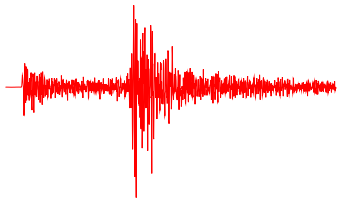
COMPUTER PROGRAMS IN SEISMOLOGY



SURFACE WAVES, RECEIVER FUNCTIONS AND CRUSTAL STRUCTURE

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PREFACE

CHAPTER 1
CRUSTAL STRUCTURE?

1. Introduction

This volume discusses techniques for estimating shallow earth structure that use different parts of the seismogram - the surface wave and the time series associated with some body-wave arrivals. Techniques for deriving surface wave dispersion from the recorded surface wave and for defining the receiver function for initial P- or S-waves will be presented along with programs to invert these data sets for shalloEarth structure velocity models.

Seismic data sets are never complete enough to uniquely define earth structure because of the effects of noise, limited frequency band or other reasons for lack of observations. To impress this reality, perfect data sets will be inverted for earth structure and their differences in predicting independent observations will be compared.

2. A Simple Example

The chosen earth model is a simple, single layer over a halfspace representation of the crust and upper mantle:

Simple Model

H (km)	V_P (km)	V_S (km/s)	ρ (km/s)	Q_P	Q_S	η_P	η_S	f_{ref_p} (Hz)	f_{ref_S} (Hz)
40	6.0	3.5	2.5	200.0	100.0	0.0	0.0	1.0	1.0
	8.0	4.7	3.3	900.0	500.0	0.0	0.0	1.0	1.0

Theoretical surface wave dispersion curves were generated using the surface-wave codes in *Computer Programs in Seismology - Overview*. The Love and Rayleigh, fundamental and higher mode phase velocities, group velocities and anelastic attenuation coefficients are displayed in Figures 1 and 2.

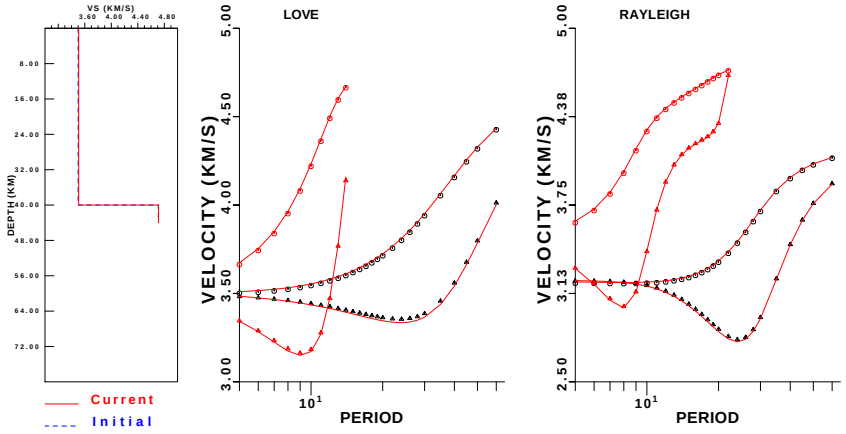


Fig. 1. Earth model and dispersion points used for inversion test.

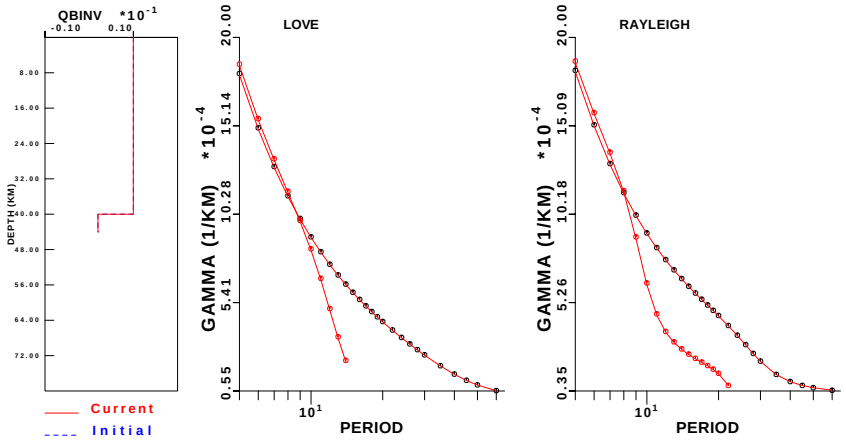


Fig. 2. $\mathbf{Q_S^{-1}}$ model and anelastic attenuation coefficietns, γ, for Love and Rayleigh, fundamental and 1st higher modes.

The period range chosen for inversion, 5 - 60 seconds, represents the author's (RBH) experience in determining dispersion for small earthquakes in eastern North America. The lower period limit is controlled by source excitation and anelastic attenuation and the upper period limit by the low signal noise. In reality one would almost never have all these data points - perhaps the group velocity values for the fundamental mode, and a few other data points.

For the same model, P-wave receiver functions were computed using the program **`hrftn96`** for a teleseismic signal with ray parameter **`p`** = 0.07 sec/km and Gaussian filter parameters of 0.50 and 1.0. Figure 3 presents the earth model again together with the receiver functions.

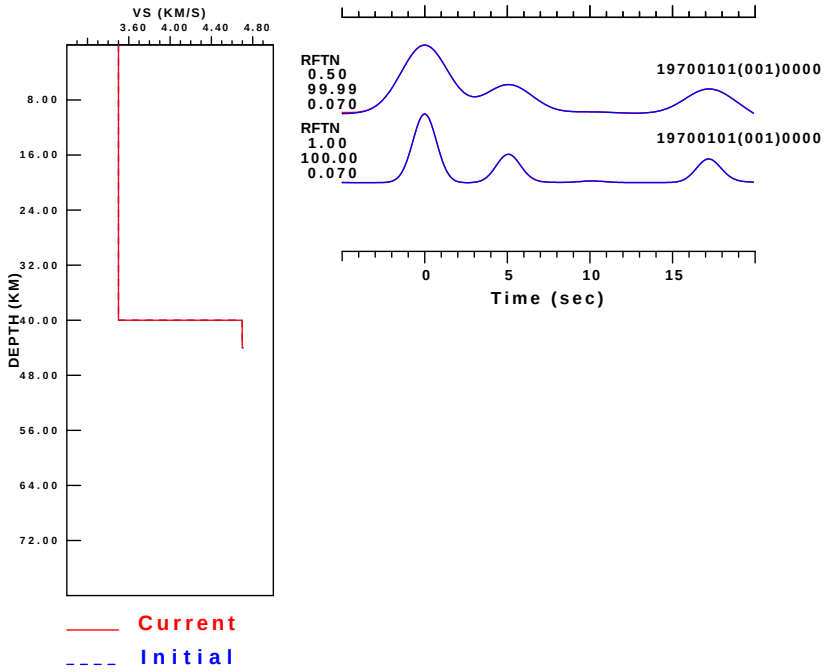


Fig. 3. Earth model receiver functions

2.1 Surface-wave inversion

The program **`surf96`** is used to invert for earth structure from the dispersion curves. An iterative, weighted inversion is used so that one can, for example, force velocity discontinuities in the resultant model. In addition, one can fix the layer thicknesses and invert for layer velocity, or fix the velocities and invert for layer thickness.

To present the variability of the results, six different inversion runs of 20 - 40 iterations each were performed using different starting models, smoothing, and damping. The final models fit the data well as evidenced by the overlay show in Figure 4.

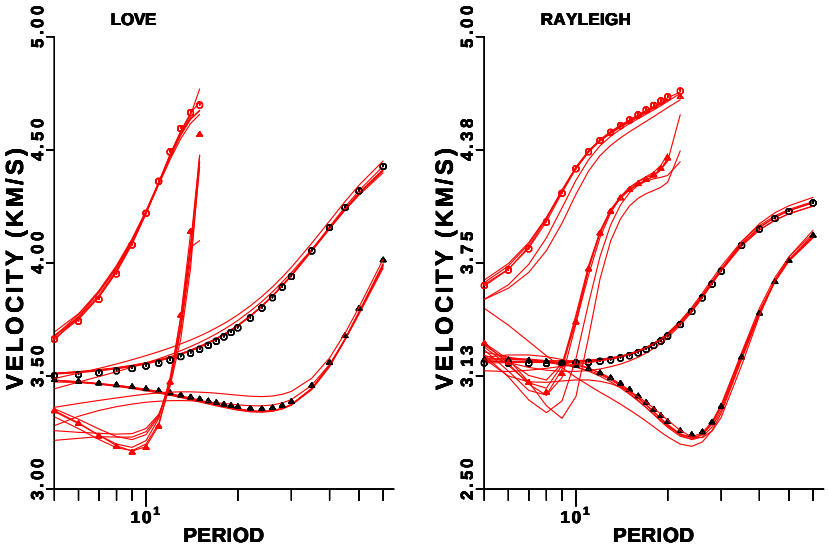


Fig. 4. Fit to dispersion data for 6 different inversions. The solid curves are the model predictions. The symbols represent the dispersion data.

However, Figure 5 shows the different models obtained for each inversion as well as true model.

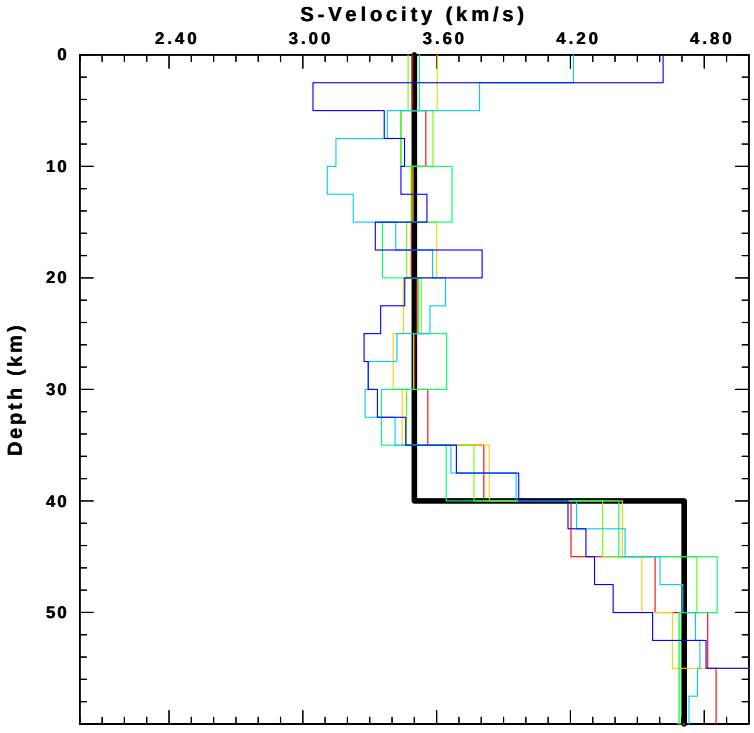


Fig. 5. Models obtained by the **surf96** inversions. The wide black line is the true model.

The lack of fit is not surprising since 4 of the 6 models had uniform smoothing constraints which preclude matching a very sharp discontinuity, in much the same manner that a finite Fourier series cannot accurately represent a periodic step function. The unifying feature of all inversions is the fact that the upper crust well defined and that the Moho is approximately at the center of the crust-mantle transition. Part of the reason for the lack

of fit was that the upper mantle velocity was free to change. In the actual earth, we would expect upper mantle velocities to be known or at least bounded so that they can be fixed.

The impact of these different earth models is seen if the models are used for something other than predicting surface-wave dispersion in the 5 - 60 second period range, such as predicting first arrival times for use in event location. The true model and the 6 inverted models were used as input to the program **`timmod96`** to predict the P-wave first arrival times from 0 - 800 km for a surface event. The results of these computations are shown in Figure 6. All but one model predicts the direct arrival well. However, the variability in the lower crust leads to predicted travel time differences of several seconds.

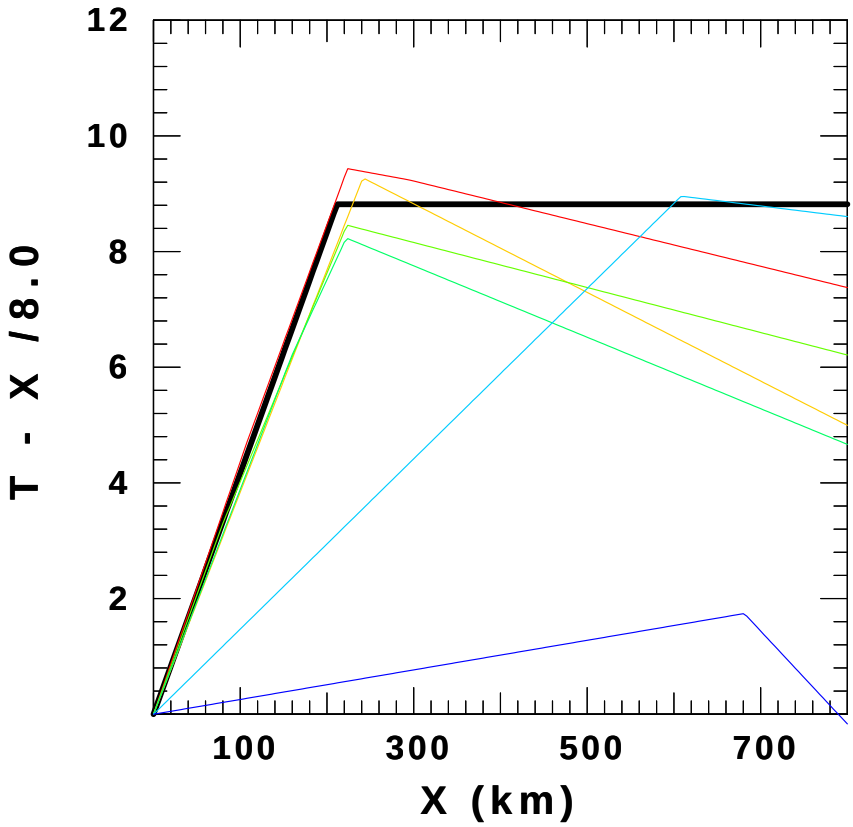


Fig. 6. P-wave first arrival times for the true model, solid black line, and inverted models.

2.2 P-wave receiver function inversion

The next exercise is to invert the P-wave receiver function data using the program **`rftn96`**. The results of 5 inversions are presented. Figure 7 presents the data fits, Figure 8 the models and Figure 9 the predicted travel times.

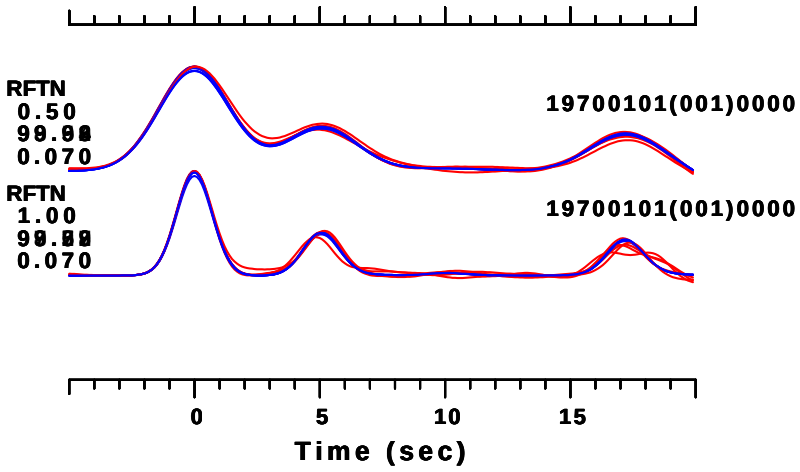


Fig. 7. Fit to receiver function for 5 different inversions.

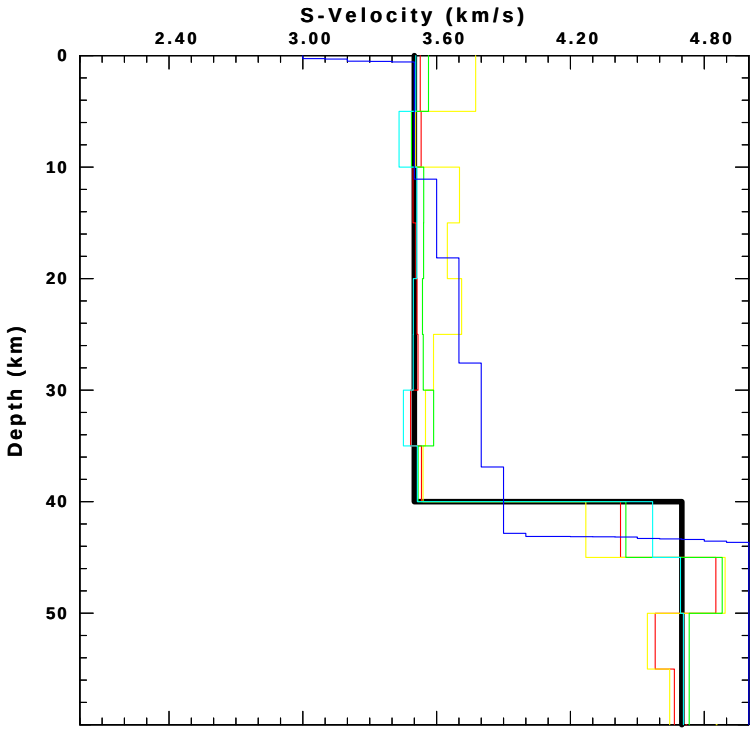


Fig. 8. Models obtained by the **`rftn96`** inversions. The wide black line is the true model.

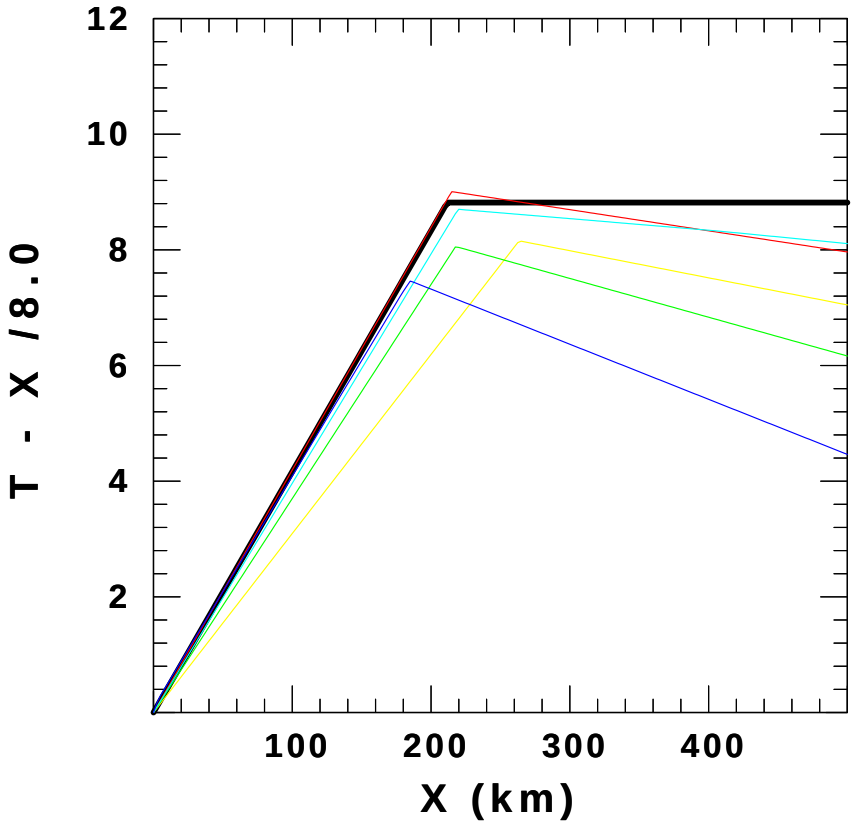


Fig. 9. P-wave first arrival times for the true model, solid black line, and inverted models.

2.3 Joint inversion

The final exercise is to invert the surface-wave dispersion and P-wave receiver function data simultaneoously using the program **`joint96`**. The results of 5 inversions are presented. Figure 10 the models and Figure 11 the predicted travel times.

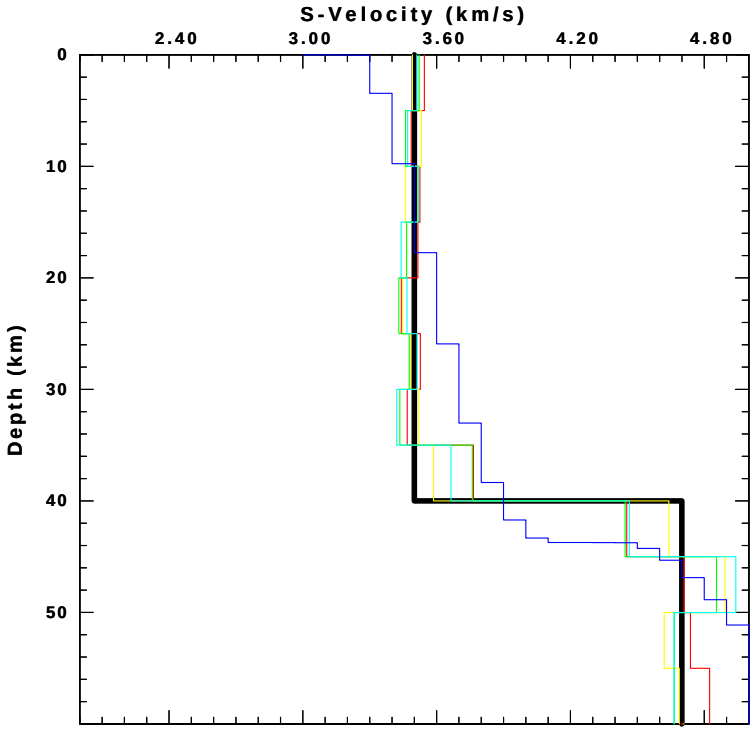


Fig. 10. Models obtained by the **`joint96`** inversions. The wide black line is the true model.

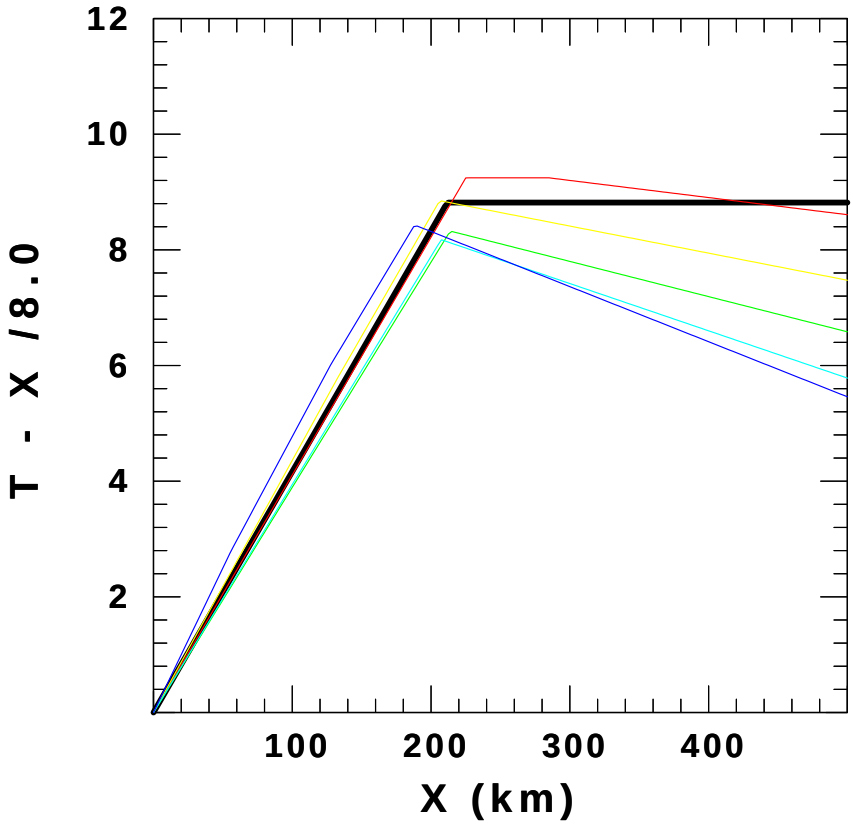


Fig. 11. P-wave first arrival times for the true model, solid black line, and inverted models.

2.4 Discussion

Seemingly the joint inversion is able to reproduce the initial model better than either of the separate inversions. One reason for this success is that many iterations were performed and the data were noise free. All the programs, **`surf96`**, **`rftn96`** and **`joint96`** save the models after each iteration in the `model96` file format, under the name `tmp-mod96.XXX`, where `XXX` is the iteration number. One can overlay the models as in Figure 12.

Starting from the same halfspace model for each inversion, this figure gives an insight into how each inversion technique moves toward the final solution. The surface-wave inversion, figure 12(a), immediately moves toward a model with an increase of velocity with depth. As a matter of fact the first iteration overshoots the final model, and slowly returns. It seems as if the upper crust is well defined after the first few iterations, but that more iterations are required to change the upper mantle velocities. This may be related to the observation that the majority of dispersion points are at the shorter periods.

The receiver function inversion, Figure 12(b), first defines the surface velocity fo the model. This is not unexpected since the largest arrival on the receiver-function occurs near zero lag - and only shallow structure can affect this time window. As the receiver function iteration continues, later arrivals, which provide information on the moho, are

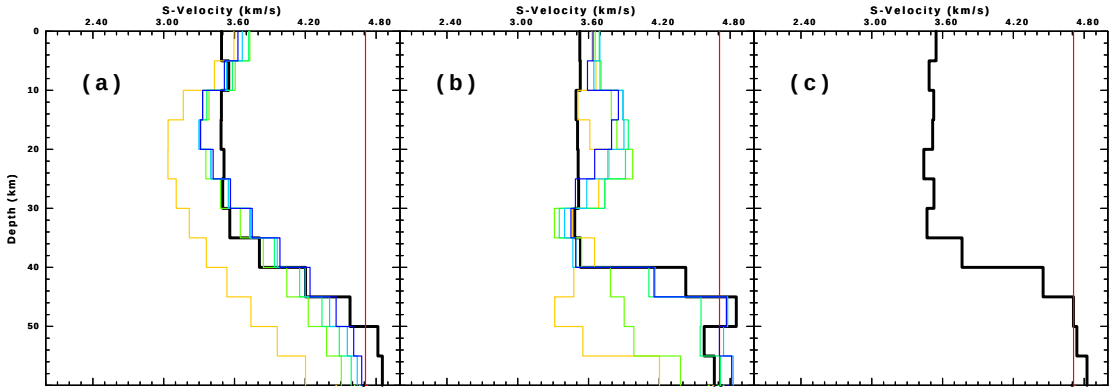


Fig. 12. Model change for first five iterations under each inversion technique. (a) **`surf96`**; (b) **`rftn96`**; (c) **`joint96`**. The final model after 20 - 40 iterations is given by the thick black line.
fit.

