

Implementation of the **gsac** command *rotate3*

The Sac header values *cmpinc* and *cmpaz* describe the direction of ground motion corresponding to increasing (more positive) values of a trace. *cmpinc* is an angle measured with respect to the upward vertical: *cmpinc* = 0 corresponds to ground motion upward, *cmpinc*=90 to horizontal motion, and *cmpinc*=180 to downward motion. *cmpaz* provides the horizontal component of the motion vector: *cmpaz*=0 corresponds to North and *cmpaz*=90 corresponds to East. The angles corresponding to positive ground motion in the up, north and east directions are

Positive motion direction	<i>cmpinc</i>	<i>cmpaz</i>
Up	0	0
North	90	0
East	90	90

This is a simple way to describe three-component orthogonal components. However, the orientation need not be in the geographical directions, just that they must form the basis for a three-component coordinate system. An example of deviation from a ZNE coordinate system would be a borehole instrument for which horizontal orientations will rarely be in the north and east directions.

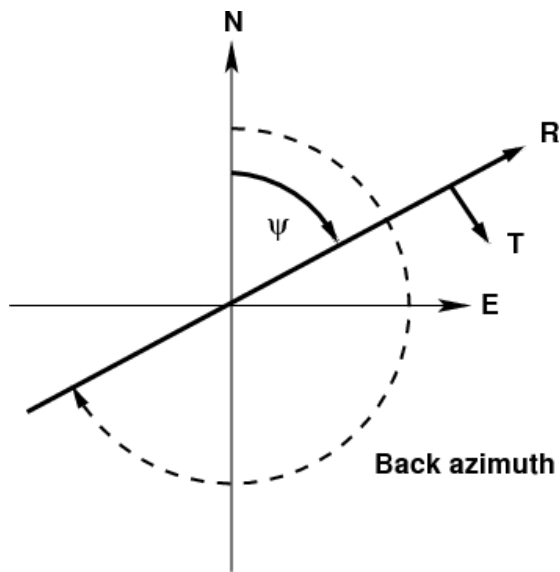
In general, the relation between the U_n component positive motion defined by the *cmpinc* (θ_n) and *cmpaz* (ϕ_n) angles, for $n=1, 2$ and 3 , to the Up, North and East components is given by the matrix relation

$$\begin{bmatrix} U_E \\ U_N \\ U_Z \end{bmatrix} = \begin{bmatrix} \sin \phi_1 \sin \theta_1 & \sin \phi_2 \sin \theta_2 & \sin \phi_3 \sin \theta_3 \\ \cos \phi_1 \sin \theta_1 & \cos \phi_2 \sin \theta_2 & \cos \phi_3 \sin \theta_3 \\ \cos \theta_1 & \cos \theta_2 & \cos \theta_3 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} \quad (1)$$

To convert these to radial (R), transverse (T) and up (Z) components (Fig. 1), the **gsac** command *rotate3 to gc* uses the transformation matrix (2) where $C = \cos(\text{Back_azimuth} + 180)$ and $S = \sin(\text{Back_azimuth} + 180)$.

If one wishes to apply the **gsac** command *rotate3 to ψ* to compute the motions in the ψ -direction (U_ψ), the $\psi+90$ ($U_{\psi+90}$) and up (Z) directions, the vector on the left would be $[U_\psi, U_{\psi+90}, U_Z]^T$ and $C = \cos \psi$ and $S = \sin \psi$ in the above expression.

One use of the option *rotate3 to ψ* is to rotate traces through a sequence of angles to find the angle that will cause the P-wave particle motion to be zero on the $U_{\psi+90}$. This is an empirical way to determine or to verify the back-azimuth. Another use is in studies of teleseismic S-wave anisotropy by rotating the horizontal components to separate the fast- and slow-directions of the S-wave.



$$\begin{bmatrix} U_R \\ U_T \\ U_Z \end{bmatrix} = \begin{bmatrix} S & C & 0 \\ C & -S & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} U_E \\ U_N \\ U_Z \end{bmatrix} \quad (2)$$

Fig. 1. Orientation of R-T coordinates with respect to N-E coordinates as function of back azimuth or rotation angle.

Some modern broadband sensors, such as the STS-2 and Trillium consist of three sensors which are not mounted in the conventional xyz directions. Instead all are tilted toward the vertical and aligned every 120 degrees about the central vertical axis. An advantage of this arrangement is that all three sensors are identical, instead of having two identical horizontal and a separate vertical sensor. A disadvantage of the STS-2 or Trillium arrangement, though, is that if one sensor fails, then the Z, N and E outputs are all affected.

gsac implements two new rotation commands of the form *rotate to uvwsts2* and *rotate to uvwtril* which will take the Z, N and E output to create the U, V and W outputs. The application of this command is seen in a simple example.

Assume two three-component broadband sensors, one the standard reference and the other the test system, are on the same pier and both oriented so that the expected output is Z, N and E for both. Further assume that the instrument responses are the same. Thus we would expect the Z, N and E channels of each instrument to be identical. If the Z, N and E channels do not agree, then it is possible to identify which of the U, V or W internal single-component sensors in the test system is not working properly. The way to accomplish this is to convert the Z, N and E channels of each to U, V and W, and then compare the U, V and W of the two systems to isolate the problem. It is not necessary for the reference system to even have the internal UVW sensors, since the objective is to focus on the test system. If the two three-component sensors have different responses, then this comparison can be made after removing the instrument responses to give the Z, N and E ground motions.

Since one starts with the ZNE motions, the ZNE output of both sensors are used to emulate the internal UVW channels of the test system. Because the STS-2 and Trillium have different orientations of the internal UVW internal sensors, **gsac** implements two options depending upon which is the tested

sensor.

Trillium being tested (rotate to uvwtri):

If installed as recommended, the Trillium X, Y and Z axes correspond to East, North and Up respectively. The transformation and inverse transformations are

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \frac{1}{\sqrt{6}} \begin{bmatrix} 2 & 0 & \sqrt{2} \\ -1 & \sqrt{3} & \sqrt{2} \\ -1 & -\sqrt{3} & \sqrt{2} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \qquad \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{\sqrt{6}} \begin{bmatrix} 2 & -1 & -1 \\ 0 & \sqrt{3} & -\sqrt{3} \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

The Sac headers for the UVW components would have the following values for component orientation:

Positive motion direction	<i>cmpinc</i>	<i>cmpaz</i>
U	54.7	90
V	54.7	330
W	54.7	210

STS-2 being tested (rotate to uvwsts2):

If installed as recommended, the STS-2 X, Y and Z axes correspond to East, North and Up respectively. The transformation and inverse transformations are

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \frac{1}{\sqrt{6}} \begin{bmatrix} -2 & 0 & \sqrt{2} \\ 1 & \sqrt{3} & \sqrt{2} \\ 1 & -\sqrt{3} & \sqrt{2} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \qquad \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{\sqrt{6}} \begin{bmatrix} -2 & 1 & 1 \\ 0 & \sqrt{3} & -\sqrt{3} \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

The Sac headers for the UVW components would have the following values for component orientation:

Positive motion direction	<i>cmpinc</i>	<i>cmpaz</i>
U	54.7	270
V	54.7	30
W	54.7	150