right)_i (1/2a\omega)^2 \left(1 + (c_f / 2a\omega)^2\right)^{-1/2} \right\} \frac{a}{r_i}$$

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DEFINITIONS

Gamma Values - a measure of the non-geometrical attenuation of a signal due to anelastic processes which is of the form $\exp(-\gamma r)$. If a two-station technique is used, and if surface-wave propagation is assumed, then the interstation γ is defined by the relation:

$$\gamma = \ln\left(A_1 r_1^{1/2} / A_2 r_2^{1/2}\right) / (r_2 - r_1)$$

where A_j is the instrument corrected surface-wave spectral amplitude observed at distance r_j. Both spectral amplitude observations are measured at the same frequency. To obtain clean estimates of the spectral amplitude estimates, multiple filter analysis or phase match filter techniques can be used.

The units are km^{-1}, m^{-1} or ft^{-1}. These must be in the same units as the velocities input.

Group Velocity - velocity of energy propagation. This can be estimated graphically or by using multiple filter techniques, which bandpass filter the surface wave, and compute the envelope of the resulting function. The group velocity is the obtained by dividing the epicentral distance by the time of arrival of the envelope maximum.

The units are km/sec or m/sec or sec.

Phase Velocity - velocity of a given phase. Usually measured from phase spectrum at a given frequency. If source and instrument phase is known, a single station technique can be used. For data acquired along the same azimuth, two station interstation phase is easily obtained if the instrument phase is known. If more than two stations are available, a stacking technique can be used to reduce the problems of spatial aliasing.

The units are km/sec or m/sec or sec.