The joint inversion progression, Figure 12(c), seems to miror that of the receiver function, in this case. It is difficult to see how the surface wave information improves the solution. A different model is obtained if the time window used for fitting the receiver function is $[-5, 10]$ instead of $[-5, 20]$ seconds. In this case just the first two pulses in the receiver functions are fit, which are due to the direct P and the converted Ps incident on the free surface. The later arrivals in the $10, 20]$ window control the sharpness of the crust-mantle discontinuity.

3. Caveats

These programs have just been implemented. Do not believe the inversion results just because the fit looks good. Seek other independent constraints to permit evaluation of the model.

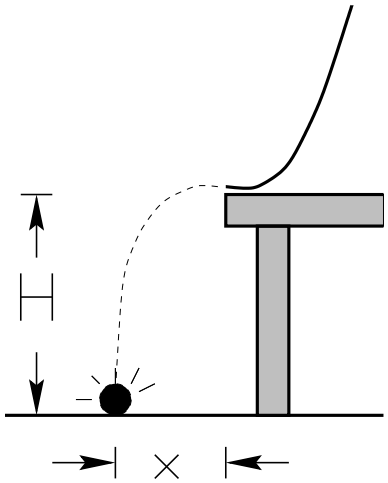
CHAPTER 2
REVIEW OF INVERSION THEORY

1. Introduction

Having discussed the separate inversion of receiver functions and surface-wave dispersion for earth structure, we now present the joint inversion of these two data sets. This section will review basic concepts of regression analysis. Such a review is appropriate since regression analysis defines the coefficients of a chosen model that best fit observational data. These concepts will be illustrated using simple models. Hopefully, a good understanding of simple regression problems will serve as a basis for understanding inversion theory.

2. Means, variances and standard deviations

Assume that we can observe some phenomena and make repeated measurements. Also assume that we expect to obtain the same observation, but that there is some unknown, random observational error that prevents this.



An example of this is to imagine an inclined ramp on a table. A metal spherical ball rolls down the incline. At the bottom of the incline the ball travels horizontally with a velocity V. From elementary physics, we know the ball will impact the floor after a time

$$t = \left(2gH\right)^{1/2},$$

where H is the vertical distance from the floor to the base of the ramp. At the same time the ball will be a horizontal distance

$$X = V\,t$$

from the base of the ramp. As a matter of fact, we would expect that the path of the ball from the table to the floor would be a parabola if we do not have to worry about air resistance.

If the ball is repeatedly released at the top of the ramp, the pattern of impacts will generally be at a distance of X from the base of the ramp, with some impacts at shorter and greater distances. The variation may be due to variations over which the experimenter has no control: e.g., the ball is not perfectly spherical, the ramp may distort with changes in room temperature, there may be different patterns of swirling air currents, or the ball is not released in exactly the same way.

Let x_i be the i′th measured distance. Let N be the number of times the experiment is repeated. Let the expected value of x, E(x), be μ, which we call the mean. Finally let ε_i be the random error of the i′th observation. Thus the i′th observation is

$$x_i = \mu + \varepsilon_i$$

At this point an important assumption is made about the random error process -- this process has a zero mean, i.e., $E(\varepsilon) = 0$. This can be written as

$$\lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \varepsilon_i \to 0$$

That there is no bias in the measurements, such as might arise from a bad measuring scale, is an article of faith. Further we assume that the errors are truly random and not correlated. Although not necessary here, the error is often assumed to arise from a normal, or Gaussian, distribution with zero mean and variance σ^2

$$\varepsilon_i \sim N(0, \sigma^2).$$

For such a distribution, we expect about 68% if the observations to lie within the range $(\mu - \sigma, \mu + \sigma)$, and 95% within the range $(\mu - 2\sigma, \mu + 2\sigma)$.

Mathematically the normal distribution $N(z, \sigma^2)$ is defined as

$$N(z, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-z^2 6/2\sigma^2}$$

This is a probability distribution and the $\int_{-\infty}^{Z} N(z, \sigma^2) dz$ is called the cumulative probability, which varies from 0 at $Z = -\infty$ to 1 at $Z = +\infty$.

Our task is to use all observations to estimate the μ and the variance σ^2. We acknowledge that we cannot determine the μ, but only estimate an $\bar{x}$ since we have only a finite number of observations. One way to accomplish this is by trying to find an a that minimizes the sum of square residuals

$$S = \sum_{i=1}^{N} (x_i - a)^2$$

This value is determined by requiring $\frac{\partial S}{\partial \bar{x}} = 0$ Solving gives

$$a = \frac{1}{N} \sum_{i=1}^{N} x_i$$

The standard deviation s, an estimate of σ, is defined by

$$s^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2 \qquad \text{(A.2.1)}$$

$$= \frac{1}{N-1} \left(\sum_{i=1}^{N} x_i^2 - N\bar{x}^2 \right)$$

(the $N-1$ is used instead of N since $\bar{x}$ has already been specified and only $N-1$ pieces of independent information are available to estimate σ^2. This also guarantees that $E(s^{2)} = \sigma^2)$

The symbol ~ means "is distributed as".

Because we have only a finite set of observations, the $\bar{x}$ estimate of μ is not perfect. We estimate the standard error of the distribution of $\bar{x}$ by the relation

$$s_{\bar{x}}^2 = \frac{s^2}{N} = \frac{1}{N(N-1)} \sum_{i=1}^{N} (x_i - \bar{x})^2 \qquad \text{(A.2.2)}$$

At this stage, we can examine the residuals, $x_i - \bar{x}$, and test whether we can reject the hypothesis that the random error process is normal. We could also test the inappropriateness of other distributions.

The meaning of the estimated values is simple. If we perform the experiment once by collecting N samples, we are able to estimate the true μ, the error process variance σ^2 and the variance on the mean, $s_{\bar{x}}^2$.

If we perform the experiment again by collecting additional samples from the same noise contaminated population, we would expect the new $\bar{x}$ to lie about the true μ with a distribution controlled by $s_{\bar{x}}^2$. The s^2 indicates the spread in future observations, and the $s_{\bar{x}}^2$ indicates the spread in the $\bar{x}$ estimates of μ.

Finally as the number of observations N increases, we expect that $\bar{x} \to \mu$, $s^2 \to \sigma^2$, and $s_{\bar{x}} \to 0$, in probability.

If the noise process is assumed to be Gaussian (normal) then confidence limits can be placed on the measured quantities. *Discuss confidence limits on the x etc to give meaning to the x bar sigmas perhaps give a tabular example and show a histogram*

3. Linear Regression

Assume now that the observed data are generated by a true linear process, e.g.,

$$Y = A + Bx \qquad \text{(A.3.1)}$$

The observations are again affected by a zero mean random error:

$$y_i = A + Bx_i + \varepsilon_i$$

Our objective is to use the data to estimate the true values A and B as well as some properties of the ε process.

The least squares problem is to find the a and b that minimizes

$$S(a, b) = \sum_{i=1}^{N} \varepsilon_i^2 = \sum_{i=1}^{N} \left(y_i - a - bx_i \right)^2, \qquad \text{(A.3.2)}$$

where the a and b are estimates of A and B.

The condition that the a and b make S a minimum requires $\frac{\partial S}{\partial a} = 0$ and $\frac{\partial S}{\partial b} = 0$. These conditions yield two linear equations, the normal equations, in the unknowns a and b:

$$\begin{bmatrix} N & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \end{bmatrix} \qquad \text{(A.3.3)}$$

(for simplicity the summation indices are dropped). The solution of this linear equation

is obtained by taking the inverse of the square matrix which leads to

$$\begin{bmatrix}\mathbf{a}\\ \mathbf{b}\end{bmatrix}=\frac{\mathbf{1}}{\mathbf{N}\sum\mathbf{x_i^2}-(\sum\mathbf{x_i})^2}\begin{bmatrix}\sum\mathbf{x_i^2} & -\sum\mathbf{x_i}\\ -\sum\mathbf{x_i} & \mathbf{N}\end{bmatrix}\begin{bmatrix}\sum\mathbf{y_i}\\ \sum\mathbf{x_i y_i}\end{bmatrix} \tag{A.3.4}$$

We can easily show that the a and b values arising from the normal equations gives

$$S(a,b)=\sum y_i^2-a\sum y_i-b\sum x_i y_i \tag{A.3.5}$$

An alternative way of writing (A.3.4) explicitly is

$$b=\frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sum(x_i-\bar{x})^2}$$

and

$$a=\bar{y}-b\bar{x}$$

The estimated variance of the error process is

$$s^2=\frac{1}{N-2}S(a,b) \tag{A.3.6}$$

The confidence limits on a and b are given through the use of the t-distribution:

$$\Delta a=t(N-2,1-\alpha/2)\left[s^2\frac{\sum x_i^2}{N\sum x_i^2-(\sum x_i)^2}\right]^{1/2} \tag{A.3.7}$$

and

$$\Delta b=t(N-2,1-\alpha/2)\left[s^2\frac{N}{N\sum x_i^2-(\sum x_i)^2}\right]^{1/2} \tag{A.3.8}$$

where $t(N-2,1-\alpha/2)$ is the Student-t distribution for $N-2$ degrees of freedom and the $1-\alpha/2$ confidence level [For 95% confidence, $\alpha=0.05$ and $t(\infty,0.975)=1.96$]. If these confidence bounds are interpreted in the same sense as for the simple example of section 2, these are the confidence that the true value of A lies within $a\pm\Delta a$, and similarly the value of B lies within $b\pm\Delta b$. There is one slight complication, and that is that the error estimates Δa and Δb may be interrelated.

The confidence limits that the predicted regression line $y=a+bx$ lies near the true line $y=A+Bx$ are

$$\pm t(N-2,1-\alpha/2)\left[s^2\left(\frac{\sum x_i^2-2x\sum x_i+Nx^2}{N\sum x_i^2-(\sum x_i)^2}\right)\right]^{1/2} \tag{A.3.9}$$

$$\pm t(N-2,1-\alpha/2)\left[s^2\left(\frac{\sum(x-\bar{x})^2}{\sum(x_i-\bar{x})^2}\right)\right]^{1/2}$$

The confidence limits on the distribution of the data (or future data) about the regression line $y=a+bx$ are

$$\pm t(N-2,1-\alpha/2)\left[s^2\left(1+\frac{\sum x_i^2-2x\sum x_i+Nx^2}{N\sum x_i^2-(\sum x_i)^2}\right)\right]^{1/2} \qquad (A.3.10)$$

The first equation gives two hyperbolas about the regression line whose asymptotes are $y=(a+\Delta a)+(b-\Delta b)x$ and $y=(a-\Delta a)+(b+\Delta b)x$. If the experiment were repeated, there is a $(1-\alpha/2)\times 100\%$ chance that the resultant regression line will lie within these limits. The second equation indicates where future data may lie. The hyperbolic nature of the error bound is interesting since it indicates that the prediction error increases as one gets away from the centroid $(\bar{x},\bar{y})$ of the data set; this is to be expected when extrapolating beyond the data set.

The interrelationship of error in a and b can be examined by searching through possible values of A and B, comparing the sum of squared residuals to that of the least squares solution(Draper and Smith, 1966, §2.6 and §10.3):

$$S(A,B)=S(a,b)\left(1+\frac{2}{N-2}F(2,N-2,1-\alpha)\right) \qquad (A.3.11)$$

Rearranging, one would contour the following function of A and B, which is related to the F-statistic:

$$\left(\frac{S(A,B)-S(a,b)}{S(a,b)}\ \frac{N-2}{2}\right)=F(2,N-2,1-\alpha) \qquad (A.3.12)$$

Because our model was linear, the contours in the (A, B) space satisfying this relation will be ellipses. In general the major axis of the elliptical contour may be inclined, indicating some interdependence between the a and b values. In this case, a change in the value of b causes a change in a.

Since Draper and Smith (1966) may be the only ones to define confidence ellipses in this manner, an alternative expression is

$$[A-a,B-b]\begin{bmatrix}N & \sum x_i\\ \sum x_i & \sum x_i^2\end{bmatrix}\begin{bmatrix}A-a\\ B-b\end{bmatrix}=2s^2F(2,N-2,1-\alpha)$$

Example 1

Consider the following data set of 10 points:

Table A.3.1.

x_i	y_i	x_i	y_i
1	1	7	8
2	1	9	7
1	2	10	8
3	3	12	14
6	5	14	16

From this data set we form the sums

$$\begin{array}{ll} N = 10 & \sum y_i = 65 \\ \sum x_i = 65 & \sum y_i^2 = 669 \\ \sum x_i^2 = 621 & \sum x_i y_i = 635 \end{array}$$

For a 95% confidence level, $t(8, 0.975) = 2.306$. (The Fischer, Student t-distribution has the property that $t(\inf, \alpha)$ = Normaldistribution. Using these sums and the t-value, we obtain

$$\begin{array}{l} a = -0.4584 \pm 1.9884 \\ b = 1.0705 \pm 0.2523 \\ s^2 = 2.3765 \end{array}$$

Figure 1 shows the regression line and the 95% confidence hyperbolas on the regression line. The confidence hyperbolas mean that another data set from the same data population would lead to a regression line which has a 95% chance of being within these hyperbolas. (These statements are repetitious but the distinction between scatter in data and model parameters is important).

Figure 2 shows the regression line and the 95% confidence limits on the data. If we select additional samples from the population, then 95% of them should lie within the hyperbolas.

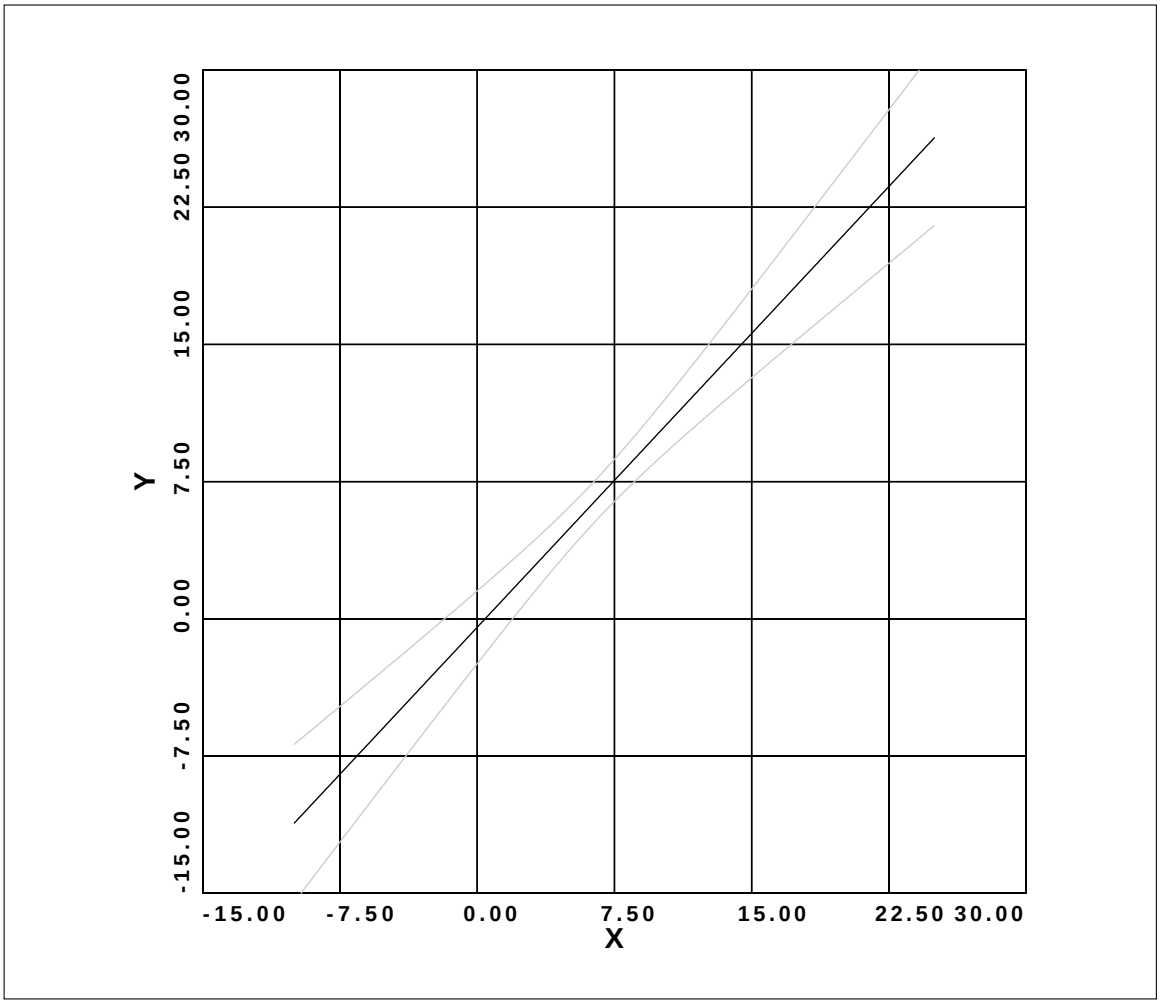


Fig. 1. Regression line (black) and 95% confidence bounds (light gray) on the regression line.

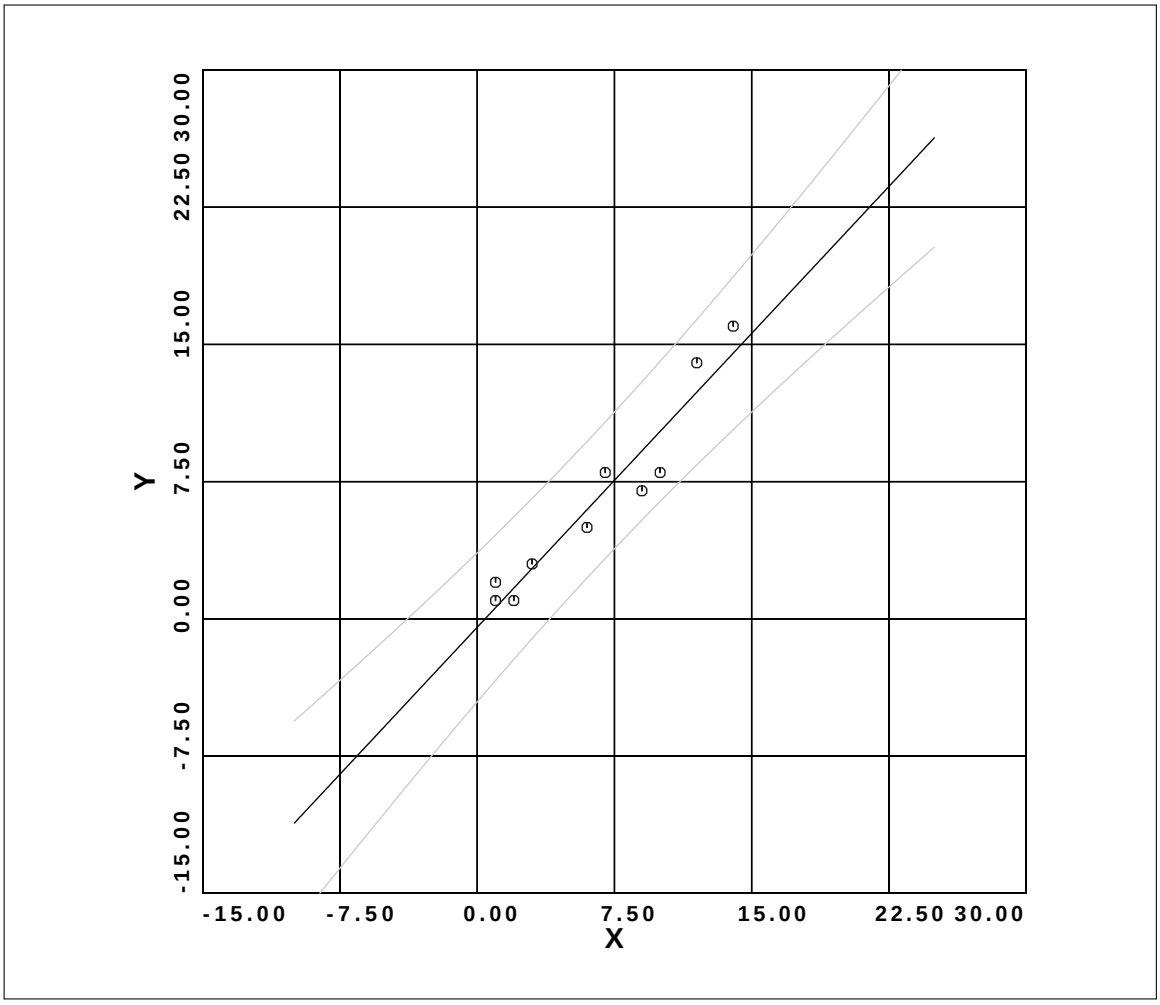


Fig. 2. Regression line (black) and 95% confidence bounds (light gray) on the data set. The data are shown by the points.

Figure 3 plots the F-statistic at 50%, 75%, 90%, 95%, 97.5% and 99% levels. The smallest region, centered on the regression values $(a, b) = (-0.458, 1.071)$, indicates that there is only a 50% chance that the true values of (a, b) are located within this region. As the confidence increases, the confidence region increases. We can always state with 100% confidence that the true value is somewhere within the (a, b) plane, if our original assumption of a linear model is correct.

Example 2

Consider the same data set of 10 points, except that the origin is shifted to the $(\bar{x}, \bar{y})$ of the previous data set. Subtracting the $\bar{x} = 6.5$ from all x and $\bar{y} = 6.5$ from all y values gives:

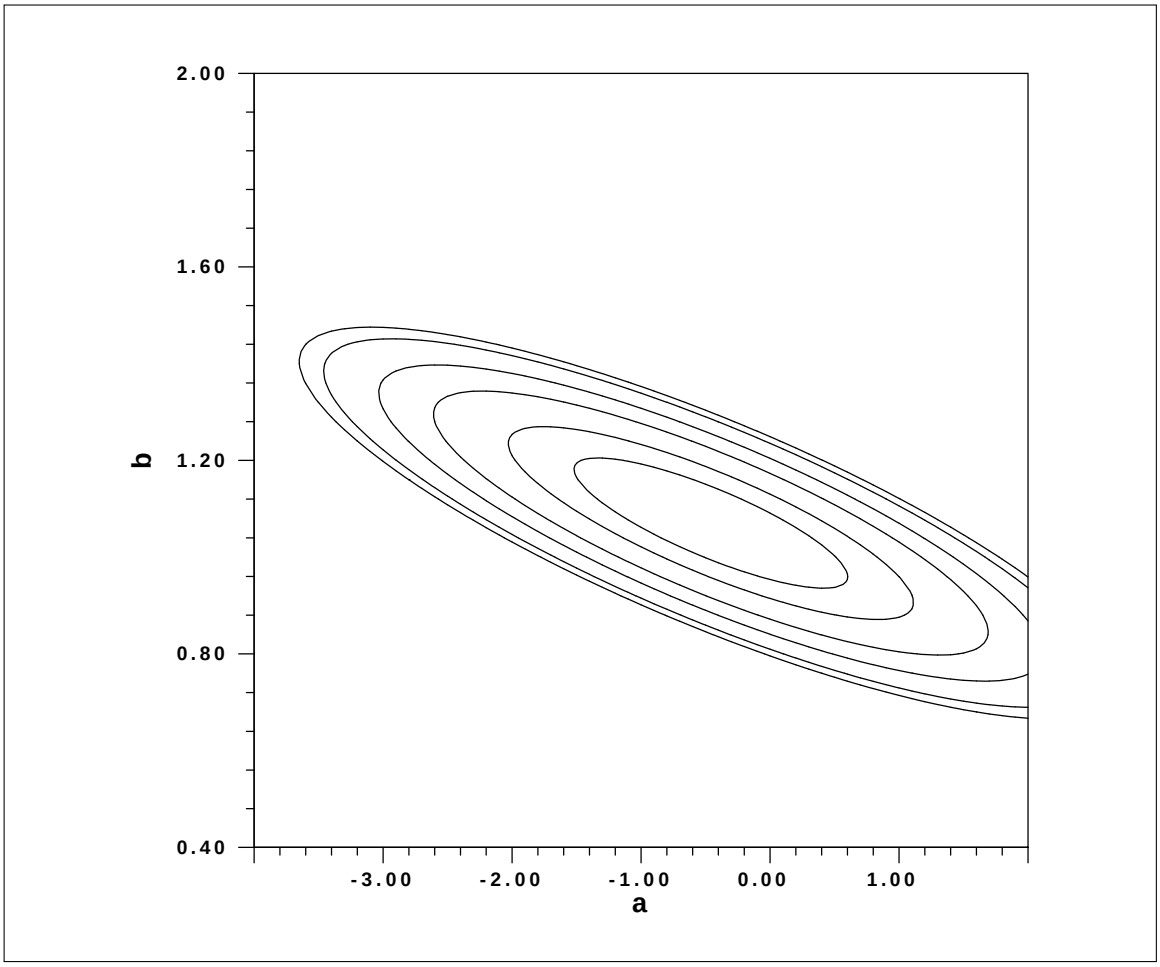


Fig. 3. Plot of the F-statistic using S(a, b). Contours are drawn at F(2, 8, p) = 0.757 for 50%, 1.66 for 75%, 3.11 for 90%, 4.46 for 95%, 6.06 for 97.5% and 6.85 for 99%.

Table A.3.2			
x_i	y_i	x_i	y_i
-5.5	-5.5	0.5	1.5
-4.5	-5.5	2.5	0.5
-5.5	-4.5	3.5	1.5
-3.5	-3.5	5.5	7.5
-0.5	-1.5	7.5	9.5

From this data set we obtain the sums:

$N = 10$	$\sum y_i = 0$
$\sum x_i = 0$	$\sum y_i^2 = 246.5$
$\sum x_i^2 = 198.5$	$\sum x_i y_i = 212.5$

Since the $\sum x_i = 0$, the matrices in (A.3.3) and (A.3.4) are diagonal. For a 95%

confidence level, $\mathbf{t(8, 0.975) = 2.306,}$ and these we obtain

$$\mathbf{a = 0.000 \pm 1.1242}$$

$$\mathbf{b = 1.0705 \pm 0.2523}$$

$$\mathbf{s^2 = 2.3766}$$

Note that only the intercept a has changed. In addition its confidence bound is smaller. The Δa obtained for this data set is the same as (A.3.7) evaluated at $x = 6.5$ for the Example 1 data set.

Figure 4 shows the regression line and the 95% confidence hyperbolas on the regression line. The confidence hyperbolas mean that another data set from the data population would lead to a regression line which has a 95% chance of being within the hyperbolas.

Figure 5 shows the regression line and the 95% confidence limits on the data. If we select additional samples from the sample population, then 95% of them should lie within the hyperbolas.

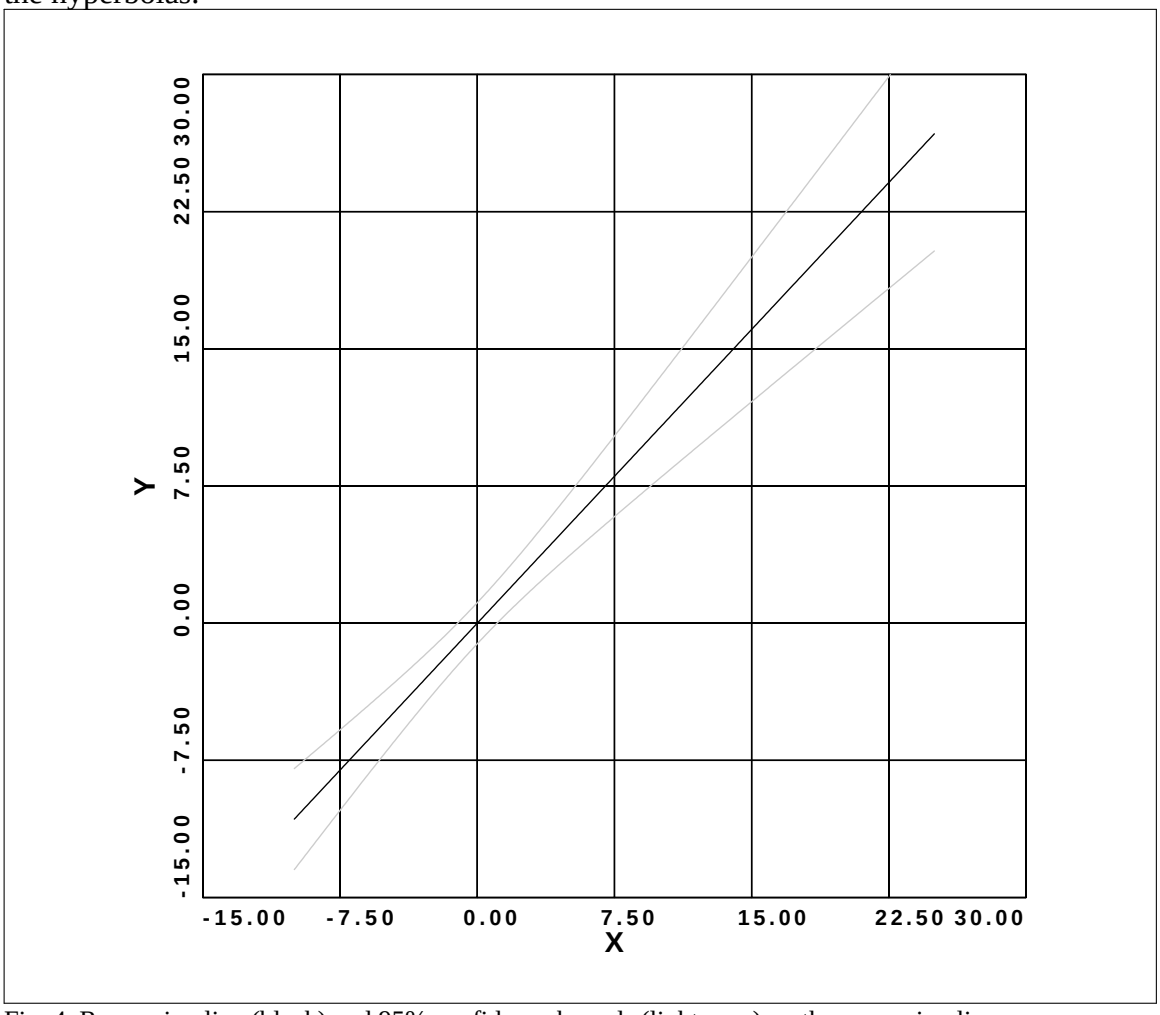


Fig. 4. Regression line (black) and 95% confidence bounds (light gray) on the regression line.

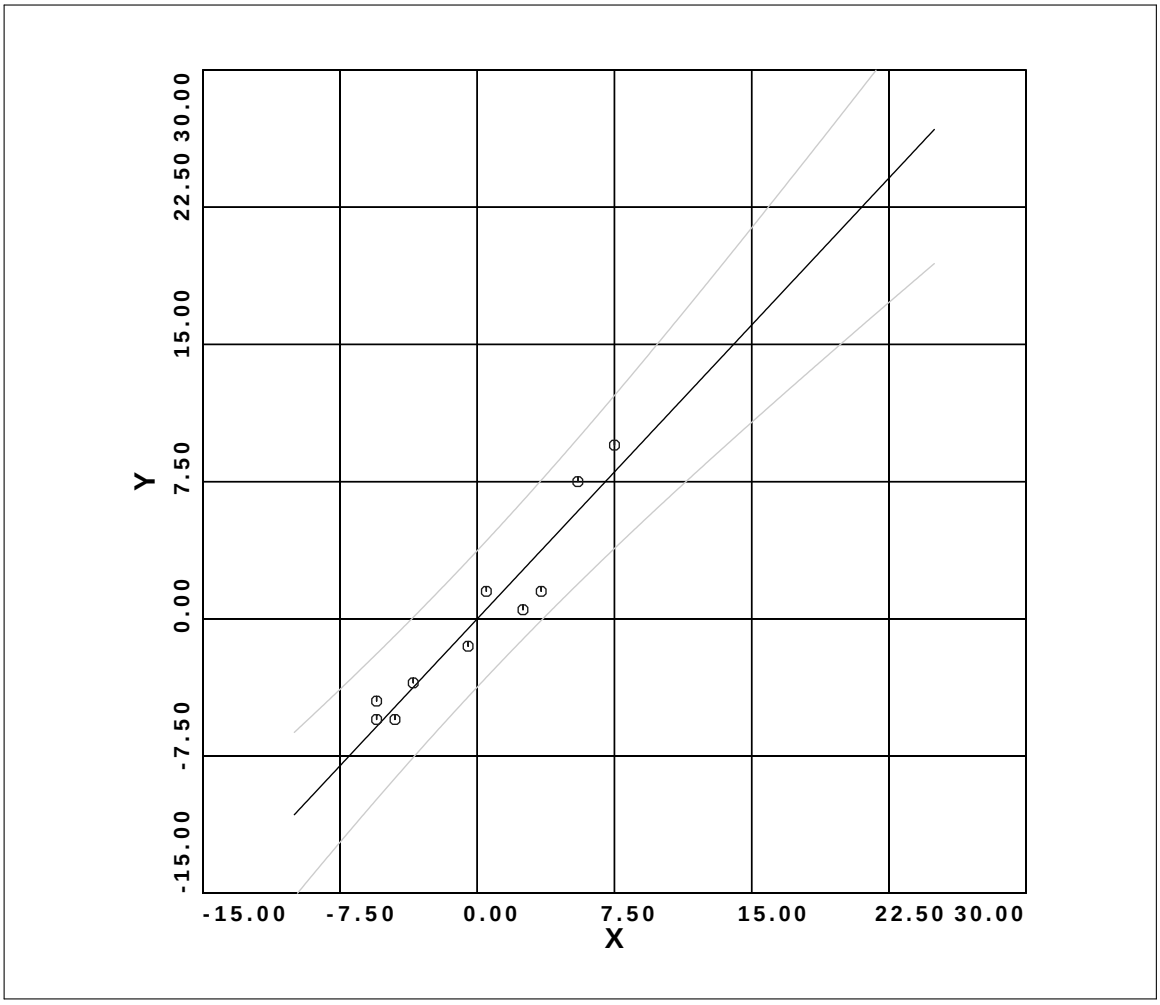


Fig. 5. Regression line (black) and 95% confidence bounds (light gray) on the data set. The data are shown by the points.

Figure 6 plots the F-statistic at 50%, 75%, 90%, 95%, 97.5% and 99% levels. The smallest region, centered on the regression values $(a, b) = (0.0, 1.071)$, indicates that there is only a 50% chance that the true values of (a, b) are located within this region. We note now that the confidence bounds are no longer inclined ellipses - the major and minor axes are aligned with the a and b axes. This is a direct consequence of the fact that $\sum x_i = 0$. This inclination of the ellipse indicates that there is no trade-off, or co-variance between the a and b values for this data set. In the previous data set, increasing the slope b required a smaller a to continue to have the line pass through the centroid of the data.

4. Linear Regression - Known, but Uniform Variance

The next way to perform the same regression analysis uses a priori knowledge of the variance, σ^2, of the random error as a weight. For the straight line problem, we now minimize

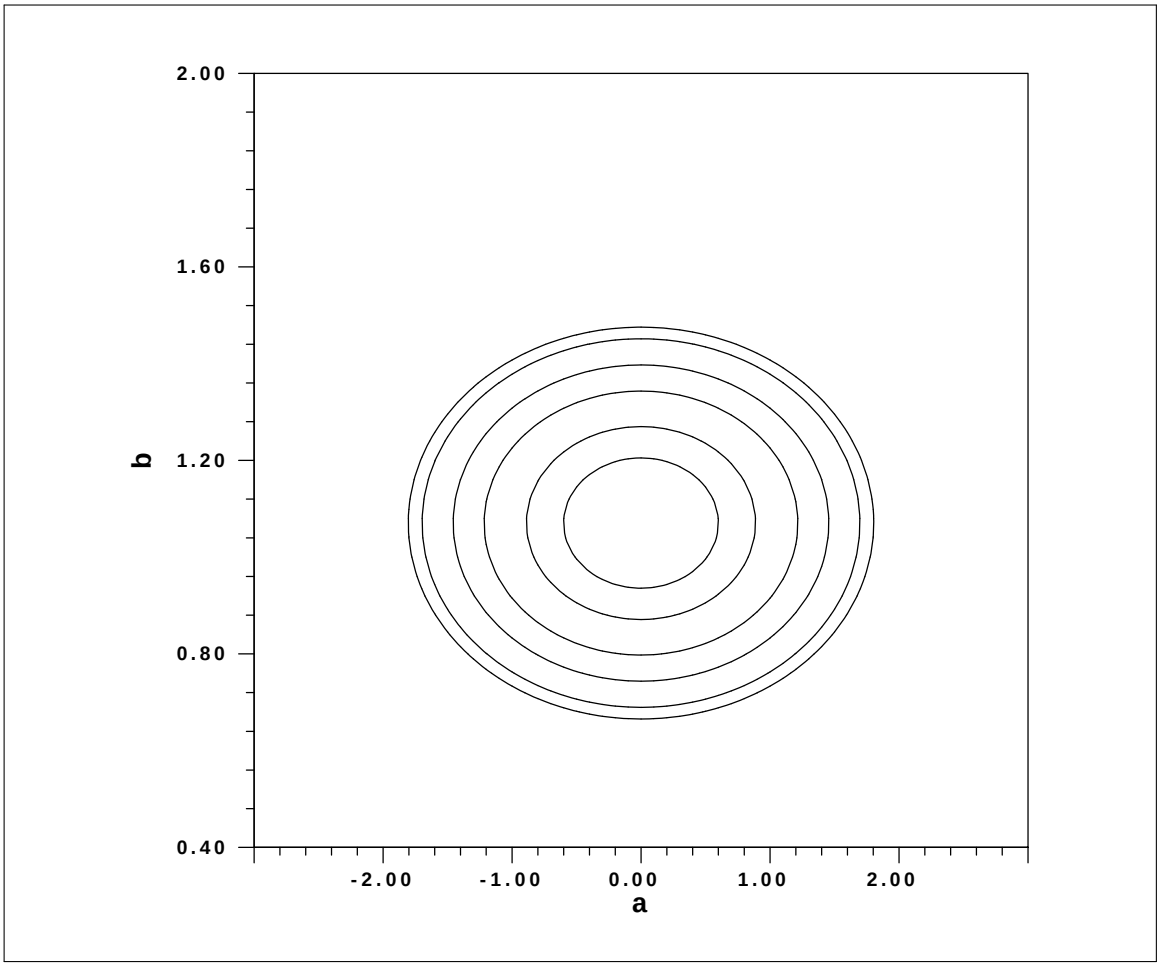


Fig. 6. Plot of the F-statistic using S(a, b). Contours are drawn at F(2, 8, p) = 0.757 for 50%, 1.66 for 75%, 3.11 for 90%, 4.46 for 95%, 6.06 for 97.5% and 6.85 for 99%.

$$S(a,b)=\sum_{i=1}^{N}\left(\frac{y_i-a-bx_i}{\sigma}\right)^2 \tag{A.4.1}$$

As $\lim_{N\to\infty}$, we expect that the minimum value is $S(a,b)=N$ because of the definition of σ as the limit of s as the number of observations increase. Applying the necessary conditions for a minimum, $\frac{\partial S}{\partial a}=0$ and $\frac{\partial S}{\partial b}=0$, yields the two linear equations

$$\begin{bmatrix} \sum\frac{1}{\sigma^2} & \frac{\sum x_i}{\sigma^2} \\ \frac{\sum x_i}{\sigma^2} & \frac{\sum x_i^2}{\sigma^2} \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}=\begin{bmatrix} \frac{\sum y_i}{\sigma^2} \\ \frac{\sum x_i y_i}{\sigma^2} \end{bmatrix} \tag{A.4.2}$$

The solution of the linear equation is simply

$$\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{\sum \frac{1}{\sigma^2} \sum \frac{x_i}{\sigma^2} - (\sum \frac{x_i}{\sigma^2})^2} \begin{bmatrix} \sum \frac{x_i^2}{\sigma^2} & -\sum \frac{x_i}{\sigma^2} \\ -\sum \frac{x_i}{\sigma^2} & \sum \frac{1}{\sigma^2} \end{bmatrix} \begin{bmatrix} \sum \frac{y_i}{\sigma^2} \\ \sum \frac{x_i y_i}{\sigma^2} \end{bmatrix} \qquad \text{(A.4.3)}$$

The confidence limits on a and b are given through the use of the z-distribution:

$$\Delta a = z(1-\alpha/2) \left[\frac{\sum \frac{x_i^2}{\sigma^2}}{\sum \frac{1}{\sigma^2} \sum \frac{x_i}{\sigma^2} - (\sum \frac{x_i}{\sigma^2})^2} \right]^{1/2} \qquad \text{(A.4.4)}$$

and

$$\Delta b = z(1-\alpha/2) \left[\frac{\sum \frac{1}{\sigma^2}}{\sum \frac{1}{\sigma^2} \sum \frac{x_i}{\sigma^2} - (\sum \frac{x_i}{\sigma^2})^2} \right]^{1/2} . \qquad \text{(A.4.5)}$$

The expression for the confidence bounds on a and b is identical to that of Section 3 if we replace s^2 by σ^2. The confidence on the regression line is

$$\pm z(1-\alpha/2) \left[\left(\frac{\sum \frac{x_i^2}{\sigma^2} - 2x \frac{\sum x_i}{\sigma^2} + \sum \frac{1}{\sigma^2} x^2}{\sum \frac{1}{\sigma^2} \sum \frac{x_i}{\sigma^2} - (\sum \frac{x_i}{\sigma^2})^2} \right) \right]^{1/2} \qquad \text{(A.4.6)}$$

and the confidence limits on the distribution of the data (or future data) about the regression line $y = a + bx$ are

$$\pm z(1-\alpha/2) \left[\left(\sigma^2 + \frac{\sum \frac{x_i^2}{\sigma^2} - 2x \frac{\sum x_i}{\sigma^2} + \sum \frac{1}{\sigma^2} x^2}{\sum \frac{1}{\sigma^2} \sum \frac{x_i}{\sigma^2} - (\sum \frac{x_i}{\sigma^2})^2} \right) \right]^{1/2} \qquad \text{(A.4.7)}$$

The reason that the normal, z-, distribution is used instead of the t-distribution is that the variances are known.

5. Weighted Linear Regression

During the acquisition of data, more may be known qualitatively or quantitatively about an observation. This may occur if a data pair is repeated in the observations, or if the data variance is better known. We denote this by assigning each (x_i, y_i) data pair with a weight w_i or a data variance σ_i . In its simplest form the w_i may indicate that there were repeated (x_i, y_i) values in the data set, and that we wish to use only one pair but indicate frequency of repeated values. In another form, the w_i may reflect a subjective assessment of data quality, perhaps varying between [0,1]. Finally if σ_i is known, $w_i = \frac{1}{\sigma_i^2}$. The

object of this section is to show how presentations in Sections 4 and 5 are to be modified and also how they are related.

5.1 Weights

In the case of a given w_i , the data variance is unknown and must be estimated from the residuals. To simplify the confidence limit estimates, let N be the total number of (x_i, y_i) pairs, each with a corresponding w_i. Also *require* that the w_i be *normalized* such that

$$\sum_{i=1}^{N} w_i = N \tag{A.5.1}$$

This normalization is introduced to simplify the estimation of data variance. The sum of squares to be minimized is

$$S(a, b) = \sum w_i (y_i - a - bx_i)^2 \tag{A.5.2}$$

The normal equation (A.3.3) becomes

$$\begin{bmatrix} \sum w_i & \sum w_i x_i \\ \sum w_i x_i & \sum w_i x_i^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum w_i y_i \\ \sum w_i x_i y_i \end{bmatrix} \tag{A.5.3}$$

(for simplicity the summation indices are dropped). The solution of this linear equation is

$$\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{\sum w_i \sum w_i x_i^2 - (\sum w_i x_i)^2} \begin{bmatrix} \sum w_i x_i^2 & -\sum w_i x_i \\ -\sum w_i x_i & \sum w_i \end{bmatrix} \begin{bmatrix} \sum w_i y_i \\ \sum w_i x_i y_i \end{bmatrix} \tag{A.5.4}$$

We can easily show that the a and b values arising from the normal equations makes

$$S(a, b) = \sum w_i y_i^2 - a \sum w_i y_i - b \sum w_i x_i y_i \tag{A.5.5}$$

Because of the normalization requirement that $\sum w_i = N$, the estimated variance of the error process is still

$$s^2 = \frac{1}{N - 2} S(a, b) \tag{A.5.6}$$

The confidence limits on a and b are given through the use of the t-distribution:

$$\Delta a = t(N - 2, 1 - \alpha/2) \left[s^2 \frac{\sum w_i x_i^2}{\sum w_i \sum w_i x_i^2 - (\sum w_i x_i)^2} \right]^{1/2} \tag{A.5.7}$$

and

$$\Delta b = t(N - 2, 1 - \alpha/2) \left[s^2 \frac{\sum w_i}{\sum w_i \sum w_i x_i^2 - (\sum w_i x_i)^2} \right]^{1/2} \tag{A.5.8}$$

The confidence limits on the predicted regression line $\mathbf{y = a + bx}$ are

$$\mathbf{\pm t(N-2,1-\alpha/2)}\left[\mathbf{s^2}\left(\frac{\mathbf{\sum w_i x_i^2 - 2x \sum w_i x_i + \sum w_i x_i^2}}{\mathbf{\sum w_i \sum w_i x_i^2 - (\sum w_i x_i)^2}}\right)\right]^{\mathbf{1/2}} \qquad \text{(A.5.9)}$$

The confidence limits on the distribution of the data (or future data) about the regression line $y = a + bx$ are

$$\pm t(N-2,1-\alpha/2)\left[s^2\left(1+\frac{\sum w_i x_i^2 - 2x\sum w_i x_i + \sum w_i x^2}{\sum w_i \sum w_i x_i^2 - (\sum w_i x_i)^2}\right)\right]^{1/2} \qquad \text{(A.5.10)}$$

The confidence contours in the (a, b) plane are still given by (A.3.12). Note that the t-distribution was used here since the variances are not known, only estimated.

5.2 Stochastic Weights

This section is very similar to that of §4, but with the distinction that the known variances of each observation may be different. The effective weighting of the observations is similar to that of §5.1. The task here is to minimize

$$S(a,b) = \sum_{i=1}^{N}\left(\frac{y_i - a - bx_i}{\sigma_i}\right)^2 \qquad \text{(A.5.11)}$$

The equations of Section 4 continue to apply with the slight modification that all σ values are replaced by σ_i. We again note that for N large, (A.5.11) should approach N and that

$$s^2 = \frac{S(a,b)}{N-2} \to 1 \text{ as } N \to \infty$$

Thus (A.4.3) becomes

$$\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{\sum \frac{1}{\sigma_i^2}\sum \frac{x_i^2}{\sigma_i^2} - (\sum \frac{x_i}{\sigma_i^2})^2}\begin{bmatrix} \sum \frac{x_i^2}{\sigma_i^2} & -\sum \frac{x_i}{\sigma_i^2} \\ -\sum \frac{x_i}{\sigma_i^2} & \sum \frac{1}{\sigma_i^2} \end{bmatrix}\begin{bmatrix} \sum \frac{y_i}{\sigma_i^2} \\ \sum \frac{x_i y_i}{\sigma_i^2} \end{bmatrix} \qquad \text{(A.5.12)}$$

and the confidence limits on a an b given in (A.4.4) and (A.4.5) become

$$\Delta a = z(1-\alpha/2)\left[\frac{\sum \frac{x_i^2}{\sigma_i^2}}{\sum \frac{1}{\sigma_i^2}\sum \frac{x_i^2}{\sigma_i^2} - (\sum \frac{x_i}{\sigma_i^2})^2}\right]^{1/2} \qquad \text{(A.5.13)}$$

and

$$\Delta b = z(1-\alpha/2)\left[\frac{\sum \frac{1}{\sigma_i^2}}{\sum \frac{1}{\sigma_i^2}\sum \frac{x_i^2}{\sigma_i^2} - (\sum \frac{x_i}{\sigma_i^2})^2}\right]^{1/2} . \qquad \text{(A.5.14)}$$

6. Weighted Linear Regression - Combining Data

One may collect sufficient data in an experiment to perform some preprocessing before regression. An example might be repeated surface-wave dispersion measurements along the same path for the same frequencies. One could use all data or one could use just the mean observations at each frequency, the latter may be preferable since it reduces the work required in an inversion scheme since a smaller number of data points are processed. The question arises of how to determine the correct error estimates.

To develop this topic, construct a data set consisting of 1000 observations at each of 10 abscissa, x_i. The observations are from the model

$$y_{ij} = 0.0 + 1.0\, x_i + \varepsilon_{ij}$$

for $i = 1, \cdots, 10$ and $j = 1, \cdots, 1000$. The ε_{ij} is from a normal distribution of zero mean and variance σ_i^2 (Press *et al*, 19XX). Table A.6.1 gives the ten x_i, the target y_i and σ assigned to the Gaussian error process. The computed $\bar{y}$, s_y and $s_{\bar{y}}$ are derived from the data set for each x_i. Even with 1000 observations at each x_i, we see that the observed mean is not exactly the target value, but that 9 of the 10 $\bar{y}$'s are within one $s_{\bar{y}}$ of the target y.

Table A.6.1 Statistics of Test Data

i	x	y	σ	$\bar{y}$	s_y	$s_{\bar{y}}$
1	0	0	4.00	0.080	4.144	0.131
2	1	1	4.00	0.965	4.071	0.129
3	2	2	4.00	1.993	4.117	0.130
4	3	3	2.00	2.985	1.985	0.063
5	4	4	2.00	4.047	2.023	0.064
6	5	5	2.00	5.035	1.928	0.061
7	6	6	1.00	6.043	0.946	0.030
8	7	7	1.00	7.032	0.996	0.031
9	8	8	1.00	8.011	0.984	0.031
10	9	9	1.00	9.015	0.985	0.031

Given this data set of 10,000 observations or the 10 reduced observations from the above table, we consider three minimization problems:

a) Using the entire data set, minimize:

$$\sum_{i=1}^{10}\sum_{j=1}^{1000}(y_{ij} - a - bx_i)^2$$

b) Using the entire data set and the s_y values for each i:

$$\sum_{i=1}^{10}\sum_{j=1}^{1000}\left(\frac{y_{ij}-a-bx_i}{\sigma_{y_i}}\right)^2$$

c) Using the reduced data set consisting of the 10 $x_i, \bar{y}_i, s_{\bar{y}_i}$:

$$\sum_{i=1}^{10}\left(\frac{\bar{y}_i-a-bx_i}{\sigma_{\bar{y}_i}}\right)^2$$

Case a) is solved using (A.3.4) - (A.3.8), Case b) is solved using §5.2 (A.5.12 - A.5.14) and Case c) is also solved using §5.2 except that y_i and σ_i in (A.5.11) are replaced by $\bar{y}_i$ and $s_{\bar{y}_i}$, respectively. To understand how this may relate to actual data sets, for which we do know not the σ's, we replace each σ in these minimization problems by the corresponding estimate s also given in Table A.6.1. This is essential for Case c) because the $\sigma_{\bar{y}_i}$ must be zero since we do not assume an error in the fundamental data set.

Table A.6.2

$\sum_{i=1}^{10}\sum_{j=1}^{1000}(y_{ij}-a-bx_i)^2$	$\sum_{i=1}^{10}\sum_{j=1}^{1000}\left(\frac{y_{ij}-a-bx_i}{s_{y_i}}\right)^2$	$\sum_{i=1}^{10}\left(\frac{\bar{y}_i-a-bx_i}{s_{\bar{y}_i}}\right)^2$
$N = 10000$	$N = 10000$	$N = 10$
	$\sum \frac{1}{s_{y_i}^2} = 5542.32$	$\sum \frac{1}{s_{\bar{y}_i^2}} = 5132.48$
$\sum x_i = 45000$	$\sum \frac{x_i}{s_{y_i}^2} = 36590.06$	$\sum \frac{x_i}{s_{\bar{y}_i^2}} = 34568.24$
$\sum x_i^2 = 285000$	$\sum \frac{x_i^2}{s_{y_i}^2} = 264022.57$	$\sum \frac{x_i^2}{s_{\bar{y}_i^2}} = 252523.32$
$\sum y_i = 45209.243$	$\sum \frac{y_i}{s_{y_i}^2} = 36728.95$	$\sum \frac{\bar{y}_i}{s_{\bar{y}_i^2}} = 34696.42$
$\sum y_i^2 = 353317.02$	$\sum \frac{y_i^2}{s_{y_i}^2} = 277354.62$	$\sum \frac{\bar{y}_i^2}{s_{\bar{y}_i^2}} = 254195.59$
$\sum x_i y_i = 285988.53$	$\sum \frac{x_i y_i}{s_{y_i}^2} = 264915.86$	$\sum \frac{x_i \bar{y}_i}{s_{\bar{y}_i^2}} = 253356.99$
$a = 0.0184 \pm 0.0478$	$a = 0.0320 \pm 0.0461$	$a = 0.0351 \pm 0.0500$
$b = 1.0006 \pm 0.0089$	$b = 0.9989 \pm 0.0067$	$b = 0.9985 \pm 0.0071$
	$\sum \varepsilon_i^2 = 11542.16$	$\sum \varepsilon_i^2 = 1.6784$
$s^2 = 6.624882$	$s^2 = 1.15445$	$s^2 = 0.2098$
t(1000-2,0.975)=1.96	t(10000-2,0.975)=1.96	t(10-2,0.975)=2.31

For simplicity the ± error bounds are given assuming that the Student-t distribution is t( )=1, e.g., they are one-sigma bounds and not a given probability. The probability comes

from the t-distribution.

A comparison of the results of this table indicates that all a, b estimates of are within one sigma of the assumed true A, B values of 0 and 1, respectively. We also note that the $\sum \varepsilon_i^2$ values for Case b) is very close to the number of observations N! On the other hand, the $\sum \varepsilon_i^2 \ll N$ for Case c).

As mentioned in §5 we might consider the s_i as weights, and then apply (A.5.4), (A.5.7) and (A.5.8). We must be careful though. If we use the definition $w_i = 1/{s_{y_i}}^2$, and the formula

$$\sum_{i=1}^{10} \sum_{j=1}^{1000} w_i \left(y_{ij} - a - bx_i \right)^2$$

we obtain for the data of Case b)

$$a = 0.0320 \pm 0.0494 \quad b = 0.9989 \pm 0.0071$$

which is essentially the same as that in the center column of Table A.6.1 Use of the equations of §5.1 requires that the error estimate be based on the lack of fit to the observations, and our sample of 10,000 points seems sufficiently large for this to be stably estimated here.

However, if we apply this to the $x_i, \bar{y}_i, s_{\bar{y}_i}$ and minimize

$$\sum_{i=1}^{10} w_i \left(\bar{y}_i - a - bx_i \right)^2$$

using $w_i = 1/{\sigma_{\bar{y}_i}}^2$, we obtain

$$a = 0.0351 \pm 0.0228 \quad b = 0.9985 \pm 0.0033$$

The a and b values agree with those of the third column of Table (A.6.2), but the error estimates Δa and Δb are approximately two times smaller. The reason for that is related to the fact that we have only 10 data points to fit, and the fit, which is used to estimate the s^2, happens to be better than with the original 10,000 points. Thus the s^2 is underestimated. From our discussion of the statistical weighting, we would expect $\sum \varepsilon_i^2$ should be 10, or equivalently that statistical weighting should give $s^2 = 1$ for large numbers of observations. Thus the errors are underestimated by a factor of *sqrt(10.0/1.67). This suggests that if statistical weighting is used, that we base the error estimate on* s^2*, but that we never permit this value to be less than* 1.0.

By performing this exercise, we have learned the following:

- All three procedures yield that same results.
- If the observations at one x_i are collapsed into a single $\bar{y}_i$, then statistical weighting is correct if we use the $\sigma_{\bar{y}}$. *We cannot use the* σ_y *is this case!*
- If statistical weighting is used, the relation of $\sum \varepsilon_i^2$ to N can be used to determine if the data set can be used to estimate the parameter errors from the actual residuals of fit.

this is the same as saying that we base error estimates on the lack of fit *unless* the fit is too good to be true. The initial σ value forces a minimum error on the solution.

7. One Model - Two Different Data Sets

Often two different physical quantities can be measured that are functions of the same parameters. In seismology these may be body-wave travel times and surface-wave dispersion or teleseismic receiver functions and surface-wave dispersion. Or we may have observations of surface-wave dispersion and anelastic attenuation and which to derive the shear-wave velocities.

7.1 Development

The problem is that we wish to determine A and B such that the observations are

$$z_i = Ax_i + By_i + N(0, \sigma_{z_i^2}) \qquad \text{(A.7.1)}$$

$$t_i = Au_i + Bv_i + N(0, \sigma_{t_i^2})$$

The observations z_i and t_i have different units and different variances.

Let us assume further that we desire to use a parameter p such that $p = 0$ implies the use of only the $z(x, y)$ data set and $p = 1$ implies the use of only the $t(u, v)$ data set. A choice $p = 0.5$ will imply that we desire the data sets to have equal influence on the solution.

Keeping in mind the desire that the problem reduce to the familiar solution of Section 4 , we construct the functional $S(a, b)$ to be minimized:

$$S(a, b) = \left(\frac{(1-p)}{N} \sum_{i=1}^{N} \left(\frac{z_i - ax_i - by_i}{\sigma_{zi}} \right)^2 + \frac{p}{M} \sum_{j=1}^{M} \left(\frac{t_j - au_j - bv_j}{\sigma_{tj}} \right)^2 \right) \qquad \text{(A.7.2)}$$

Note that this functional does reduce to the correct form for p = 0 or p = 1. The use of the statistical weighting, e.g., division by the respective σ ensures that the dimensions have been accounted for. In addition, for large sets of observations, we expect that $S(a, b) = 1$ by construction.

7.2 Actual Data Processing

A problem with actual data is that the number of observations may not be large compared to the model parameters. Thus we may not be able to obtain a good estimate of the data variances from the model misfit. On the other hand we may be able to establish reasonable lower bounds on the expected error by carefully studying the data processing that leads to the observations.

The following strategy may be acceptable.

1. For each data set, z_i and t_i, define a minimum value of σ_z and σ_t

2. Use these to form weights and solve the weighted least squares normal equations for a and b.

3. Compute $S(a,b)/N$. If this is less than 1, use 1 for the error estimates, other wise use this value for the error estimates. Continue to search for the minimum. The error estimate is a modification of (A.5.13)

$$\Delta a = t(N-2, 1-\alpha/2)\,\frac{S(a,b)}{N}\left[\frac{\sum \frac{x_i^2}{\sigma_i^2}}{\sum \frac{1}{\sigma_i^2}\sum \frac{x_i}{\sigma_i^2} - (\sum \frac{x_i}{\sigma_i^2})^2}\right]^{1/2}$$

4. Compute the misfit of the model to each data set, giving s_z and s_t. This may be used in the future as better initial estimates of the errors

5. Since we often solve non-linear problems by iterative application of least squares, we repeat steps 2 - 4. For error analysis, adjust

This procedure combines the concepts of *a priori* knowledge of σ and data estimated σ from the *s*. Step 4 ensures that a small data set, for which $s \approx 0$, will not give artificially small estimates of errors in *a* and *b*.

7.3 Reduction to a Single System

For large N and M in (A.7.2) and the correct a and b the minimum value of S(a, b) is 1. If we define

$$w_{z_i}{}^2 = (1-p)/N\sigma_{z_i}{}^2$$

$$w_{t_j}{}^2 = p/M\sigma_{t_j}{}^2$$

then (A.7.2) the minimization problem looks like that discussed in Section 5, e.g., minimize

$$S(a,b) = \sum w_i(p_i - aq_i - br_i)^2$$

and similar equations for (A.5.3 - A.5.9)

The formulation discussed in this section assumes that the parameter p is *a priori* specified, and is not a free parameter. We assume that $p = 1/2$ implies equal contribution of each data set to the final model. (A.7.2) was carefully constructed to reduce to simpler statistical weighting for the end cases of $p = 0$ or $p = 1$. If we solve for the *a* and *b* for each fixed *p*, we can then plot the *S(a(p), b(p))* as a function of *p* as an exercise, but this should not be used in the selection of *p*.

In terms of earth model determination, the formulation permits us to appreciate the range of earth models as a function of p. Comparing the p = 1/2 solution to the p = 0 and p = 1 solutions may give us an insight on how the two independent data sets interact with each other to give a, perhaps, more realistic model that builds upon the strengths and sensitivity of each data set.

8. Matrix Formulation

The least squares problem can be stated on that solves the problem

$$|Ax - b| = MIN$$

where A is an mxn matrix, x is an nx1 matrix and b is an mx1 matrix representing m equations in n unknowns. The classic least-squares solution of this problem is given by

$$x = (A^T A)^{-1} A^T b$$

which is adequate if $(A^T A)^{-1}$ exists.

It may be desirable to place a constraint on the problem to solve the related problem

$$|Ax - b| + |\sigma x| = MIN$$

which is the same as

$$(Ax - b)^T (Ax - b) + \sigma^2 x^T x = MIN$$

This problem can be solved using the singular value deconposition of A as $A = U\Lambda V^T$, $A^T = V\Lambda U^T$, and the definitions $UU^T = I$ and $V^T V = I$. These lead to the solution vector x, the variance-covariance matrix C and the resolution matrix R (Crosson, 1976):

$$x = V(\Lambda^2 + \sigma^2 I)^{-1} \Lambda U^T b$$

$$= Hb$$

$$C = HH^T = V(\Lambda^2 + \sigma^2 I)^{-1} \Lambda^2 (\Lambda^2 + \sigma^2 I)^{-1} V^T$$

$$R = V(\Lambda^2 + \sigma^2 I)^{-1} \Lambda^2 V^T \qquad (6.5)$$

This modified problem, known as the Levenberg-Marquardt generalized inverse, determines the best solution to the Ax = b problem subject to the constraint that the size of x is kept small. For a well behaved A matrix, $\sigma^2 = 0$ gives the classic least squares solution. Larger values of σ^2 force the solution to be one of steepest descent if the algorithm is applied iteratively.

The solution vector x now depends upon the parameter σ. If we consider the x for $\sigma = 0$ to be the true value, and the x for a given σ to be an estimate of the true value, one can replace b by the definition Ax to yield

$$x_{est} = V(\Lambda^2 + \sigma^2 I)^{-1} \Lambda U^T A x_{true}$$

The resolution R is defined by the relation $x_{est} = R x_{true}$. The resolution matrix is symmetric in this case and is equal to the identy matrix if $\sigma^2 = 0$.

Consider now a slightly different problem. Introduce another variable y related to the solution vector x by

$$Wx = y \quad \text{or} \quad x = W^{-1} y$$

Let us now state that we wish a value of x or equivalently y that minimizes

$$|Ax - b| + |\sigma W x| = \text{MIN}$$

or

$$|AW^{-1} y - b| + |\sigma y| = \text{MIN}$$

Define $\mathsf{A} = AW^{-1} = \mathsf{U}\lambda\mathsf{V}^T$. Here U, λ and V are matrices. Helvetica font rather than italic font is used because these matrices will be different than those of the previous problem The solution to this problem is just

$$x = W^{-1}\mathsf{V}(\lambda^2 + \sigma^2 I)^{-1}\lambda\mathsf{U}^T b$$

with resolution and variance-covariance matrices

$$R(x) = W^{-1}\mathsf{V}(\lambda^2 + \sigma^2 I)^{-1}\lambda^2\mathsf{V}^T W$$

$$C = W^{-1}\mathsf{V}(\lambda^2 + \sigma^2 I)^{-1}\lambda^2(\lambda^2 + \sigma^2 I)^{-1}\mathsf{V}^T(W^{-1})^T$$

Note now that the resolution matrix is not symmetric. Also note that the least squares problem solved here differs significantly from the first example. The length of the vector x is not minimized but rather the weighted vector Wx.

The matrix W can be the effect of several cumulative contraints, $W = W_1 \cdots W_n$. If we only use $W = W_1 W_2$, and W_2 is

$$W_2 = \begin{bmatrix} 1 & -1 & 0 & .. & 0 \\ 0 & 1 & -1 & .. & 0 \\ 0 & 0 & 1 & .. & 0 \\ .. & .. & .. & .. & .. \\ 0 & 0 & 0 & .. & 1 \end{bmatrix}$$

with

$$W_2^{-1} = \begin{bmatrix} 1 & 1 & 1 & .. & 1 \\ 0 & 1 & 1 & .. & 1 \\ 0 & 0 & 1 & .. & 1 \\ .. & .. & .. & .. & .. \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

the minimization constraint attempts to minimize the difference between adjacent values of $\mathbf{x_i}$. In an inversion for an earth model, where the $\mathbf{x}$ represents the change in velocity from an initial model, this form of $\mathbf{W_2}$ forces a degree of smoothness on the changes in velocity.

If $W_2 = I$, then there is no smoothness constraint in an interative non-linear inversion, just a restriction that the changes in x be small.

A useful functional form for W_1 is that of a diagonal matrix

$$W_1 = \text{diag}\left\{\sigma_1^{-1}, \sigma_2^{-1}, \ldots, \sigma_n^{-1}\right\}$$

The effet of this is to apply a relative weight to the constraints. For example, to force a sharp discontinuity at a given boundary when using the smoothing constraint, make the σ_k^{-1} small.

It is possible to create a W_2 matrix that combines smoothness and lack of smoothness with a suitable W_1 that fixes x in certain layers.

9. General Linear Least Squares

Sections 2 - 8 used the example of the simple linear model

$$Y = A + BX$$

which was both linear in terms of the A and B coefficients but also in terms of the independent variable X. This model was used to illustrate data sets. The only requirement for linear least squares is that the predicted value be a linear function of the unknown model coefficients. Thus data models such as

$$\log Y = A + BX,$$

$$Y = A + B \sin X + C \cos X,$$

or

$$\frac{1}{Y} = A + \frac{B}{X}$$

are linear in the coefficients **A** and **B**.

Since the number of coefficients may exceed two, and since the dependence on the independent variable or variables may be complicated, plots such as those of Figures 1 and 2 are not useful, even though F-statistic plots in the manner of Figure 3 will continue to be useful.

9.1 Correlation Coefficient

A visual measure of goodness of fit is still required. The *correlation coefficient r* is useful. Let the observed data be y_i and let the model prediction using the determined linear coefficients be Y_i. The correlation coefficient is defined by the equation

$$r = \frac{\sum (y_i - \bar{y})(Y_i - \bar{Y})}{\left(\sum (y_i - \bar{y})^2 \sum (Y_i - \bar{Y})^2 \right)^{1/2}} \qquad \text{(A.9.1)}$$

or

$$r = \frac{\left(\sum Y_i y_i - N\bar{Y}\bar{y} \right)}{\left(\left(\sum y_i^2 - N\bar{y}^2 \right) \left(\sum Y_i^2 - N\bar{Y}^2 \right) \right)^{1/2}} \qquad \text{(A.9.2)}$$

where $\bar{y} = \sum y_i / N$ and $\bar{Y} = \sum Y_i / N$. The correlation coefficient r lies in the range [–1,1]. If the data have no error and the model is correct then we expect $r = 1$.

As with many items in statistics, a distribution as associated with r. Under the assumption of a Gaussian normal distribution of error, the statistic is.

The value of this statistic is that it permits a rejection of a data model. *expand on this*

Robert Shumway corrected the draft and assigned a B+ to this effort.

10. Non-Linear Least Squares by Linearization

11. L-1 Norms

Acknowledgements

Robert Shumway corrected the draft and assigned a B+ TO THIS EFFORT.

12. Problems

1. Prove that (A.3.5) follows from (A.3.2) and (A.3.3). Hint: do not substitute the a and b values from (A.3.4). Rather use the algebraic property that $(A+B)(A+B) = A(A+B) + B(A+B)$.

13. References

Crosson, R. S. (1976). Crustal structure modeling of earthquake data I. Simultaneous least squares estimation of hypocenter ande velocity parameters, J. Geophys. Res. 81, 3036-3046.

Draper, N. R., and H. Smith (1966). Applied Regression Analysis, John Wiley & Sons, New York, 407 pp.

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CHAPTER 3
SURFACE WAVE ANALYSIS

1. Introduction

The study of surface waves holds a special place in seismology for several reasons. Because of their large amplitudes compared to body waves for teleseisms, these are the most recognizable part of the seismogram, especially along paths between the earthquake and seismograph station that cross deep oceans. These waves are also significant because they arise from boundary conditions near the Earth's surface - a low velocity waveguide for Love waves and a stress free surface for Rayleigh waves - hence their name of *surface wave*.

Because of their ubiquitous presence in data, they are an obvious choice for the study of primarily shear-wave velocity structure near the surface.

We will discuss the data processing of observational data to provide phase velocity, c, and group velocity, U, velocity dispersion curves as well as anelastic attenuation coefficients, γ, which can be inverted for the elastic wave velocities and Q^{-1}. The organization of this chapter address basic surface wave theory, methods for determination of phase velocity and group velocity, and finally the inversion for earth structure.

2. The Surface Processing

2.1 Surface wave representation

The surface-wave in a flat-layered earth is usually the largest low frequency signal at large distances because its geometrical spreading is less than that of the direct body waves. The Fourier transform of the surface-wave signal for a single-mode observed at a distance r from the source is written as

$$\frac{1}{\sqrt{(}\mathrm{r})}\,\mathrm{S}(\omega)\,\mathrm{A}(\omega)\mathrm{e}^{-\mathrm{i}(\mathrm{kr}+\psi)}$$

where ω is the angular frequency, k is the horizontal wavenumber which is related to the

phase velocity **c** by the definition $\omega = \mathbf{kc}$. **S(ω)** is complex source spectrum, and **A(ω) exp(−iψ)** represents the complex excitation of surface waves for a point source. The excitation is a function of frequency, source depth and properties of the elastic medium.

The Fourier transform of the surface wave which includes the higher modes is given by

$$\frac{1}{\sqrt{r}} S(\omega) \sum_m A_m(\omega) e^{-i(k_m r + \psi_m)}$$

where the index m is the mode number. The presence of high modes complicates the interpretation of phase or group velocities, especially because the excitation of each mode depends upon frequency, which may may make mode identification difficult.

2.2 Phase Velocity Determination

McMechan and Yedlin (1981) described a technique to obtain phase velocity dispersion from an array of seismic traces. They proposed first performing a $p - \tau$ stack followed by a transformation into the $p - \omega$ domain. A separate stacking procedure is not required to accomplish this if operations are performed in the frequency domain.. Mokhtar *et al* (1988) describe how this can be done. Let the observed Fourier spectrum of a seismic signal at distance r_n be

$$A(\omega, r_n) e^{i\phi(\omega)_n} \tag{1}$$

One possible $p - \omega$ stack of N traces at different distances from the same source is defined by the relation

$$F(p, \omega) = \sum_{n=1}^{N} C(\omega, r_1, r_n)^{-1} A(\omega, r_n) e^{i\phi_n} e^{i\omega p r_n}. \tag{2}$$

where

$$C(\omega) = A(\omega, r_1, r_n) e^{i\phi_1} \sqrt{\frac{r_n}{r_1}}$$

Division by $C(\omega, r_1, r_n)$ is a simple artifice to remove the source spectrum from the observations, and to correct for geometrical spreading. If the signal is only that of a single mode, then the depth dependent excitation is also removed.

Since the observed spectrum is assumed to be the superposition of M surface-wave modes such that

$$A(\omega, r_i) e^{j\phi_i} = \frac{S(\omega}{\sqrt{r_i}} \sum_{m=1}^{M} A_m(\omega, r_i) e^{j[\psi_{0m}(\omega) - \omega p_{0m}(\omega) r_i]}, \tag{3}$$

the operation in (2) does correct for geometrical spreading and the source excitation. In this notation the $\psi_{\mathbf{0m}}$ is the actual excitation phase of the **k**'th mode, and $\mathbf{k_m} = \omega \mathbf{p_{0m}}$. This expression separates the phase into distance dependent and independent contributions. Interpretation of (2) is difficult unless the is only a single mode or the amplitude of one mode is so large at a given frequency that its contribution outweighs that of other modes.

If the signal consists of a single noise free surface-wave mode, then the quantity $F(p, \omega)$ will have a maximum when $p = p_{0k}$. Searching for the maxima of

$$|F(p, \omega)| \tag{4}$$

yields the possible dispersion curves. Since there are N distances, the maximum value of the quantity $|F(p, \omega)|$ should be equal to N.

If this value is less than N, then we can attribute this to an error in the ray parameter between the stations. If we assume that in the neighborhood of a maximum of the stack, $\Delta p = p - p_{0k}$ has a normal distribution with zero mean and a variance σ^2, then the expected value of any term in (1) is

$$E[e^{-j2\pi f \Delta p r_i}] = e^{-2\sigma^2 \pi^2 f^2 r_i^2} \tag{5}$$

where we used the definition $\omega = 2\pi f$.

Since each term in (4) is always positive, the expected value of the stack (1) of a single mode is just

$$\left| \sum_{i=1}^{N} e^{-2\sigma^2 \pi^2 f^2 (r_i - r_1)^2} \right| \tag{6}$$

Given the stack value (4), a Newton-Raphson technique is used to find the value of σ from (6) that corresponding to this value. The error in phase velocity is obtained using the definition $p = 1/c$ and the relation

$$\Delta c = \sigma c^2. \tag{7}$$

This relation was tested by numerically modeling a stacking operation in which the ray parameter error had the assumed distribution.

In the more realistic case of multimode surface waves, (4) will not yield a maximum independent of the amplitude spectrum of the other modes. Thus the stack value will typically be largest for one mode and smaller for others. The simplistic error analysis will yield larger, perhaps unwarranted, errors for the other modes. This is an inherent problem with this technique, that can only be resolved if a phase matched filter technique is first applied to each input spectra to isolate a single mode before the phase velocity stack is performed.

The program **sacpom96** implements this technique. Figure 1 presents the processing flow for this program:

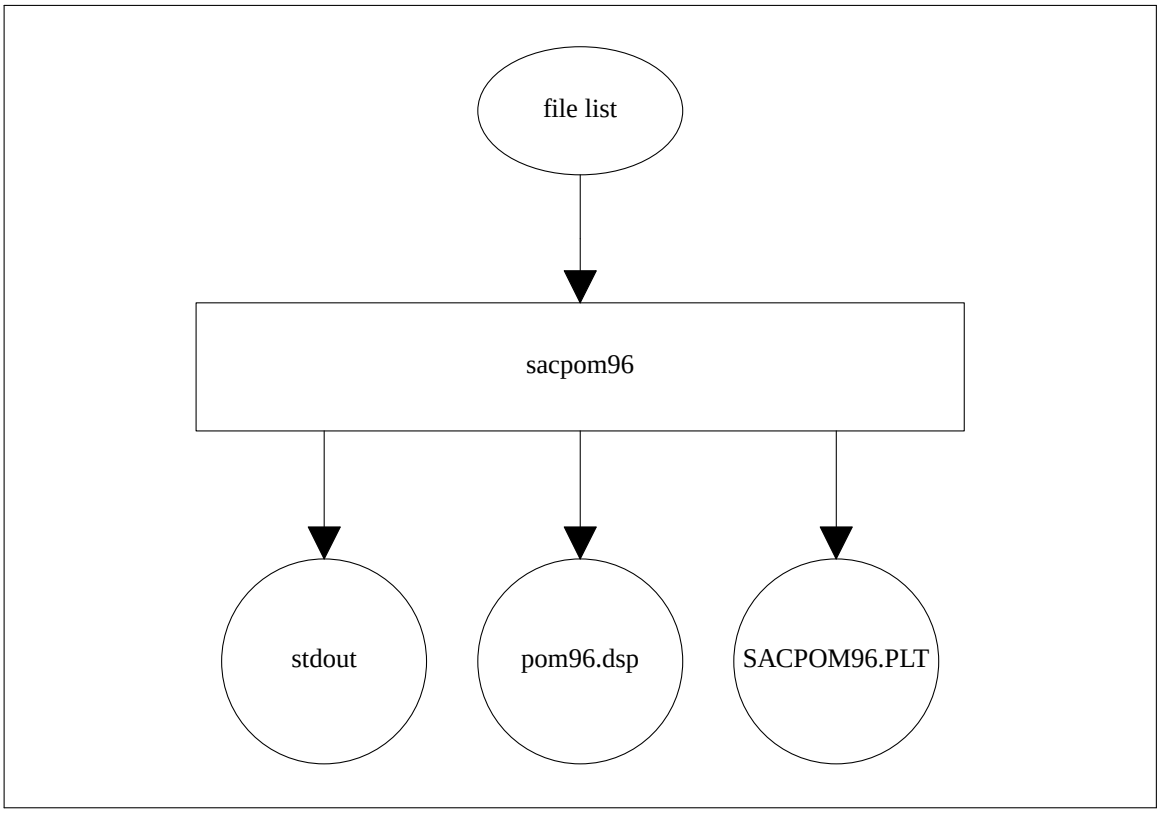


Fig. 1. Processing flow for **sacpom96**

Program control is through the command line:

sacpom96 *[flags]*, where the command flags are

-C [spcfil]	*(default stdin) : Input data file name*
-ci [ci]	*(default=2.0) : starting phase velocity*
-ce [ce]	*(default=5.0) : ending phase velocity*
-nray [nray]	*(default = 20) : number of ray parameters*
-fmin [fmin]	*(default=0.02) : minimum frequency for plot*
-fmax [fmax]	*(default=0.25) : maximum frequency for plot*
-vmin [vmin]	*(default 2.0) : minimum velocity for plot*
-vmax [vmax]	*(default 5.0) : maximum velocity for plot*
-xlin	*(default= false) : linear frequency axis*
-xlog	*(default= true) : logarithmic frequency axis*
-ylin	*(default = false): linear velocity axis*
-ylog	*(default = true): logarithmic velocity axis*
-V	*(default = false): verbose output*
-O	*(default = false): c-f values from maxima on STDOUT*
-E	*(default = false): plot error bars*
-T	*(default = false): x-axis is period not f*

-R *(default = true): data are Rayleigh*
-L *(default = false): data are Love*
-S *(default = false): contour shading*

The output is a tabulation of peak motions from the stack:

```
POM96 R C -1    45.511        4.0000    0.0026       6.0000    1
POM96 R C -1    7.9380        4.0000    0.3301       2.0351    2
POM96 R C -1    5.2648        4.0000    0.4403       1.2643    3
```

The output consists of 9 columns with the following meaning:

Col	Description
1	*SACPOM96* Name of the generating program
2	*R* Wave type, either *L* for Love or *R* for Rayleigh.
3	*C* Dispersion measurement. Always *C* for phase velocity
4	*0* Mode identification. *0* for fundamental, *1* for 1'st, etc. Note the program **do_pom** is available to run **sacpom96** and to interactively identify the modes. A value of *-1* indicates that no mode identification has been made
5	*1.6787* Period of observation in seconds. The **surf96** dispersion format required periods in seconds, and dispersion in km/s.
6	*0.66658* Phase velocity in *km/s*.
7	*0.16019E-02* Error estimate of velocity in *km/s*. From equation (7).
8	*1* Amplitude order of the stack. The *1* indicates that for this phase velocity, this periods had the largest stack amplitude. this may permit one to follow the modes.
9	*48.940* The value of equation (4). The maximum value of this is the number of traces.

One of the difficulties of the phase velocity stack is due to the discrete sampling in frequencies and the analyst's expectation of a single phase velocity for each frequency in the manner of the multiple filter analysis program **sacmft96** for group velocity. This program actually starts with a given phase velocity and then searches for the 8 frequencies for which the stack is a maximum. The presentation will then show many phase velocities for each frequency. To assist in organizing the results it may be useful to sort the output by period. The command

```
sort -n +4 < pom96.dsp > pom.sort
```

will give

```
POM96 R C -1    23.011        2.0843    0.8231       1.0353    7
POM96 R C -1    23.011        3.4699    0.0010       6.0000    1
```

On the basis of the stack amplitude, last column, I would associate the velocity *3.4699* km/s with the period of *3.011* seconds and ignore the other value.

Example

The following example demonstrates the use of this program using a simple crustal earth model.

```
#!/bin/sh

#this depth and model gives a spectra hole at 15-20 sec
HS=10
STK=0
RAKE=0
DIP=90
AZ=45
NMODE=10
######
#             changing the RAKE to 45 removes some of the spectral hole
#             = 0 Strike slip if dip is 90
#             = 90 dip slip is dip is 45  -- good hole
######
#####
#             create file of distances for synthetics
#             DISTANCE  DT NPTS T0 VRED
#####
cat > dfile << EOF
1000.0     1.000   2048    -1.0 8.0
1050.0     1.000   2048    -1.0 8.0
1100.0     1.000   2048    -1.0 8.0
1150.0     1.000   2048    -1.0 8.0
1200.0     1.000   2048    -1.0 8.0
1250.0     1.000   2048    -1.0 8.0
EOF


#####
#             create the earth model
#####
cat > model.d << EOF
MODEL
TEST MODEL
ISOTROPIC
KGS
FLAT EARTH
1-D
CONSTANT VELOCITY
LINE08
LINE09
LINE10
LINE11
HR      VP      VS  RHO QP  QS  ETAP ETAS FREFP FREFS
40.     6.0     3.5 2.8 0.0 0.0 0.0  0.0  1.0   1.0
00.     8.0     4.7 3.3 0.0 0.0 0.0  0.0  1.0   1.0
EOF

#####
#             Calculate multimode dispersion and make synthetics
#####

sprep96 -M model.d -NMOD NMODE – HS{HS} -HR 0 -d dfile -L -R
sdisp96
sregn96
slegn96
sdpegn96 -R -C -PER -TXT -XLOG -YMIN 2.5 -YMAX 5
sdpegn96 -L -C -PER -TXT -XLOG -YMIN 2.5 -YMAX 5
spulse96 -d dfile -D -p -EQ -2 > file96
fmech96 -A AZ – ROT – D{DIP} -R RAKE – S{STK} -M0 1.0E+20 < file96 > 3.96
```

```
f96tosac -B 3.96
cp B0101Z00.sac Z1.sac
cp B0201Z00.sac Z2.sac
cp B0301Z00.sac Z3.sac
cp B0401Z00.sac Z4.sac
cp B0501Z00.sac Z5.sac
cp B0601Z00.sac Z6.sac
```

These synthetics represent the recordings of a regional earthquake by a modern set of broadband seismic stations. The traces generated are presented in Figure 2.

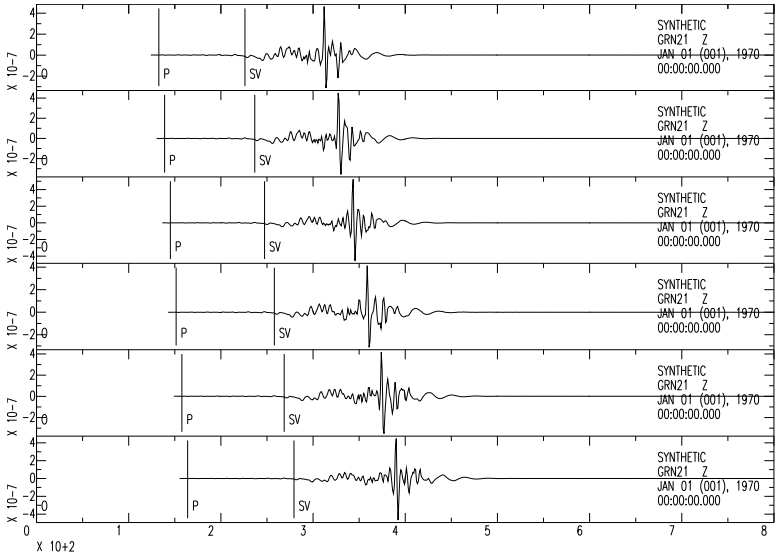


Fig. 2. Record section plot of the synthetics

To run **`sacpom86`** one performs the following operations

```
#####
#           create a list of SAC file to be processed
#####

ls Z?.sac  > cmdfil

sacpom96 -C cmdfil -PMIN 4.0 -PMAX 100.0 -nray 100 \
      -A -VMIN 2.00 -VMAX 5.00  -R -S

#####
#           the output in the file pom96.dsp. consists of
#
#POM96 L C 0 period phase_velocity err_phase_vel no_peak stack_amplitude
# or
#POM96 R C 0 period phase_velocity err_phase_vel no_peak stack_amplitude
#
# the CALPLOT graphics file POM96.PLT, a control file for do_pom
# named  pom96.ctl, and a shell script POM96CMP
#####
rm cmd
```

the graphic output in *POM96.PLT* can be merged with the predicted Rayleigh wave phase velocity values in *SREGNC.PLT* to create the *CALPLOT* file by doing

```
POM96CMP
cat POM96.PLT SREGNC.PLT > BIG.PLT
```

if the data are for Rayleigh waves. Of course an earth model must exist and theoretical eigenfunction files must have been created. The purpose of doing this here is to indicate how well the technique works in obtaining correct phase velocities.

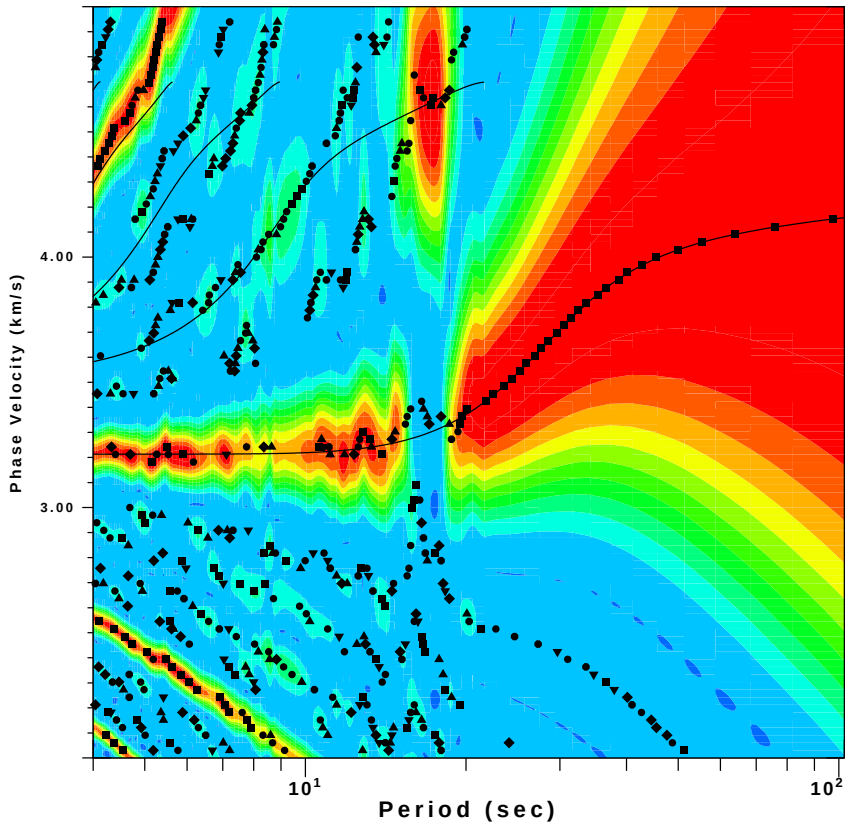


Fig. 3. Phase velocity stack values overlain by model predicted phase velocity dispersion curves. The colors indicate the stack value, with red corresponding to the largest. The black symbols represent the chosen peaks from the stack after a two dimensional search over the phase velocity - period grid. The light black curves are the theoretical dispersion values. Note the difficulty in identifying the higher modes.

2.3 Multiple filter analysis

The purpose of this note is to develop an analytic expression for a Gaussian filtered dispersed surface wave into order to assess the effects of signal spectrum shape on the dispersion. The impetus for this is the recommendation by Levshin (19XX) that the instantaneous frequency should be used rather than the filter frequency when determining group velocity dispersion. Block *et al.* (1969) and Herrmann (1973) ignored the effect of surface wave amplitude spectrum on the interpretation of the results.

Bhattacharya (1983) studied the bias effect in detail and recommended two applications of multiple filter analysis to obtain bias free estimates of group velocity and spectral amplitude. This note follows the presentation by Bhattacharya (1983) except that a Gaussian signal amplitude spectrum will be assumed rather than a linear shape and attention will be given to the use of the instantaneous frequency in the interpretation. The objective is to avoid specifying the source signal spectrum.

Let the dispersed signal mode surface-wave signal be

$$\mathbf{s(t)} = \frac{\mathbf{1}}{\mathbf{2\pi}} \int_{-\infty}^{\infty} \mathbf{A}(\omega)\mathbf{e}^{\mathbf{i}(\omega\mathbf{t}-\mathbf{kr}+\phi)}\mathbf{d}\omega \tag{1}$$

A filtered signal, resulting from the action of the filter $H(\omega)$ on s(t) is

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A(\omega)H(\omega)e^{i(\omega t-kr+\phi)}d\omega \tag{2}$$

For ease in deriving an analytic solution define the filter as

$$H(\omega) = e^{-\alpha(\omega-\omega_0)^2/\omega_0^2}$$

and approximate the signal in the neighborhood of the filter center frequency ω_0 as

$$k(\omega) = k_0 + (\omega - \omega_0)\frac{1}{U_0} + \tfrac{1}{2}(\omega - \omega_0)^2 k_0''$$

$$\phi(\omega) = \phi_0 + (\omega - \omega_0)\phi_0' + \tfrac{1}{2}(\omega - \omega_0)^2 \phi_0''$$

and

$$A(\omega) = A_0 e^{-\gamma(\omega-\Omega)^2/\omega_0^2} e^{+\gamma(\omega_0-\Omega)^2/\omega_0^2}$$

We used the definition of group velocity to introduce $1/U_0 = k_0' = (dk/d\omega)_{\omega_0}$ and note that $A(\omega_0) = A_0$ by construction. Also $A(\omega)$ peaks at $\omega = \Omega$ and γ controls the width of the signal spectrum.

Now introduce the change of variable $\omega = \omega_0 + x$ into (2). For a sharp filter cutoff at $\omega = \omega_0 \pm \omega_c$ where $H(\omega_0 \pm \omega_c) = e^{-\alpha\omega_c^2/\omega_0^2}$, where $H(\omega \pm \omega_c) = \exp(-\alpha\omega_c^2/\omega_0^2)$, we have

$$g(t) = \frac{1}{2\pi} A_0 e^{i(\omega_0 t - k_0 r + \phi_0)} \cdot \int_{-\omega_c}^{\omega_c} e^{-(ax^2+2bx+c)}dx$$

where we define

$$a = \frac{\gamma}{\omega_0^2} + \frac{\alpha}{\omega_0^2} + \frac{i}{2}(k_0''r - \phi_0'') \tag{3}$$

$$= \rho e^{i\psi} = a_R + ia_I$$

$$b = \frac{\gamma}{\omega_0^2}(\omega_0 - \Omega) - \frac{i}{2}\left(t - \frac{r}{U_0} + \phi_0'\right) \tag{4}$$

$$= \sigma e^{i\chi} = b_R + ib_I$$

$$c = 0$$

For later use, define $\beta = t - \dfrac{r}{U_0} + \phi_0'$.

From Abramowitz and Stegun (1965),

$$I = \int e^{ax^2+2bx+c)}dx = \tfrac{1}{2}\sqrt{\frac{\pi}{a}}\, e^{(b^2-ac)/a}\mathrm{erf}\left(\sqrt{a}x + \frac{b}{\sqrt{a}}\right)$$

Since **erf (–z) = – erf (z)**, we have for **g(t)**

$$\mathbf{g(t)} = \frac{\mathbf{1}}{\mathbf{2}\pi}\mathbf{A_0 e^{i(\omega_0 t - k_0 r + \phi_0)}} \tag{5}$$

$$\cdot\sqrt{\frac{\pi}{\rho}}\, e^{-i\frac{1}{2}\psi} e^{\frac{\sigma^2}{\rho}e^{i(2\chi-\psi)}}$$

$$\cdot\left\{\tfrac{1}{2}\,\mathrm{erf}\left(\sqrt{a}\omega_c + \frac{b}{\sqrt{a}}\right) + \tfrac{1}{2}\,\mathrm{erf}\left(\sqrt{a}\omega_c - \frac{b}{\sqrt{a}}\right)\right\}$$

or

$$g(t) = \frac{1}{2\pi}A_0 e^{i(\omega_0 t - k_0 r + \phi_0)} \tag{6}$$

$$\cdot\sqrt{\frac{\pi}{\rho}}\, e^{-i\frac{1}{2}\psi} e^{\frac{a_R(b_R^2-b_I^2)+2a_I b_R b_I}{a_R^2+a_I^2}} e^{i\frac{-a_I(b_R^2-b_I^2)+2a_R b_R b_I}{a_R^2+a_I^2}}$$

$$\cdot\left\{\tfrac{1}{2}\,\mathrm{erf}\left(\sqrt{a}\omega_c + \frac{b}{\sqrt{a}}\right) + \tfrac{1}{2}\,\mathrm{erf}\left(\sqrt{a}\omega_c - \frac{b}{\sqrt{a}}\right)\right\}$$

At this stage no approximation has been made. If we assume that the filter is narrow, then following Bhattacharya (1983), the two erf terms are replaced by the single real term

$$\left\{\mathrm{erf}\left(\sqrt{\rho}\cos\tfrac{1}{2}\psi\right)\right\}$$

We can also write g(t) as

$$g(t) = \Theta(t)\, e^{i\theta(t)}$$

where $\Theta(t) = |g(t)|$ and $\theta(t) = \arg g(t)$. The extreme positions of Θ are obtained from $d\Theta/dt = 0$, The instantaneous frequency is defined at $\omega_i = d\theta/dt$. Since time t only enters into the expression for g(t) in the $\omega_0 t$ term and in β or equivalently the b_I, we can show that the extreme values in $\Theta(t)$ occur when

$$-a_R b_I + a_I b_R = 0$$

and that the instantaneous frequency is

$$\omega_I = \omega_0 + \frac{2\left[a_R b_R + a_I b_I\right]}{a_R^2 + a_I^2}\left[-\tfrac{1}{2}\right]$$

$$= \omega_0 + \delta\omega_{0i}$$

At the envelope maximum, we have

$$\delta\omega_{0i} = -\frac{b_I}{a_I} = -\frac{b_R}{a_R} \tag{7}$$

For the special case of a flat signal spectrum, $\gamma = \mathbf{0}$, $\mathbf{b_R} = \mathbf{0}$, $\delta\omega_{\mathbf{0i}} = \mathbf{0}$ and $\mathbf{b_I} = \mathbf{0}$. Substituting into the expression for the maximum, and the factor $\mathbf{b_I}$, we have from (7)

$$\mathbf{t} - \frac{\mathbf{r}}{\mathbf{U_0}} + \phi_\mathbf{0}' \equiv -\mathbf{2b_I} = -\frac{\mathbf{2a_I b_R}}{\mathbf{a_R}} \tag{8}$$

$$= \frac{-2a_I\gamma(\omega_0 - \Omega)}{\gamma + \alpha}$$

Following Bhattacharya (1983), consider the use of two values of the filter parameter α, so that

$$t_1 - \frac{r}{U_0} + \phi_0' = \frac{-2a_I\gamma(\omega_0 - \Omega)}{\gamma + \alpha_1} = 2a_I\delta\omega_1$$

and

$$t_2 - \frac{r}{U_0} + \phi_0' = \frac{-2a_I\gamma(\omega_0 - \Omega)}{\gamma + \alpha_2} = 2a_I\delta\omega_2$$

Subtracting,

$$t_1 - t_2 = -2a_I\gamma(\omega_0 - \Omega)\left[\frac{1}{\gamma + \alpha_1} - \frac{1}{\gamma + \alpha_2}\right]$$

But from (7)

$$\delta\omega_1 = -\frac{\gamma(\omega_0 - \Omega)}{\gamma + \alpha_1} \tag{9}$$

and

$$\delta\omega_2 = -\frac{\gamma(\omega_0 - \Omega)}{\gamma + \alpha_2} \tag{10}$$

Thus

$$t_1 - t_2 = 2a_I(\delta\omega_1 - \delta\omega_2) \tag{11}$$

and

$$t_2 - \frac{r}{U_0} + \phi_0' = \frac{t_1 - t_2}{\delta\omega_1 - \delta\omega_2}\,\delta\omega_2$$

or a better estimate of U_0 is

$$U_0 = \frac{r}{t_2 - \delta\omega_2(t_1 - t_2)/(\delta\omega_1 - \delta\omega_2) + \phi_0'} \tag{12}$$

Note that we cannot resolve the time delay due to the derivative of the source phase, ϕ_0' but hope that the estimate of U_0 by assuming that ϕ_0' is closer to the true value than the simple estimate r/t_2.

We have thus paralleled the Bhattacharya formula for improved group velocity.

At this stage we have a_I from (7) which is an essential component for computing the correct spectral amplitude.

$$\mathbf{a_I} = \tfrac{\mathbf{1}}{\mathbf{2}} \frac{\mathbf{t_1} - \mathbf{t_2}}{\delta\omega_\mathbf{1} - \delta\omega_\mathbf{2}} \tag{13}$$

By the definition of $\delta\omega_{0i}$,

$$b_I = -a_I \delta\omega_{0i} \tag{14}$$

Taking the ratio of (7) to (8) solving for γ, if $\gamma \neq 0$, gives

$$\gamma = \frac{\left(\alpha_2 - \alpha_1 \frac{\delta\omega_1}{\delta\omega_2}\right)}{\left(\frac{\delta\omega_1}{\delta\omega_2} - 1\right)}$$

Thus

$$a_R = \frac{\gamma + \alpha}{\omega_0^2}, \tag{15}$$

$$b_R = -a_R \delta\omega_0 \tag{16}$$

and

$$\Omega = \omega_0 - b_R \omega_0^2 / \gamma \tag{17}$$

The analysis presented highlights several important aspects of multiple filter analysis. First, the effect of the source phase on phase and group delay cannot be eliminated using a single seismogram. Second, the spectral amplitude shape can skew the group velocity measurements. This is seen in the dependence of the instantaneous frequency on the amplitude spectrum. Levshin (19XX) recommends associating the instantaneous frequency with the group arrival time, but this is not always effective. One could use the group times associated with two filter parameters α as developed here, but the difficulty of automatically associating the corresponding envelope peaks for different α's is not trivial. The spectral amplitude estimate is good only when the α is larger as distance increases.

3. Graphical Interfaces

To assist in the task of selecting phase velocity or ground velocity dispersion points from the possibilities created by the programs **`sacpom96`** and **`sacmft96`**, respectively, two interactive display programs were created for the purpose of selecting the desired SAC file, setting processing parameters, and then interactively selecting the dispersion points.

3.1 Test of multiple filter analysis

The discussion above shows that the choice of α is crucial to a good estimate of group velocity and spectral amplitude. To test the use of the program *sacmft96*, we generate synthetics with the following shell script

```
#!/bin/sh


set -x
HS=40
#this depth and model gives a spectra hole at 33-40 sec
STK=0
RAKE=0
DIP=90
AZ=45
NMODE=10

######
#           changing the RAKE to 45 removes some of the spectral hole
#           = 0 Strike slip if dip is 90
#           = 90 dip slip is dip is 45  -- good hole
######

cat > dfile << EOF
 500.0     1.000   2048    -1.0 8.0
1000.0     1.000   2048    -1.0 8.0
2000.0     1.000   2048    -1.0 8.0
3000.0     1.000   2048    -1.0 8.0
4000.0     1.000   2048    -1.0 8.0
8000.0     1.000   2048    -1.0 8.0
EOF


cat > model.d << EOF
MODEL
TEST MODEL
ISOTROPIC
KGS
FLAT EARTH
1-D
CONSTANT VELOCITY
LINE08
LINE09
LINE10
LINE11
HR      VP      VS  RHO QP  QS  ETAP ETAS FREFP FREFS
40.     6.0     3.5 2.8 0.0 0.0 0.0  0.0  1.0   1.0
00.     8.0     4.7 3.3 0.0 0.0 0.0  0.0  1.0   1.0
```

```
EOF

#####
#           Chapter 3
#####

sprep96 -M model.d -NMOD ${NMODE} -HS ${HS} \
            -HR 0 -d dfile -L -R
sdisp96
sregn96
slegn96
sdpegn96 -R -U -PER -TXT -XLOG -YMIN 2.5 -YMAX 5
sdpegn96 -L -U -PER -TXT -XLOG -YMIN 2.5 -YMAX 5
plotxvig < SREGNU.PLT
plotxvig < SLEGNU.PLT
spulse96 -d dfile -D -p -EQ -2 > file96
fmech96 -A ${AZ} -ROT -D ${DIP} -R ${RAKE} \
            -S ${STK} -M0 1.0E+20 < file96 > 3.96
f96tosac -B 3.96
cp B0101Z00.sac Z1.sac
cp B0201Z00.sac Z2.sac
cp B0301Z00.sac Z3.sac
cp B0401Z00.sac Z4.sac
cp B0501Z00.sac Z5.sac
cp B0601Z00.sac Z6.sac
rm -f B*.sac

for MODE in 0 1 2 3 4
            do
            spulse96 -d dfile -D -p -EQ -2 -M ${MODE} | \
              fmech96 -A ${AZ} -ROT -D ${DIP} -R ${RAKE} \
              -S ${STK} -M0 1.0E+20 > 3.96.${MODE}
            f96tosac -B 3.96.${MODE}
            cp B0101Z00.sac Z1.${MODE}.sac
            cp B0201Z00.sac Z2.${MODE}.sac
            cp B0301Z00.sac Z3.${MODE}.sac
            cp B0401Z00.sac Z4.${MODE}.sac
            cp B0501Z00.sac Z5.${MODE}.sac
            cp B0601Z00.sac Z6.${MODE}.sac
done
rm -f B*sac
```

The following plots is obtained using *sacmft96* and the trace at a distance of 4000 km.

```
sacmft96 -f Z5.sac -PMIN 4.0 -PMAX 100.0 \
            -a0 100.0 -A -VMIN 2.0 -VMAX 5.0 -U cm -R -S
```

This command says to use the SAC file *Z5.sac*, use the automatically generated del periods between *4.0* and *100.0* seconds, the filter parameter $\alpha = 100.0$, contour with absolute amplitudes,-A, and the plot group velocities in the range of *.0* and *5.0 km/s*. The input traces units are in *cm*, The trace is a Rayleigh wave, *-R*. Provide a color shaded plot, *-S*. The output graphic is in the file *MFT96.PLT* which is shown in Figure 4.

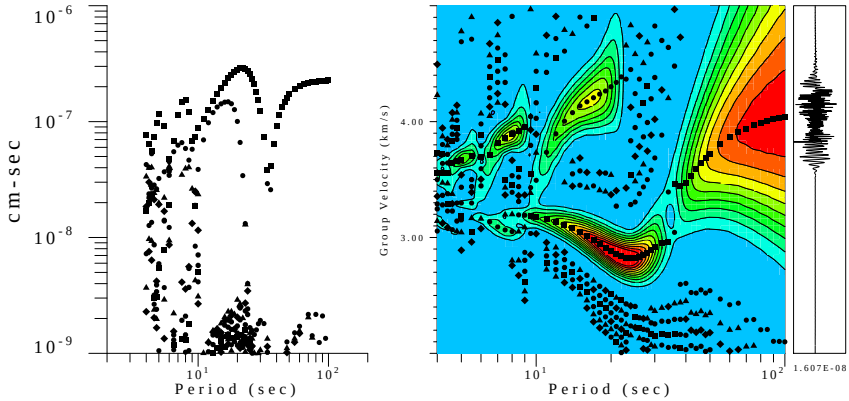


Fig. 4. Graphic output of sacmft96. An α=100 is used.

Figure 5 overlays the model predicted group velocity dispersion and the modal spectral amplitudes

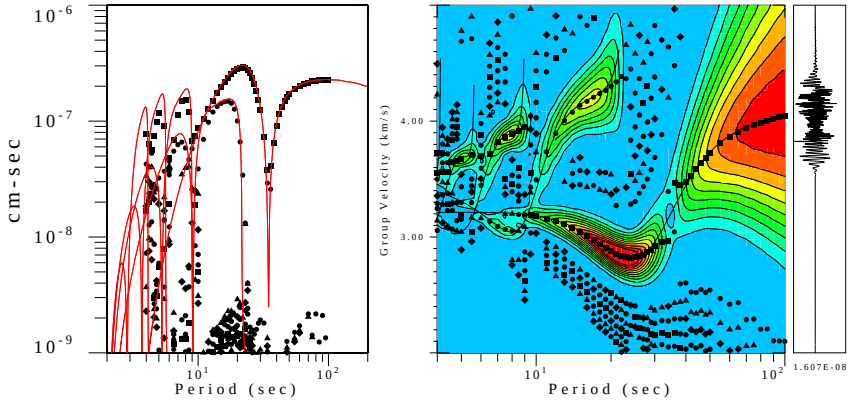


Fig. 5. Overlay of true group velocity dispersion and modal spectral amplitudes on the sacmft96 output.

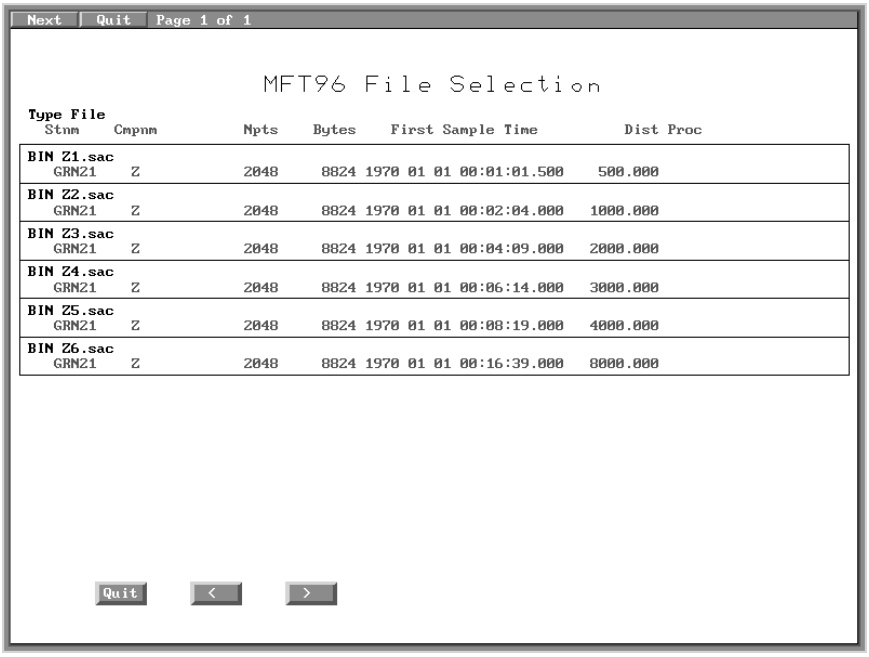
3.2 do_mft

The most time consuming part of multiple filter analysis is the need for manually selecting the correct dispersion from the program output. This is made easier by the use of the program *do_mft* what permits selection of the SAC file to process, defining the trace units and filter parameters, interactive identification of modes, choice of phase match filtering, and saving of processing results. The following figures present several of

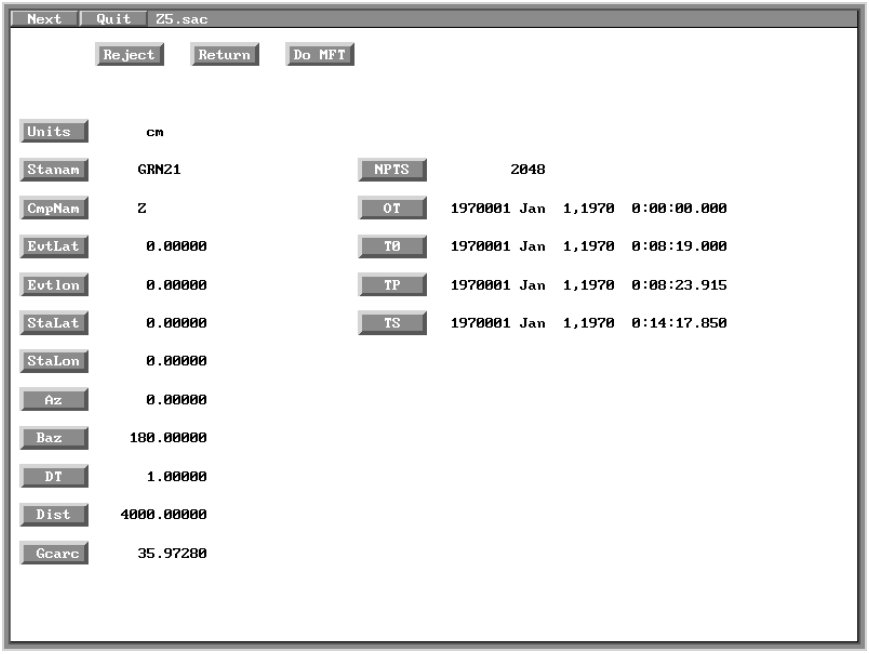
the menus. One can start the program with the simple command

```
do_mft *
```

whereby the program looks at all files in the current directory to determine whether they are SAC files. The result is the initial screen

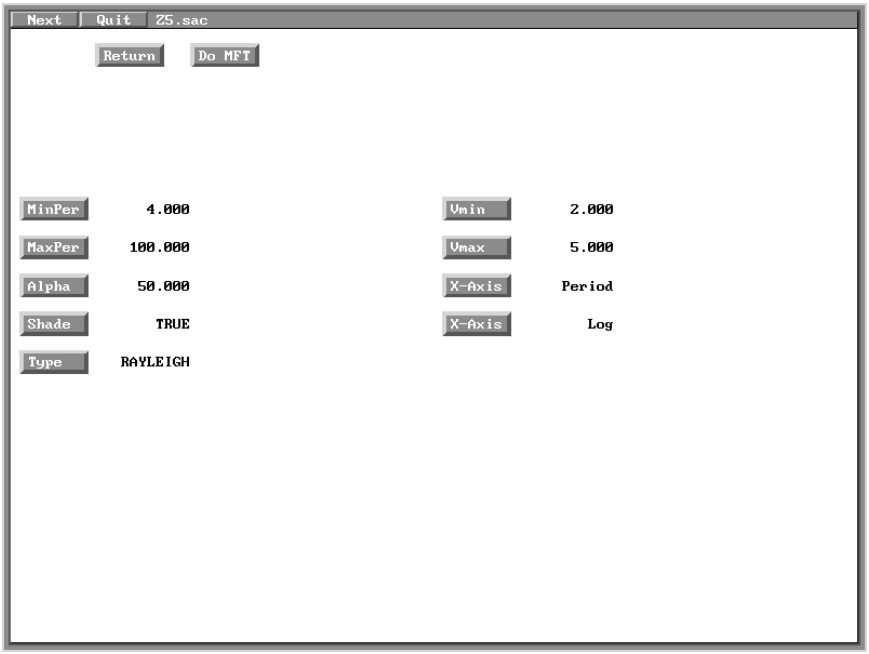


The second screen permits one to define the units and to review, but not change, the contents of the SAC header. The DIST is required.

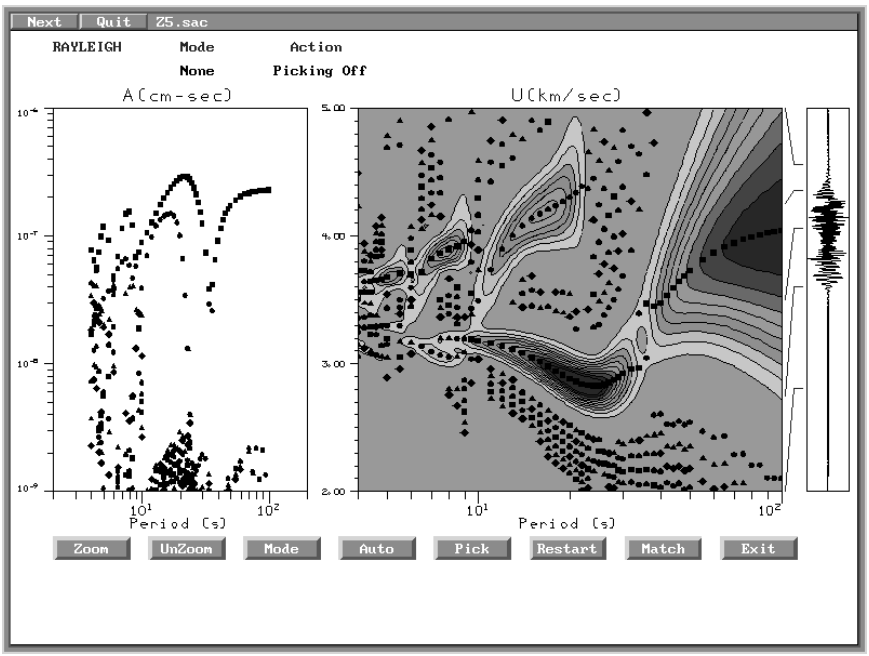


Selecting the *Do MFT* button, leads to the next stage, whereby one can change the periods for processing, the filter parameter α, the shading, and the wave type (*UNKNOWN*,

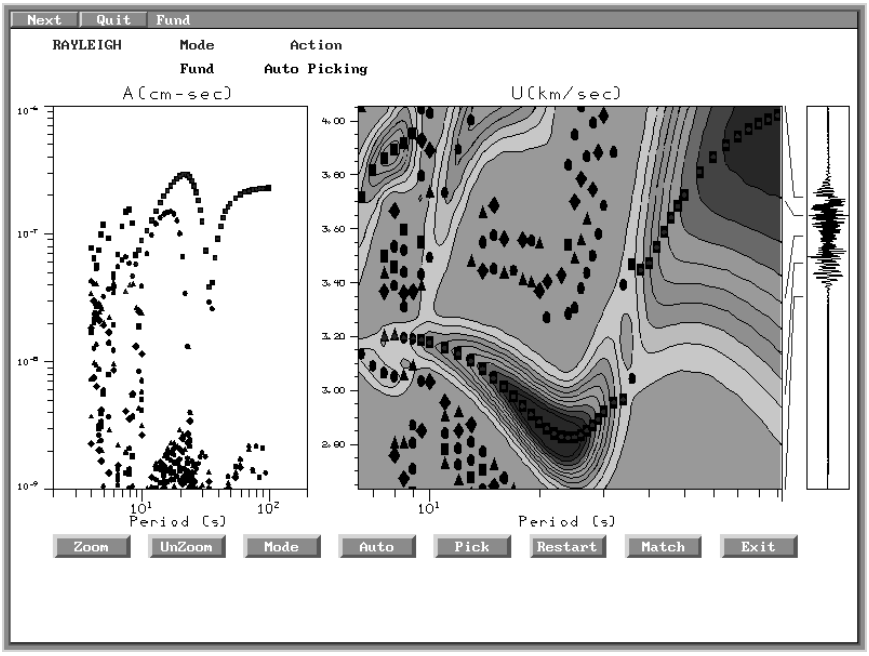
LOVE or *RAYLEIGH*), and the plotting parameters.



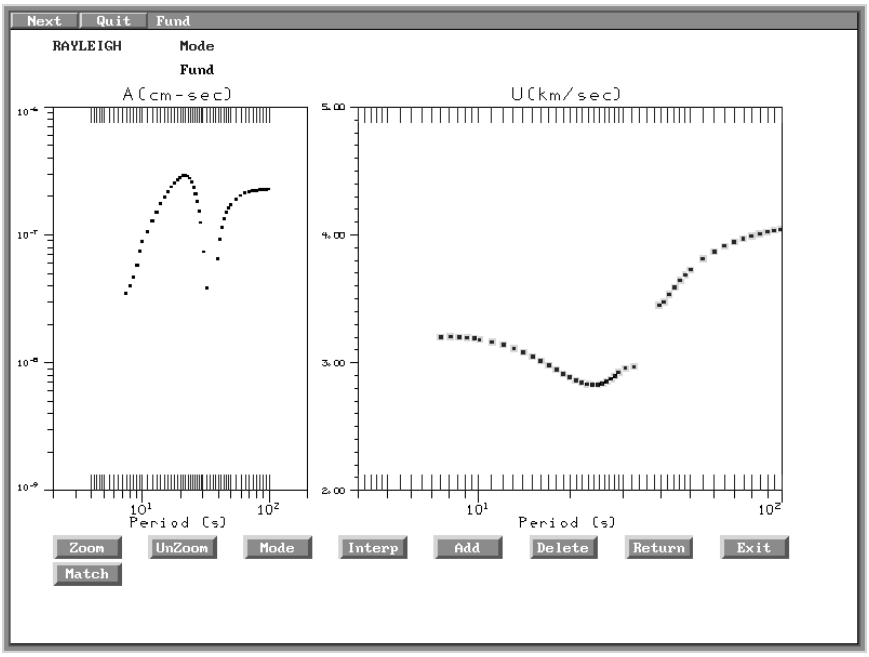
If the *Do MFT* is again pressed, the program *sacmft96* is run in the background to create three files: the dispersion file, the graphic plot, and an index file to the graphic. *do_mft* displays this together with processing buttons.



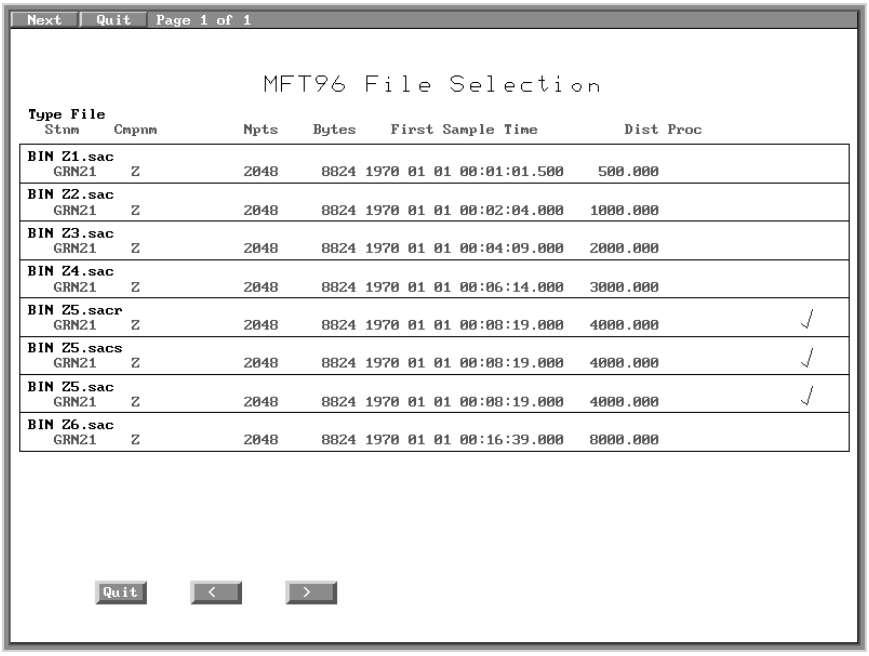
One can zoom in on a portion of the dispersion plot:



define the mode and then select the dispersion values a single point at a time, *Pick* or by finding values near a connected line, *Auto*. If desired one move to the phase match filter stage, *Match*



which requires the specification of a single mode. After phase match filtering the initial file menu is updated so that one could perform the multiple filter analysis on the presumably single mode trace.



3.3 Recommendations for multiple filter analysis

The example was purposely chosen because the focal mechanism yielded a spectra hole near a period of 35 seconds. A value of $\alpha < 100$ would have shown a greater bias in the group velocity estimate near this period. Levshin et al (19XX) recommended that the value of α change with distance. As a result of this simple experiment, the following choices may be adequate for the period range of 4 - 100 sec:

Distance Range	α
1000	25
2000	50
4000	100
8000	200

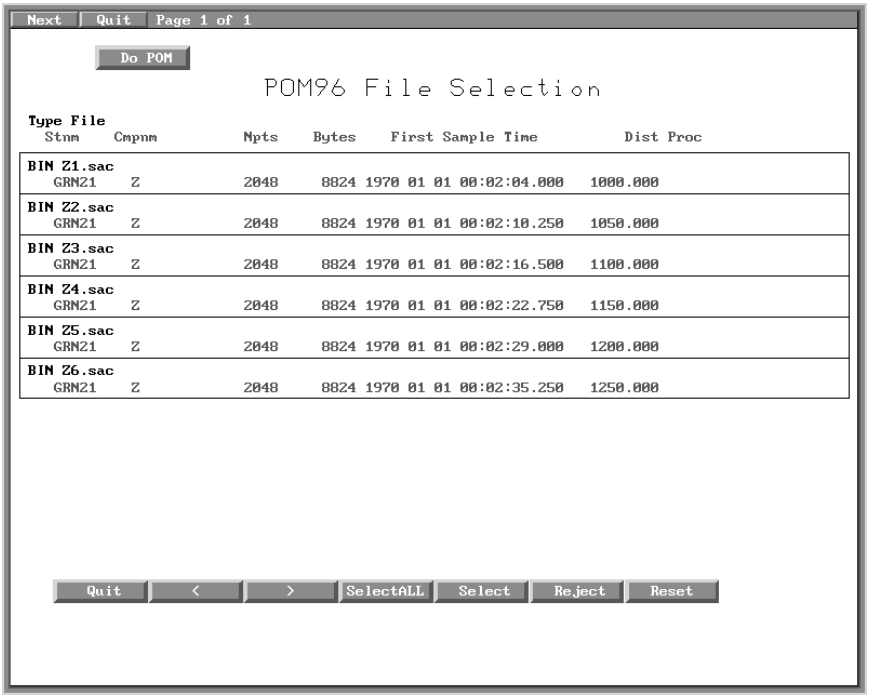
3.4 do_pom

As indicated in §2.2, the output of **`sacpom96`** consists of possible dispersion values in the file *pom96.dsp* a plot *POM96.PLT* and a control file *pom96.ctl*. The program **`do_pom`** controls the operation of **`sacpom96`** and also permits an easy selection of dispersion values. The operation of **`do_pom`** is very similar to that of **`do_mft`** since many routines are shared. The following figures provide guidance on the use of this program.

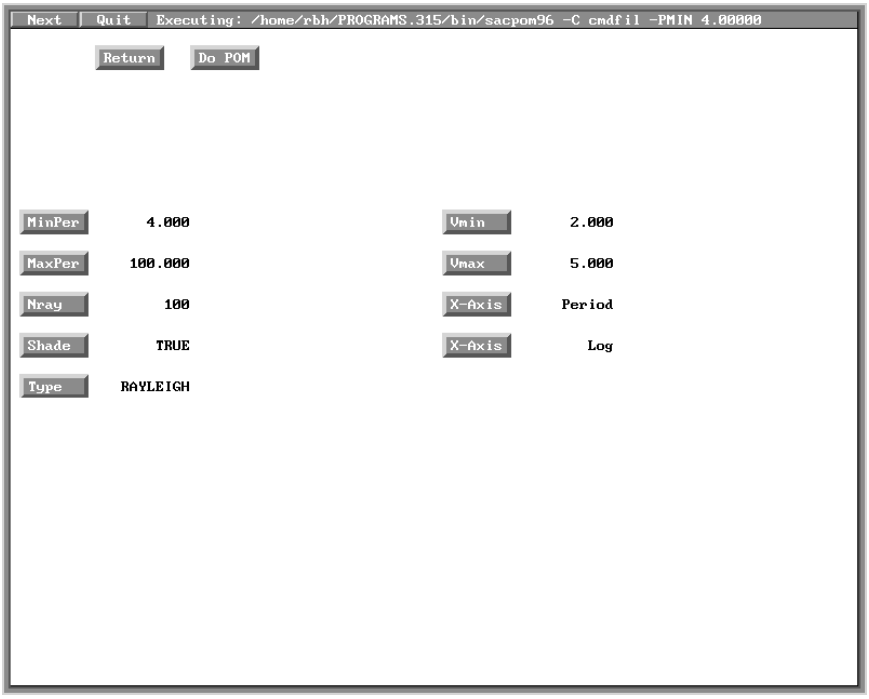
To start the program, enter the command

```
do_pom *
```

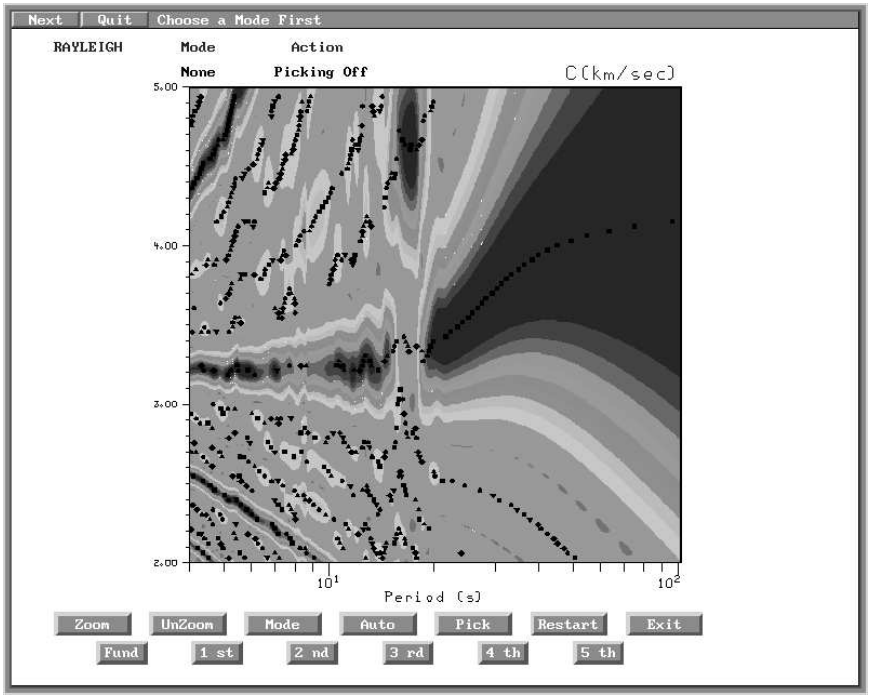
or just `do_pom sacfiles`. The program scans all programs listed on the command line to determine if they are SAC files. The initial menu displayed is



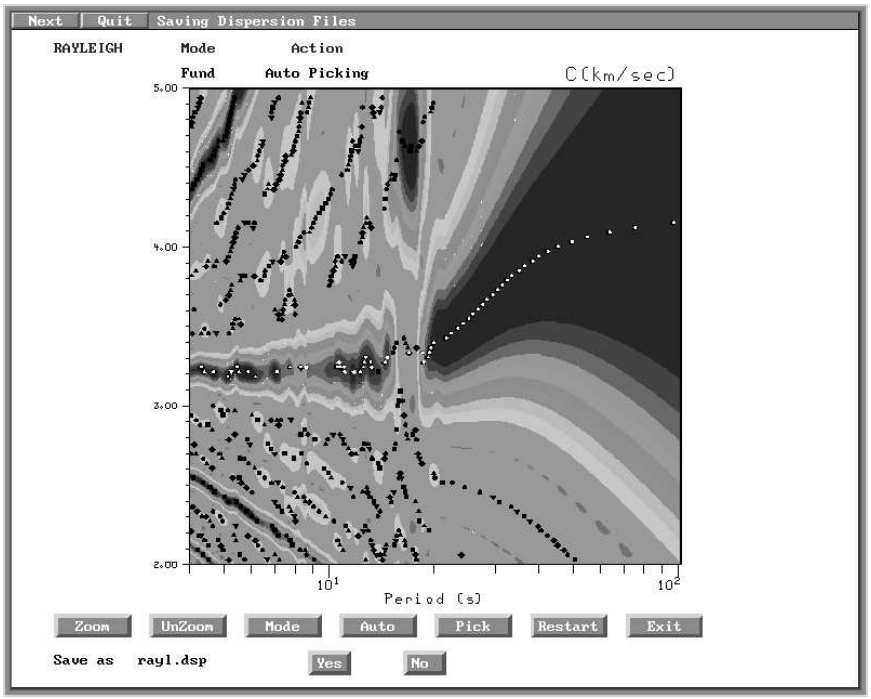
Initial display. If all SAC files are to be click on "SelectALL." If some are not desired, click "Reject" and then click on the appropriate box. An accepted file is indicated by a red ’X’ and a rejected by an ’X’. When the trace selection is complete, click on the "Do POM" button at the top.



The second page permits one to define the command line parameters for **sacpom96f1. When the "Do POM" button at the top is pressed, the programs sacpom96** is run in the background with the command line indicated in the very top margin.



This appears when **`sacpom96`** is finished. The user must choose a mode, and then may engage in interactive selection of dispersion points using the "Pick" or "Auto" modes. The "Auto" mode uses a rubber-band cursor to select groups of data points. Of course one may "Zoom" or ""UnZoom." "Exit" indicates that picking is complete and the next page appears:



Since the wave type was defined as Rayleigh on the second menu, the user may choose to save the picks in the file *rayl.dsp*, which is in the format for use by the inversion programs **`surf96`** and **`joint96`**.

4. Data Preparation

Data preparation consists of two parts: preparing the SAC trace file for analysis and performing the analysis. Multiple filter analysis requires that the instrument response be removed and that the recording be reduced to ground displacement, velocity or acceleration in some frequency range.

A typical response file representation is in terms of poles and zeros. If such a file exists, *pz_file*, SAC can be used to remove the instrument response with the following commands:

```
read SAC_file
rtr
taper
transfer from polezero subtype pz_file to none freqlimits 0.01 0.02 0.2 0.4
 new_SAC_file
```

This operation removes linear trends from the data, and then removes the complex response in the frequency domain as represented by the pole-zero description. The freqlimits, here 0.01 0.02 0.2 0.4 as an example, band pass the signal to prevent emphasizing low frequency and high frequency noise. One should note these band limits since the dispersion points should only be believed in the frequency range between 0.02 and 0.20 Hz.

If the program SAC is not available, one can use the pole-zero with the program **`sacfilt`** to accomplish the same result.

One should be careful of the meaning of the pole-zero response file. If it is obtained from AUTODRM software, then the pole-zero response in SAC format will present the displacement sensitivity in units of counts/micron. If the pole-zero response is desired in GSE format, then the units are counts/nanometer, I believe. Read the manual.

If one uses an FDSN SEED response file with the program *evalresp* from IRIS, two files will be created: one a table of the amplitude and the other a table fo the phase response as a function of frequency. The *evalresp* program permits the user to select the output in units of counts/m, counts/m/sec or counts/m/s/sec. To remove the instrument response using these tables, one can use the program **`sacevalr`** with the SAC file.

It is up to the user to know what the units actually are for the response. This is essential for source studies. The program **`do_mft`** permits the user to define the units of the trace after response correction. One must just be very careful and consistent.

Instrument correction prior to phase velocity stacking is only required if the sensors have different responses.

For the program to work, the SAC file must have the time of first sample, origin time, distance and azimuth fields filled. If one uses SAC and fills the event and station latitudes and longitudes, these latter two values will be computed automatically.

The final step before inverting for earth structure is to merge the output of **`do_pom`** and/or **`do_mft`** into a `surf96` dispersion file. The `rayl.dsp` or `love.dsp` are already in the correct format:

```
SURF96 R C T  0       25.28     3.54550     0.00190     5.9999
SURF96 R C T  0       25.92     3.57580     0.00300     5.9998
SURF96 R C T  0       26.95     3.60610     0.00160     5.9999
SURF96 R C T  0       27.68     3.63640     0.00080     6.0000
SURF96 R C T  0       28.44     3.66670     0.00330     5.9998
SURF96 R C T  0       29.68     3.69700     0.00270     5.9999
SURF96 R C T  0       30.57     3.72730     0.00150     6.0000
SURF96 R C T  0       31.51     3.75760     0.00030     6.0000
SURF96 R C T  0       32.51     3.78790     0.00130     6.0000
SURF96 R C T  0       33.57     3.81820     0.00310     5.9999
SURF96 R C T  0       35.31     3.84850     0.00200     6.0000
SURF96 R C T  0       36.57     3.87880     0.00080     6.0000
```

However the output of **do_mft** is not in the surf96 format. The following lists three lines from a file, *BLONMBHT.dsp*, created by answering *YES* to the *Save* query in **do_mft**. The \ represents an extension because of the many columns in the output.

```
MFT96 L U  0          42     3.91380     0.22821  2819.1001   76.3  3.6300e-04 37.540001 \
 -118.879997 39.171902 -86.522202 0 1 41.119999 COMMENT: BLO BHT 2007 163 7 23
MFT96 L U  0          40     3.81550     0.20656  2819.1001   76.3  3.8180e-04 37.540001 \
 -118.879997 39.171902 -86.522202 0 1 39.340000 COMMENT: BLO BHT 2007 163 7 23
```

The columns of this output represent the following:

1	MFT96 tag	9	source-to-receiver azimuth	18	COMMENT
2	Rayleigh or Love	10	Spectral amplitude (cm-sec)	19	Station, component, date
3	U for group velocity	11	Event latitude		
4	Mode (0 for fundamental)	12	Event longitude		
5	Period (sec)	13	Station latitude		
6	Group velocity (km/sec)	14	Station longitude		
7	Error in group velocity*	15	Flag to represent peak		
8	Distance km	16	Instantaneous period		

The reason for this extensive output which is much more than required by the surf96 format, is that one may wish to use the spectra amplitudes to estimate source properties or attenuation, or one may wish to incorporate the measured dispersion into a tomographic study. The error is group velocity is not a true error, since the dispersion is based on only one observation. Since the Gaussian filter has a longer impulse response at longer periods, a reading error could be associated with a misplaced maximum - the number computed here assumes that the travel time can be mis-measured by one filter period.

The table given above can be easily converted into the surf96 format by using **awk**, a standard utility on UNIX/LINUX systems, in a simple example:

```
cat GRN21Z.dsp | \
  awk '{ printf "SURF96 %s %s X %s %s %s %s\n", $2, $3, $4, $5, $6, $7}'
```

which extracts

```
SURF96 R U X 0 85 3.98400 1.34900
SURF96 R U X 0 80 3.96800 1.26000
SURF96 R U X 0 75 3.95300 1.17200
```

from the GRN21Z.dsp file.

To combine different dispersion files, e.g., for Love and Rayleigh, one just concatenates them together using the UNIX **cat** or the DOS **COPY** command into one file, perhaps called disp.d.

5. surf96

The program **surf96** is based on the program **surf** initially written by David Russell in 1985. The use of the new surf96 and model96 file formats was incorporated into the new version. In addition, the graphics interface was improved to enhance the visualization of the inversion success in fitting the data.

5.1 Interactive control file setup

The following example shows the interactive dialog for the case that the dispersion file and initial earth model file do not exist. If either or both exist, then just enter the file name and the program proceeds. The user input is indicated by the *italic* font.

```
surf96
 Enter h,dcl,dcr
    h = fraction change in period to get group vel
        (0.005 is reasonable)
    dcl, dcr are phase velocity increment in root
        search for Love and Rayl respectively
0.005 0.005 0.005
  Enter 1 if variance based on residual or
        0 if variance based on obs std err
1
  Enter maximum number of Love gamma modes to process
       0 means DO NO PROCESS LOVE gamma data
2
  Enter maximum number of Love Phvel modes to process
       0 means DO NO PROCESS LOVE phase vel data
2
  Enter maximum number of Love Gpvel modes to process
       0 means DO NO PROCESS LOVE group vel data
2
  Enter maximum number of Rayl gamma modes to process
       0 means DO NO PROCESS RAYL gamma data
2
  Enter maximum number of Rayl Phvel modes to process
       0 means DO NO PROCESS RAYL phase vel data
2
  Enter maximum number of Rayl Gpvel modes to process
       0 means DO NO PROCESS RAYL group vel data
0
  Enter inversion technique
     0   invert for Vs :Va,rho fixed
     1 : invert for Vs :Poisson fixed, rho from Vp
1
  Enter name of model file
modl.d
  Interactively setting up initial model file:
 modl.d
 Is model flat (0) or spherical (1)
```

```
0
 Enter descriptive title for this model
Test model
  Enter d,a,b,rho,qa,qb
   d=0.0 or EOF  indicates halfspace and end of input
40 6 3.5 2.5 100 100
0 8 4.7 3.3 1000 1000
  Enter name of dispersion file
disp.d
 Enter ilvry,iporg,imode,per,val,dval
 ilvry=1(Love)
     =2(Rayleigh)
 iporg=1 (phase velocity km/s)
     =2 (group velocity km/s)
     =3 (gamma 1/km)
 imode (mode number) e.g., 0=fundamental, 1=first
 per=the period
   val=dispersion value, velocity or gamma
  dval=error in dispersion value
     (Enter 1.0 if stderr from residuals)
  NOTE: Enter all zeros or negative to terminate input
1 1 0 10.0 3.5 0.01
1 2 0 15.0 3.6 0.01
2 2 0 20.0 3.0 0.01
0 0 0 0 0 0
```

If this is successful, then the initial program menu will be presented. Before presenting this, the requested input in the initial setup must be described.

```
Enter h,dcl,dcr
```

Determination of surface wave dispersion requires a search in the frequency - phase velocity space of the surface-wave period equation. Is is know that all surface wave modes are bounded at the low end by some fraction of the smallest shear-wave velocity, or in the case of a surface fluid layer, the lowest compressional-wave velocity. The upper bound of the dispersion for a fixed period, is the halfspace shear-wave velocity. **dcl** and **dcr** are the search increments to find the roots of the period equation. If these numbers are too large, modes may be missed by jumping past them. If the numbers are too small, computations take too long. The value of *0.005* km/sec are acceptable for crustal studies, but could be made smaller when studying dispersion in low velocity sediments.

The parameter **h** is used to compute group-velocity partial derivatives. For example, $\partial \mathbf{U}/\partial \mathbf{V_S}$ can be computed by using $\partial \mathbf{c}/\partial \mathbf{V_S(f)}$ and $\partial \mathbf{c}/\partial \mathbf{V_S((1+h)f)}$. The recommended value of *0.005* seems adequate.

```
Enter 1 if variance based on residual or
      0 if variance based on obs std err
```

This relates to the computation of confidence limits in the earth model and also the weighting of the individual data. *Currently* only the surface wave data permits a tabulation of the confidence of any observation.

```
Enter maximum number of Love gamma modes to process
     0 means DO NO PROCESS LOVE gamma data
```

This sequence of 6 questions permits the user to invert subsets of the data contained within the dispersion file.

```
Enter inversion technique
   0   invert for Vs :Va,rho fixed
   1 : invert for Vs :Poisson fixed, rho from Vp
```

The first option, *0* was introduced for the special case of investigating receiver function in deep soil layers for which the P-velocity was known. The second case, *1*, iteratively inverts for the S-wave velocity and then updates the P-velocity using the Vp/Vs ratio of the initial model; the new density is computed from the new Vp using the Nafe-Drake relation.

After this the names of the earth model file and dispersion file names are requested. If the files do not exist, then they are interactively constructed. Once this is done interactively, the format of each is correctly prototyped and the user may use an editor to modify the files faster than using this interactive dialog.

5.2 Main menu

```
                              SURF96 MENU
 0- Display menu                      24- ASCII Q Resolving  (file name,lam)
 1- Run Dispersion                    27- ASCII Veloc Dispersion (file name)
 2- Run Velocity Inversion            28- ASCII Model File  (file name, lam)
 3- Run Q(beta) Inversion             29- ASCII Vel Resolving(file name,lam)
 4- Run Simultaneous Veloc-Q Inversion 30- (0) Fix Vp,(1) Fix Vp/Vs
 5- Set Thick(0)/Velocity(1) Inversion 31- Change dd(i), enter i,dd(i)
 6- Update Model (need lam)           32- Enter Damping Factor (lam)
 7- Plot RFTN/Dispersion/Velocity Model 35- Inversion: (0) Non-Causal (default)
 8- Plot Gamma / Qb inverse  Model                    (1) Decoupled Causal
 9- Plot Resolution Kernel                            (2) Coupled Causal
10- List Singular Values              36- Smoothing: (0) Global reset none
11- Gamma Data Partial Derivatives                   (1) Global reset diff
12- Gamma Data Dispersion             37- Reset Number of Iterations
13- List Q Model (need lam)           38- Temporary End
14- Q(beta) Resolving Kernel (need lam) 39- Permanent End
16- Velocity Data Partial Derivatives  40- Enter Sigv minimum
17- Velocity Data Dispersion           41- Enter Sigg minimum
18- List Velocity Model (need lam)     45- Show Velocity Weights
19- Velocity Resolving Kernels(need lam)46- Show Qinv     Weights
22- ASCII Gamma Dispersion (file name) 47- Show Inversion Controls
23- ASCII Q(beta) File (file name,lam) 48- Modify Individual Layer Smoothing
Enter Command at READY Prompt
ready
```

This menu appears once the control file *sobs.d* exists. If the dispersion and initial model do not exist, then error messages will appear.

This menu is hopefully self-explanatory. Entering one of these numbers at the **ready** prompt causes an action. Actions **45**, **46** and **47** present other menus.

If **45** is entered, and the no smoothing option was chosen, e.g., **36** with option **0**, then the velocity weights shown are

```
V  Model Weighting Parameters: Large value forces change at boundary or layer
  I   DD(I) Inv S     depth/range          I   DD(I) Inv S     depth/range
--------------------------------------------------------------------------------
  1    1.00 Lyr 1      5.00             |  8    1.00 Lyr 1     40.00
  2    1.00 Lyr 1     10.00             |  9    1.00 Lyr 1     45.00
  3    1.00 Lyr 1     15.00             | 10    1.00 Lyr 1     50.00
  4    1.00 Lyr 1     20.00             | 11    1.00 Lyr 1     55.00
  5    1.00 Lyr 1     25.00             | 12    1.00 Lyr 1     60.00
  6    1.00 Lyr 1     30.00             | 13    1.00 Lyr 1     60.00-   9999.00
  7    1.00 Lyr 1     35.00             |
--------------------------------------------------------------------------------
 Lyr - get velocity change at boundary
 Lyr - get velocity in layer
 S =0 Vp fixed, S=1 Vp/Vs fixed in layer
 Use option 30 to change how Vp obtained
 Use option 31 to change layer weight
 Use option 48 to change layer smoothing
 Use option 45 to redisplay this menu
```

Here the weight, DD, is indicated together with the depth range in the model that it applies to. To permit significant change in the model velocities in the 45-50 km depth range, one would just enter from the command line

surf96 31 10 100

or interactively

ready
31
13 layers: 1-13 for Vs 13-26 for Qbinv
Enter i
10
Current dd(10)= 1.
Enter New dd(10)
100
ready

If differential smoothing is used,**36** with option **1**, then one would see

```
V  Model Weighting Parameters: Large value forces change at boundary or layer
  I   DD(I) Inv S     depth/range          I   DD(I) Inv S     depth/range
--------------------------------------------------------------------------------
  1    1.00 Bdy 1      5.00             |  8    1.00 Bdy 1     40.00
  2    1.00 Bdy 1     10.00             |  9    1.00 Bdy 1     45.00
  3    1.00 Bdy 1     15.00             | 10    1.00 Bdy 1     50.00
  4    1.00 Bdy 1     20.00             | 11    1.00 Bdy 1     55.00
  5    1.00 Bdy 1     25.00             | 12    1.00 Bdy 1     60.00
  6    1.00 Bdy 1     30.00             | 13    1.00 Lyr 1     60.00-   9999.00
  7    1.00 Bdy 1     35.00             |
--------------------------------------------------------------------------------
 Bdy - get velocity change at boundary
 Lyr - get velocity in layer
 S =0 Vp fixed, S=1 Vp/Vs fixed in layer
 Use option 30 to change how Vp obtained
 Use option 31 to change layer weight
 Use option 48 to change layer smoothing
 Use option 45 to redisplay this menu
```

The display is slightly different, since the DD(I) controls the change in velocity across a boundary. This display shows the depth of that boundary. To emphasize a Moho at a depth of 35 km, one would use

surf96 31 7 100

The end of these listing also so the meny cimmands for changing the parameters in each layer.

The menu produced using option **47** summarizes the current processing parameters.

```
Inversion controls for surf96
Cmd   Value   Description
        1       1 Variance based residual of fit
                0 Variance based on observed std observation
        2       Maximum number of Love     modes to use
        2       Maximum number of Rayleigh modes to use
        5       Current iteration
       -1       Number of receiver functions to be inverted
        2       2 last inversion for Vs
                3 last inversion for Q inverse
                4 last inversion for Vs-Q inverse
  5     1       0 Layer thickness  inversion
                1 Layer velocity/Q inversion
 32   1.000     Damping value    (default value 1.0)
 35     0       0 non-causal Vs - Q relation (default)
                1 Decoupled causal
                2 Fully coupled causal
 36     1       0 No smoothing constraint
                1 Differential smoothing constraint
 40   0.0500   Std error of fit floor for velocity disp
 41  0.500E-04  Std error of fit floor for gamma disp
--------------------------------------------------------------------------------
 Use menu command cmd to change value
```

A few control parameters listed cannot be changed interactively. They are defined during the initial creation of the `sobs.d` control file. Other parameters can be changed during the iterative inversion.

Version 3.16 introduced a new choice for option **36** and changes the meaning of this option slightly. When the program is initially interactively started, differential smoothing is assumed. At any time a **36** with choice **0** will force a no-smoothing inversion on all layers, a **36** with choice **1** will force a differential smoothing on all layers except the bottom layer.

A new option **48** - *Modify Individual Layer Smoothing* was also introduced. The purpose if this is to fine tune the type of smoothing for a given layer or boundary by switching between smoothed and non-smoothed layers/boundaries. With this option one can fix a layer velocity and still permit smoothing else where.

5.3 Program operation

The program can be run interactively or from the command line. For repeated runs or to reproduce a run running from a command line may be easier, especially if the sequence of commands is placed in a shell script (batch file)For repeated runs or to reproduce a run running from a command line may be easier, especially if the sequence of commands is placed in a shell script (batch file). The only slight difference is that creation of the output file in interactive mode, Option **28** asks the user for a command text to be placed in the **model96** header.

6. Example

This example uses the same dispersion information presented in Chapter 3 and the same receiver functions presented in Chapter 4. These synthetic data sets were created for the single layer over a halfspace model

```
MODEL
TEST MODEL
ISOTROPIC
KGS
FLAT EARTH
1-D
CONSTANT VELOCITY
LINE08
LINE09
LINE10
LINE11
HR      VP      VS RHO  QP    QS   ETAP ETAS FREFP FREFS
40          6     3.5 2.5 200.0 100.0 0.0  0.0  1.0   1.0
 0          8     4.7 3.3 900.0 500.0 0.0  0.0  1.0   1.0
```

To demonstrate operation of the program, the following shell script was run. It is assumed that the control file `jobs.d` and the initial earth model, dispersion and receiver function files exist in the current directory. The contents of `jobs.d` are

```
    0.00499999989  0.00499999989  0.  0.00499999989  0.
       1    2    2    2    2    2    2    0    1    0
 modl.d
 ../MKSURF/disp.d
 rftn.lst
```

The initial model, `modl.d` consists of a halfspace -

```
MODEL
TEST MODEL
ISOTROPIC
KGS
FLAT EARTH
1-D
CONSTANT VELOCITY
LINE08
LINE09
LINE10
LINE11
HR      VP      VS  RHO QP  QS  ETAP ETAS FREFP FREFS
 5            84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5            84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5            84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5            84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5            84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5            84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5            84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5            84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5            84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5            84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5            84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5            84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 0            84.72 3.3 0.0 0.0 0.0  0.0  1.0   1.0
```

```
#!/bin/sh

#####
#              clean up
#####
surf96 39

#####
#              define damping
#####
surf96 32 1.

#####
#              Select differential smoothing
#####
surf96 36 1

#####
#              set up repeated run for 5 iterations
#####
surf96 1 2 6 1 2 6 1 2 6  1 2 6 1 2 6 1 2

#####
#              plot the model and show the data fit after 5 iterations
#####
srfphv96
plotnps -EPS -K -F7 -W10 <  SRFPHV96.PLT > figsrf1.eps

#####
#              save current model
#####
surf96 28 modl.out

#####
#              compare the individual models from the inversion
#              to the true model
#####
shwmod96 -K 1 -W 0.05 model.true
mv SHWMOD96.PLT T.PLT

shwmod96 -K -1  tmpmod96.???
mv SHWMOD96.PLT I.PLT

cat T.PLT I.PLT > IT.PLT
plotnps -EPS -K -F7 -W10 < IT.PLT > figsrf2.eps
```

The command sequence

surf96 1 2 6 1 2 6 1 2 6 1 2 6 1 2 6 1 2

sets up five complete iterations consisting of **1** computing the partial derivatives, **2** performing the singular value decomposition, and **6** updating the model. The output for these iterations consists of the following:

```
Dispersion  fit (vel)         std err  :    0.3089 (km/s)
Dispersion  fit (vel)    mean residual :   -1.0676 (km/s)
Dispersion  fit (vel)    avg |residual|:    1.0676 (km/s)
Percent of Signal Power Fit (Disp)     :  89.05252% for   68 SW Obs
RMS change in S-wave velocity model    :   1.2439 km/sec
ITERATION     1 done: UPDATING V
------------------------------------------------------------------------
Dispersion  fit (vel)         std err  :    0.2187 (km/s)
Dispersion  fit (vel)    mean residual :    0.3442 (km/s)
Dispersion  fit (vel)    avg |residual|:    0.3443 (km/s)
Percent of Signal Power Fit (Disp)     :  98.77078% for  160 SW Obs
RMS change in S-wave velocity model    :   0.3918 km/sec
ITERATION     2 done: UPDATING V
------------------------------------------------------------------------
Dispersion  fit (vel)         std err  :    0.1029 (km/s)
Dispersion  fit (vel)    mean residual :   -0.0178 (km/s)
Dispersion  fit (vel)    avg |residual|:    0.0776 (km/s)
Percent of Signal Power Fit (Disp)     :  99.91705% for  152 SW Obs
RMS change in S-wave velocity model    :   0.0656 km/sec
ITERATION     3 done: UPDATING V
------------------------------------------------------------------------
Dispersion  fit (vel)         std err  :    0.0773 (km/s)
Dispersion  fit (vel)    mean residual :    0.0140 (km/s)
Dispersion  fit (vel)    avg |residual|:    0.0575 (km/s)
Percent of Signal Power Fit (Disp)     :  99.95382% for  156 SW Obs
RMS change in S-wave velocity model    :   0.0519 km/sec
ITERATION     4 done: UPDATING V
------------------------------------------------------------------------
Dispersion  fit (vel)         std err  :    0.0590 (km/s)
Dispersion  fit (vel)    mean residual :    0.0058 (km/s)
Dispersion  fit (vel)    avg |residual|:    0.0426 (km/s)
Percent of Signal Power Fit (Disp)     :  99.97384% for  158 SW Obs
RMS change in S-wave velocity model    :   0.0307 km/sec
ITERATION     5 done: UPDATING V
------------------------------------------------------------------------
Dispersion  fit (vel)         std err  :    0.0484 (km/s)
Dispersion  fit (vel)    mean residual :    0.0037 (km/s)
Dispersion  fit (vel)    avg |residual|:    0.0345 (km/s)
Percent of Signal Power Fit (Disp)     :  99.98243% for  158 SW Obs
```

This display shows the interation number, and parameters describing the degree of fit to the data. The best fit is defined as that for which the Dispersion fits are 0.0 and the Percent of signal power fit is 100%.

Option **1** computes the predictions and partial derivatives for the current model. The dispersion information is summarized by given the mean difference between observed and predicted dispersion, the standard error of fit and the L1 norm of the fit.

The changes in the model obtained for the current model can be seen by invoking

```
surf96 18
  INVERSION FOR S-VEL
  Estimated data standard dev.:  0.833521664
  RMS model perturbation:  0.0159389861
      DEPTH THICKNESS     S-VEL SIG DELVL RESL in H DEL (VEL)
    2.5000    5.0000    3.4950 0.654E-02 0.105E+02    0.0068
    7.5000    5.0000    3.5062 0.544E-02 0.119E+02    0.0083
   12.5000    5.0000    3.4790 0.433E-02 0.167E+02    0.0062
   17.5000    5.0000    3.4235 0.393E-02 0.227E+02   -0.0025
   22.5000    5.0000    3.3920 0.420E-02 0.263E+02   -0.0160
   27.5000    5.0000    3.4587 0.470E-02 0.282E+02   -0.0255
   32.5000    5.0000    3.6673 0.523E-02 0.287E+02   -0.0210
   37.5000    5.0000    3.9783 0.575E-02 0.275E+02   -0.0025
   42.5000    5.0000    4.2958 0.609E-02 0.249E+02    0.0198
   47.5000    5.0000    4.5383 0.609E-02 0.217E+02    0.0358
   52.5000    5.0000    4.6730 0.565E-02 0.239E+02    0.0411
   57.5000    5.0000    4.7023 0.493E-02 0.200E+02    0.0356
   62.5000    0.0000    4.6551 0.462E-02 0.516E+01    0.0214
```

This display shows that the shear-wave velocity of the first layer would be increased by 0.0068 km/sec for the current damping value (**32**). One can look at the model graphically using option **7** or **9**.

This script created two figures after the 5 iterations. Figure 6 shows the starting model, the current model and the observed and predicted dispersion.

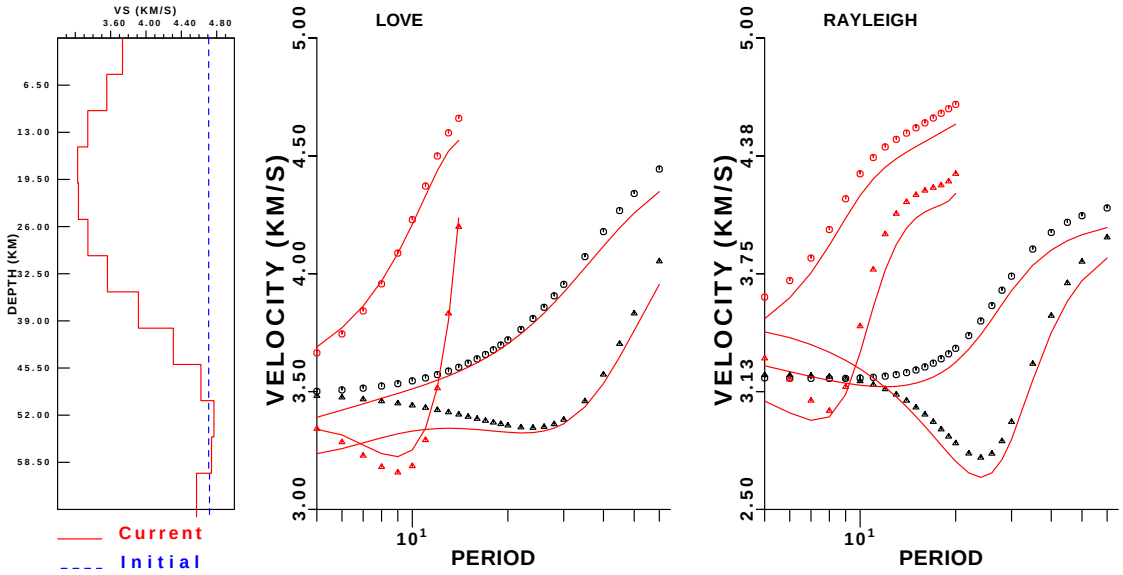


Fig. 6. Output using Option **7**. Dispersion data are indicated symbols. Predictions by the solid curves.

The earth models created at each iteration are saved and numbered with the iteration: `tmpmod96.000`, ..., `tmpmod96.005`. Since the true model is known, Figure 7 compares the true answer with the result of each step of the inversion.

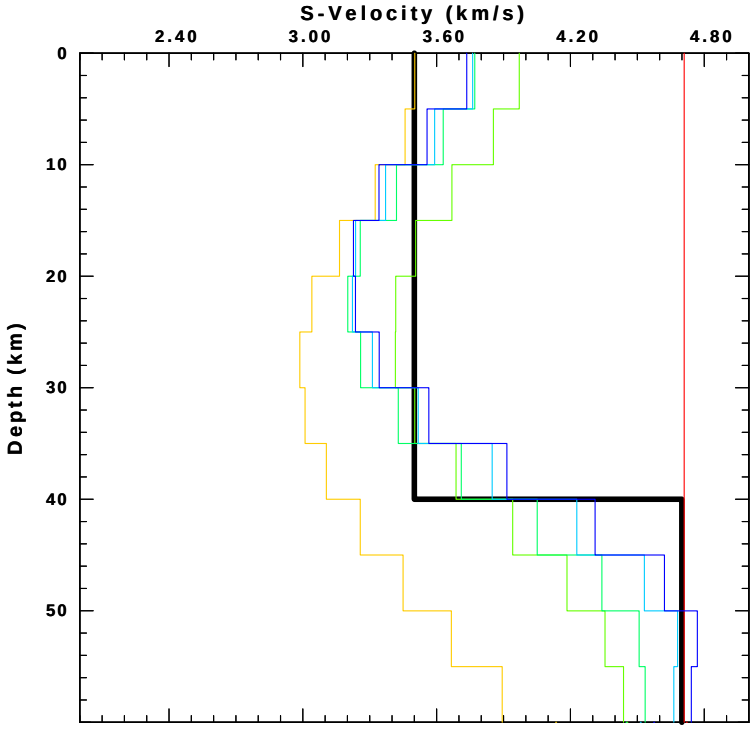


Fig. 7. True model, solid black line; iteration models, red is initial, and blue is the final model.

Figure 8 presents the resolution kernel corresponding to the last computed model. The is obtained using option **9** from within the program or the command line

surf96 9

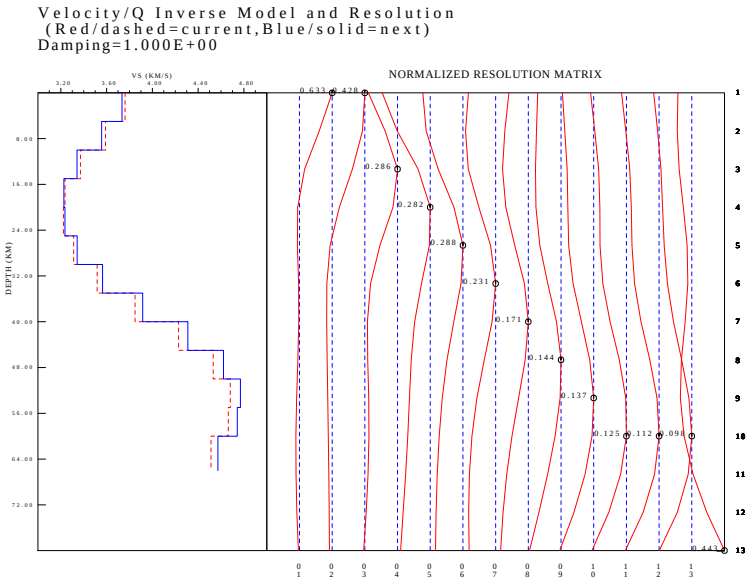


Fig. 8. Resolution kernels. Note this depends on the damping value. For a smoothed inversion, the kernels will not be symmetric.

7. Discussion

The simplest way to run the program is just to try a sequence of commands of the form

surf96 1 2 18 6

8. References

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McMechan, G. A. and M. J. Yedlin (1981). Analysis of dispersive waves by wave field transformation, *Geophysics* **46,** 869-874.

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CHAPTER 4
INVERSION OF RECEIVER FUNCTIONS

1. Introduction

A property of elastic wave propagation in an isotropic medium is that P or SV waves incident at a medium boundary are converted to P and SV waves upon reflection or transmission. This property leads to complicated surface recordings for a layered medium. The receiver function is a time series constructed from the surface recordings in a manner that focuses on the layered structure itself.

Consider the surface 3-component recording of a distant earthquake, such as the traces shown in Figure 1.

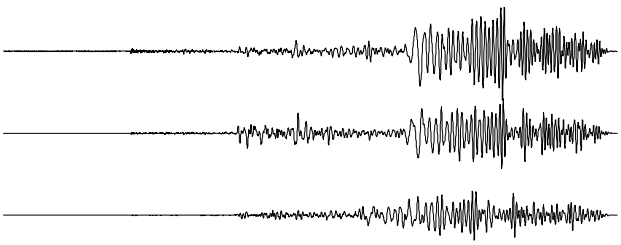

Fig. 1. SLM recordings of the destructive August 17, 1999 earthquake in Turkey. A total of 3600 seconds of ground motion is displayed. Note the small amplitude of the initial P waves.

The initial P-wave motion, shown in Figure 2, demonstrates the complexity of the motion, which is the composite effects of the source, the 84° propagation path, and the local structure beneath St. Louis, Missouri. The vertical component has the largest amplitude, with the radial motions less than one-half the vertical. The transverse component motions are even smaller. For an ideal earth model, the transverse component would be zero and the radial a fraction of the vertical, due to the teleseismic P-wave's near

vertical incidence at the surface.

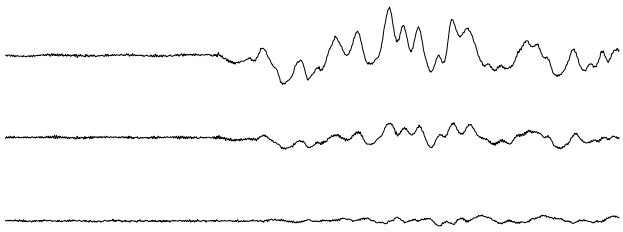

Fig. 2. Plot of initial P-wave motion at SLM.

We can ask an interesting question though, "What are the transfer functions that would convert the vertical component signal to the radial and horizontal component signals?"

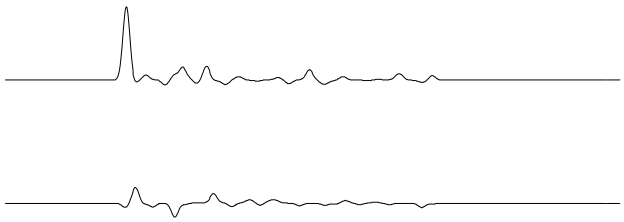

Fig. 3. Plot of Z→R and Z→T transfer functions.

2. Joint Inversion Mathematics

3. Data Preparation

3.1. Data Organization

The basic idea governing receiver-function data organization is to group the signals into "clusters" that sample the same structure. Most observations are also naturally clustered by the distance and azimuth of appropriate sources.

Waves approaching a seismometer from different directions may sample very different structures. An extreme example is shown to the right.

At most stations, the structure varies with azimuth and even in the simplest cases, the response can vary with distance from the station.

We usually group the observations by azimuth, then distance. We stack, or average, waveforms from the same azimuth and distance range, although at times, when the coverage is very broad, studying the response as a more continuous function of azimuth or distance is a nice way to study the structure.

3.2. Instrument Responses and Gains

For the Rftn analysis, you need three-component observations, preferably with a wide bandwidth. If the instrument response of the components is matched, you do not have to remove the instrument effects before proceeding, but you must insure that the gains are equalized before proceeding to the receiver function deconvolution. To correct for differences in instrument gain, use the scalar division command "div" in SAC. If the instruments are not matched, you should remove or replace them with a set of uniform instrument responses, which you can do with the "transfer" command in SAC.

For example, suppose I had three seismograms from SNZO: myevent.z myevent.n myevent.e The instrument responses for the station are matched, with the exception of a small variation in the gain. To remove the slight difference, I would execute the following SAC commands:

```
r myevent.e myevent.n myevent.z
div 6.465442 6.478608 6.307450
w over
```

The gains are actually the values above x 10^13, but the constant factor doesn't matter, only the variable coefficient.

3.3. SAC Header Values

Once you have the instrument response worked out, you next need to supply the information necessary for SAC to rotate the horizontal seismograms into the theoretically based radial and tangential directions. Specifically, you set several header variables in each of your waveforms:

- The event latitude and longitude
- The component azimuth

• The component incident angle

To set these values for each seismogram, you use the change header command:

```
SAC Commands        Comments
r my_seismogram     read in the seismogram
ch evla _____
ch evlo _____       change header values
ch evdp _____       (fill in the blanks)
ch cmpaz _____
ch cmpinc _____
wh                  Overwrite the header
```

The "r" command reads the seismogram into memory. The "wh" command writes the header, saving the appropriate information. Once this information is stored in the header, SAC will automatically compute the distance and back azimuth of the observation. Latitude is positive north of the equator, longitude is positive east of Greenwich, England. Enter the numbers in decimal degrees. The cmpaz is the component azimuth also in decimal degrees, north is 0°, east is 90 °, etc. After you have made these additions to the file header values, you can view the great-circle distance and back azimuth using the "lh" command

```
lh gcarc baz
```

3.4. Windowing the Data

The final data-preparation stage consists of windowing the P waveform from the pre-signal noise and the rest of the seismic signal. The amount of record that you use depends somewhat on the seismogram. You want to isolate the P-waveform from the remaining signal. For the usual teleseismic distances (30° to 95°) you are usually safe by using about 60 seconds of signal "leader" and 60 seconds of signal following the onset of the P wave. The precise duration can vary if needed, these are typical values. At times details in the estimated receiver function may be sensitive to substantial (10s of seconds) variations in length, and you can get a feel for the variations by comparing several lengths of signal during the source equalization procedure.

To cut the desired part of the seismogram from SAC, you use the "cut" command. The SAC time reference system is based on two different times. A reference time which is stored in the header "kzdate and kztime" and the begin time of the trace. For our purposes, the begin time is most important, since we will use that value to reference the cut. Suppose that the P-wave onset occurs at about 80 seconds into the seismogram, then we would want to window the signal between 20 and 140 seconds. We also should remove the mean and taper the ends of the signal to avoid signal processing artifacts later in the processing. The following SAC commands with perform the desired operations.

```
r  myseismogram.e myseismogram.n myseismogram.z
qdp  off
p1
cuterr fillz
cut 20 140
r
rmean
w over
```

Normally if a cut lies beyond the bounds of the data, SAC will report an error and the bad

start cut and/or end cut will be replaced with the file begin and/or file end. Further automatic use of the trace output will be in error. The *cuterr fillz* will avoid this error by placing zeros at the beginning and/or end of the trace. This is satisfactory unless the file has a DC offset, in which case an undesirable discontinuity may be introduced. In this case read the file first, remove the mean, then save the file.

Once the data are windowed, we are ready to calculate the the radial and tangential receiver functions. See the Source Equalization Page for a discussion of the procedure.

3.5 A SAC Macro For Pre-Processing Raw Observations

If you have many waveforms to prepare, the above will get tiring. Here is a SAC macro that I have used to handle cutting, detrending, tapering, and separating noisy observations from the better signals. The data are stored in files named: *BHZ, *BHN, *BHE, where the * means "whatever". Make a directory with a copy of your observations (Data files will be overwritten if you execute this macro - work with a copy of the data!) and execute this macro.

The macro is interactive - you will be picking the approximate P onset from the vertical component using the PPK command in SAC. When the cursor appears, place it at the P-onset time and enter "t" "0" (that's a zero) to set the t0 header value. Enter "q" to go onto the next part of the macro, where you decide whether to keep the data or move them into the "Noisy" directory. You only need to identify the time the time within a few seconds, don't agonize over precision.

If the data are all noisy, just enter "q" and then enter an "t" later to move the signal to the "Trash" directory.

The mean and a trend are removed from the observations and a cosine taper is applied on the left and right fifth (about 25 seconds if you keep the time limits in the macro) of the signal.

You must edit the "div" line to put in the correct instrument gains, or just delete that line and correct the gains later.

```
The SAC Macro                                                  Comments

sc mkdir Trash                                                 Make directories.
sc mkdir GoodOnes
qdp off
ygrid on
do file wild *BHZ                                              Key on the
 setbb vert $file                                              vertical...
 setbb east '( CHANGE 'BHZ' 'BHE' %vert )'
 setbb north '( CHANGE 'BHZ' 'BHN' %vert )'                    Synchronize the
 r %vert %east %north                                          file start times.
 synch
 w over
 r %vert                                                       Pick the P onset &
 rmean                                                         mark it in the
 rtr                                                           header.
 ppk
 setbb t0 &1,t0                                                Cut the data 60 s
 r %vert %east %north                                          before and 90 s
```

```
    ch t0 %t0                                                  after the P onset.
    w over
    cuterr fillz
    cut t0 -60 t0 +90
    r %vert %east %north                                       Remove the mean, a
    rmean                                                      trend, and correct
    rtr                                                        the gain of each
    taper w 0.2                                                instrument.
    div 6.307450 6.465442 6.478608
    w over
    p1
    setbb resp (REPLY "Enter t to trash the file")             Move the files into
    if %resp eq "t" then                                       the directory
      sc mv %vert Trash                                        "GoodOnes" if they
      sc mv %east Trash                                        look usable, or
      sc mv %north Trash                                       into "Trash" if
    else                                                       they look really
      sc mv %vert GoodOnes                                     bad.
      sc mv %east GoodOnes
      sc mv %north GoodOnes

    endif                                                      Turn cut off and do
    cut off                                                    the next one.
   enddo
```

3.1 Data Selection

3.2 Data Preparation

3.3 Iterative Deconvolution

4. rftn96

Operation of the program requires the existence of the `robs.d` control file, a surface-wave dispersion file, and the list of the receiver functions to be inverted. If none of these exist, the program will permit the user to interactive create the files.

4.1. Data preparation

The receiver functions must be prepared as described in Chapter 4. It is essential that the Gaussian filter parameter, $ alpha $ be placed in USER0, the ray parameter in USER4 and the receiver function delay be mirrored in the B value of the SAC header for each receiver function.

4.2. Interactive control file setup

The following example shows the interactive dialog for the case that neither the initial earth model file nor the list of receiver functions exist. The user input is indicated by the

italic font.

```
rftn96
 Enter 1 if variance based on residual or
       0 if variance based on obs std err
0
  Enter inversion technique
     0   invert for Vs :Va,rho fixed
     1 : invert for Vs :Poisson fixed, rho from Vp
1
  Enter name of model file
modl.d
  Interactively setting up initial model file:
 modl.d
 Is model flat (0) or spherical (1)
0
 Enter descriptive title for this model
Test model
  Enter d,a,b,rho,qa,qb
   d=0.0 or EOF  indicates halfspace and end of input
40 6 3.5 2.5 100 100
0 8 4.7 3.3 1000 1000
  Enter name of receiver function  file list
rftn.lst
 Interactively setting up receiver function file list:
rftn.lst
Enter receiver function SAC binary file name, EOF to end
../MKRFTN/05.rfn
../MKRFTN/10.rfn
CTRL D        (for UNIX/LINUX, CTRL Z for DOS)
```

If this is successful, then the initial program menu will be presented. Before presenting this, the requested input in the initial setup must be described.

```
Enter 1 if variance based on residual or
      0 if variance based on obs std err
```

This relates to the computation of confidence limits in the earth model and also the weighting of the individual data. *Currently* only the surface wave data permits a tabulation of the confidence of any observation.

```
Enter inversion technique
   0   invert for Vs :Va,rho fixed
   1 : invert for Vs :Poisson fixed, rho from Vp
```

The first option, *0* was introduced for the special case of investigating receiver function in deep soil layers for which the P-velocity was known. The second case, *1*, iteratively inverts for the S-wave velocity and then updates the P-velocity using the Vp/Vs ratio of the initial model; the new density is computed from the new Vp using the Nafe-Drake relation.

After this the names of the earth model file, dispersion and receiver function list are requested. If the files do not exist, then they are interactively constructed. Once this is done interactively, the format of each is correctly prototyped and the user may use an editor to modify the files faster than using this interactive dialog.

4.3. Main menu

```
                                  RFTN96 MENU
 0- Display menu                       35- Inversion: (0) Non-Causal (default)
 1- Run Dispersion                                    (1) Decoupled Causal
 2- Run Velocity Inversion                            (2) Coupled Causal
 5- Set Thick(0)/Velocity(1) Inversion  36- Smoothing: (0) Global reset none
 6- Update Model (need lam)                           (1) Global reset diff
 7- Plot RFTN/Dispersion/Velocity Model 37- Reset Number of Iterations
 9- Plot Resolution Kernel              38- Temporary End
10- List Singular Values                39- Permanent End
18- List Velocity Model (need lam)      42- Enter Sigr minimum
19- Velocity Resolving Kernels(need lam)43- Joint Weighting: 0=RFTN <--> 1=SRFW
28- ASCII Model File  (file name, lam)  44- 2x RFTN computation (0) no, (1) yes
29- ASCII Vel Resolving(file name,lam)  45- Show Velocity Weights
30- (0) Fix Vp,(1) Fix Vp/Vs            47- Show Inversion Controls
31- Change dd(i), enter i,dd(i)         48- Modify Individual Layer Smoothing
32- Enter Damping Factor (lam)          49- Show RFTN information and weight
33- Enter Tmin for RFTN (default  -5 s) 50- Change individual RFTN weight
34- Enter Tmax for RFTN (default  20 s)
Enter Command at READY Prompt
                                  RFTN96 MENU
ready
```

This menu appears once the control file *robs.d* exists. If the dispersion, initial model and file listing receiver functions do not exist, then error messages will appear.

This menu is hopefully self-explanatory. Entering one of these numbers at the **`ready`** prompt causes an action. Actions **45**, **46** and **47** present other menus.

If **45** is entered, and the no smoothing option was chosen, e.g., **36** with option **0**, then the velocity weights shown are

```
V  Model Weighting Parameters: Large value forces change at boundary or layer
  I   DD(I) Inv S     depth/range          I   DD(I) Inv S     depth/range
-------------------------------------------------------------------------------
  1    1.00 Bdy 1      5.00             |  8    1.00 Bdy 1     40.00
  2    1.00 Bdy 1     10.00             |  9    1.00 Bdy 1     45.00
  3    1.00 Bdy 1     15.00             | 10    1.00 Bdy 1     50.00
  4    1.00 Bdy 1     20.00             | 11    1.00 Bdy 1     55.00
  5    1.00 Bdy 1     25.00             | 12    1.00 Bdy 1     60.00
  6    1.00 Bdy 1     30.00             | 13    1.00 Bdy 1     60.00-   9999.00
  7    1.00 Bdy 1     35.00             |
-------------------------------------------------------------------------------
 Bdy - get velocity change at boundary
 Lyr - get velocity in layer
 S =0 Vp fixed, S=1 Vp/Vs fixed in layer
 Use option 30 to change how Vp obtained
 Use option 31 to change layer weight
 Use option 48 to change layer smoothing
 Use option 45 to redisplay this menu
```

Here the weight, DD, is indicated together with the depth range in the model that it applies to. To permit significant change in the model velocities in the 45-50 km depth range, one would just enter from the command line

rftn96 31 10 100

or interactively

`ready`
31
13 layers: 1-13 for Vs 13-26 for Qbinv
Enter i
10

```
Current dd( 10)=  1.
Enter New dd( 10)
100
ready
```

If differential smoothing is used,**36** with option **1**, then one would see

```
V  Model Weighting Parameters: Large value forces change at boundary or layer
  I   DD(I) Inv S     depth/range          I   DD(I) Inv S     depth/range
-------------------------------------------------------------------------------
  1    1.00 Bdy 1      5.00              |  8    1.00 Bdy 1     40.00
  2    1.00 Bdy 1     10.00              |  9    1.00 Bdy 1     45.00
  3    1.00 Bdy 1     15.00              | 10    1.00 Bdy 1     50.00
  4    1.00 Bdy 1     20.00              | 11    1.00 Bdy 1     55.00
  5    1.00 Bdy 1     25.00              | 12    1.00 Bdy 1     60.00
  6    1.00 Bdy 1     30.00              | 13    1.00 Lyr 1     60.00-   9999.00
  7    1.00 Bdy 1     35.00              |
-------------------------------------------------------------------------------
 Bdy - get velocity change at boundary
 Lyr - get velocity in layer
 S =0 Vp fixed, S=1 Vp/Vs fixed in layer
 Use option 30 to change how Vp obtained
 Use option 31 to change layer weight
 Use option 48 to change layer smoothing
 Use option 45 to redisplay this menu
```

The display is slightly different, since the DD(I) controls the change in velocity across a boundary. This display shows the depth of that boundary. To emphasize a Moho at a depth of 35 km, one would use

```
rftn96 31 7 100
```

Version 3.16 introduced a new choice for option **36** and changes the meaning of this option slightly. When the program is initially interactively started, differential smoothing is assumed. At any time a **36** with choice **0** will force a no-smoothing inversion on all layers, a **36** with choice **1** will force a differential smoothing on all layers except the bottom layer.

A new option **48** - *Modify Individual Layer Smoothing* was also introduced. The purpose if this is to fine tune the type of smoothing for a given layer or boundary by switching between smoothed and non-smoothed layers/boundaries. With this option one can fix a layer velocity and still permit smoothing else where.

The menu produced using option **47** summarizes the current processing parameters.

```
Inversion controls for rftn
Cmd   Value   Description
        1       1 Variance based residual of fit
                0 Variance based on observed std observation
        5       Current iteration
        4       Number of receiver functions to be inverted
        2       2 last inversion for Vs
                3 last inversion for Q inverse
                4 last inversion for Vs-Q inverse
  5     1       0 Layer thickness  inversion
                1 Layer velocity/Q inversion
 32   1.000     Damping value    (default value 1.0)
 33  -5.000     Minimum window for RFTN (default -5.0 s)
 34  20.000     Maximum window for RFTN (default 20.0 s)
 35     0       0 non-causal Vs - Q relation (default)
                1 Decoupled causal
                2 Fully coupled causal
 36     1       0 No smoothing constraint
                1 Differential smoothing constraint
 42   0.0500   Std error of fit floor for RFTN
 44     0       0 Match observed RFTN window
                1 Use 2x times series to compute RFTN
--------------------------------------------------------------------------------
 Use menu command cmd to change value
 ready
```

A few control parameters listed cannot be changed interactively. They are defined during the initial creation of the `robs.d` control file. Other parameters can be changed during the iterative inversion. The most interesting parameter is the "Joint inversion influence parameter," option **`43`** which is used to change the relative importance of the receiver function and surface wave dispersion data sets for determining the "best model"

4.4. Program operation

The program can be run interactively or from the command line. For repeated runs or to reproduce a run running from a command line may be easier, especially if the sequence of commands is placed in a shell script (batch file)For repeated runs or to reproduce a run running from a command line may be easier, especially if the sequence of commands is placed in a shell script (batch file). The only slight difference is that creation of the output file in interactive mode, Option **`28`** asks the user for a command text to be placed in the **`model96`** header.

5. Example

This example computes P-wave receiver functions for a single layer over a halfspace. These synthetic data sets were created for the single layer over a halfspace model

```
MODEL
TEST MODEL
ISOTROPIC
KGS
FLAT EARTH
1-D
CONSTANT VELOCITY
LINE08
LINE09
LINE10
LINE11
HR  VP  VS  RHO QP  QS  ETAP ETAS FREFP FREFS
40  6   3.5 2.5 200.0 100.0 0.0  0.0  1.0   1.0
 0  8   4.7 3.3 900.0 500.0 0.0  0.0  1.0   1.0
```

If this model is called `model.true`, then the commands for create the receiver functions using **`hrftn96`** are

```
#!/bin/sh
P=0.10

for CALP in 05 10 25 50
do
case ${CALP} in
        05) ALP=0.5 ;;
        10) ALP=1.0 ;;
        25) ALP=2.5 ;;
        50) ALP=5.0 ;;
esac
hrftn96    -P -ALP ${ALP} -DT 0.1 -D 10. -RAYP ${P} -M model.true  -2
mv hrftn96.sac ${CALP}.rfn
done
```

To demonstrate operation of the program, the following shell script was run. It is assumed that the control file `robs.d` and the initial earth model, dispersion and receiver function files exist in the current directory. The contents of `robs.d` are

```
   1   0   0   0   0   0   0   0   1   0
modl.d
rftn.lst
```

```
#!/bin/sh
#####
#              clean up
#####
rftn96 39
#####
#              define damping
#####
rftn96 32 1.
#####
#              Select differential smoothing
#####
rftn96 36 1
#####
#              define TMIN and TMAX for receiver function fit
#####
rftn96 33 -5.0 34 20.0
#####
#              set up repeated run for 5 iterations
#####
rftn96 1 2 6 1 2 6 1 2 6  1 2 6 1 2 6 1 2
#####
#              show menus
#####
rftn96 0 > menu00.txt
rftn96 45 > menu45.txt
rftn96 47 > menu47.txt
#####
#              save last model
#####
rftn96 28 modl.out
#####
#              plot resolution kernel
#####
srfphr96
mv SRFPHR96.PLT R.PLT
plotnps -EPS -K < R.PLT > figrfnr.eps
#####
#              plot the model and show the data fit after 5 iterations
#####
rftnpv96
plotnps -EPS -K -F7 -W10 <  RFTNPV96.PLT > figrfn1.eps
#####
#              compare the individual models from the inversion
#              to the true model
#####
shwmod96 -K 1 -W 0.05 model.true
mv SHWMOD96.PLT T.PLT
shwmod96 -K -1  tmpmod96.???
mv SHWMOD96.PLT I.PLT
cat T.PLT I.PLT > IT.PLT
plotnps -EPS -K -F7 -W10 < IT.PLT > figrfn2.eps
```

The command sequence

rftn96 1 2 6 1 2 6 1 2 6 1 2 6 1 2 6 1 2

sets up five complete iterations consisting of **1** computing the partial derivatives, **2** performing the singular value decomposition, and **6** updating the model. The output for these iterations consists fo the following:

```
Processing RFTN Partials for Layer Velocity - Window [     -5.00,     20.00]
IncP Delay    Dt     Rayp   Gauss  Npts 2x   Fit(%) Sta   Weight File
 T   10.00   0.10   0.100    0.50   512  F  36.8486 RFTN    2.00 ../MKRFTN/05.rfn
 T   10.00   0.10   0.100    1.00   512  F  34.8793 RFTN    1.00 ../MKRFTN/10.rfn
 T   10.00   0.10   0.100    2.50   512  F  36.4601 RFTN    0.40 ../MKRFTN/25.rfn
 T   10.00   0.10   0.100    5.00   512  F  38.1616 RFTN    0.20 ../MKRFTN/50.rfn
Receiver function fit         std err  :    0.010452
Percent of Signal Power Fit (RFTN)     :  36.32523% for    4 RFTNs  1004 points
RMS change in S-wave velocity model    :   0.7285 km/sec
ITERATION     1 done: UPDATING V
--------------------------------------------------------------------------------
Processing RFTN Partials for Layer Velocity - Window [     -5.00,     20.00]
IncP Delay    Dt     Rayp   Gauss  Npts 2x   Fit(%) Sta   Weight File
 T   10.00   0.10   0.100    0.50   512  F  92.4736 RFTN    2.00 ../MKRFTN/05.rfn
 T   10.00   0.10   0.100    1.00   512  F  89.3117 RFTN    1.00 ../MKRFTN/10.rfn
 T   10.00   0.10   0.100    2.50   512  F  84.5864 RFTN    0.40 ../MKRFTN/25.rfn
 T   10.00   0.10   0.100    5.00   512  F  82.5842 RFTN    0.20 ../MKRFTN/50.rfn
Receiver function fit         std err  :    0.002016
Percent of Signal Power Fit (RFTN)     :  90.29457% for    4 RFTNs  1004 points
RMS change in S-wave velocity model    :   0.2156 km/sec
ITERATION     2 done: UPDATING V
--------------------------------------------------------------------------------
Processing RFTN Partials for Layer Velocity - Window [     -5.00,     20.00]
IncP Delay    Dt     Rayp   Gauss  Npts 2x   Fit(%) Sta    Weight File
 T   10.00   0.10   0.100    0.50   512  F  97.6074 RFTN    2.00 ../MKRFTN/05.rfn
 T   10.00   0.10   0.100    1.00   512  F  96.5370 RFTN    1.00 ../MKRFTN/10.rfn
 T   10.00   0.10   0.100    2.50   512  F  92.0701 RFTN    0.40 ../MKRFTN/25.rfn
 T   10.00   0.10   0.100    5.00   512  F  86.1939 RFTN    0.20 ../MKRFTN/50.rfn
Receiver function fit         std err  :    0.001088
Percent of Signal Power Fit (RFTN)     :  96.18314% for    4 RFTNs  1004 points
RMS change in S-wave velocity model    :   0.0446 km/sec
ITERATION     3 done: UPDATING V
--------------------------------------------------------------------------------
Processing RFTN Partials for Layer Velocity - Window [     -5.00,     20.00]
IncP Delay    Dt     Rayp   Gauss  Npts 2x   Fit(%) Sta    Weight File
 T   10.00   0.10   0.100    0.50   512  F  97.8811 RFTN    2.00 ../MKRFTN/05.rfn
 T   10.00   0.10   0.100    1.00   512  F  97.4736 RFTN    1.00 ../MKRFTN/10.rfn
 T   10.00   0.10   0.100    2.50   512  F  93.5274 RFTN    0.40 ../MKRFTN/25.rfn
 T   10.00   0.10   0.100    5.00   512  F  86.5704 RFTN    0.20 ../MKRFTN/50.rfn
Receiver function fit         std err  :    0.000958
Percent of Signal Power Fit (RFTN)     :  96.77037% for    4 RFTNs  1004 points
RMS change in S-wave velocity model    :   0.0180 km/sec
ITERATION     4 done: UPDATING V
--------------------------------------------------------------------------------
Processing RFTN Partials for Layer Velocity - Window [     -5.00,     20.00]
IncP Delay    Dt     Rayp   Gauss  Npts 2x   Fit(%) Sta    Weight File
 T   10.00   0.10   0.100    0.50   512  F  98.0282 RFTN    2.00 ../MKRFTN/05.rfn
 T   10.00   0.10   0.100    1.00   512  F  97.7485 RFTN    1.00 ../MKRFTN/10.rfn
 T   10.00   0.10   0.100    2.50   512  F  93.9118 RFTN    0.40 ../MKRFTN/25.rfn
 T   10.00   0.10   0.100    5.00   512  F  86.3817 RFTN    0.20 ../MKRFTN/50.rfn
Receiver function fit         std err  :    0.000931
Percent of Signal Power Fit (RFTN)     :  96.96196% for    4 RFTNs  1004 points
RMS change in S-wave velocity model    :   0.0165 km/sec
ITERATION     5 done: UPDATING V
--------------------------------------------------------------------------------
Processing RFTN Partials for Layer Velocity - Window [     -5.00,     20.00]
IncP Delay    Dt     Rayp   Gauss  Npts 2x   Fit(%) Sta    Weight File
 T   10.00   0.10   0.100    0.50   512  F  98.2216 RFTN    2.00 ../MKRFTN/05.rfn
 T   10.00   0.10   0.100    1.00   512  F  97.9625 RFTN    1.00 ../MKRFTN/10.rfn
 T   10.00   0.10   0.100    2.50   512  F  94.0691 RFTN    0.40 ../MKRFTN/25.rfn
 T   10.00   0.10   0.100    5.00   512  F  86.1935 RFTN    0.20 ../MKRFTN/50.rfn
Receiver function fit         std err  :    0.000914
Percent of Signal Power Fit (RFTN)     :  97.13911% for    4 RFTNs  1004 points
```

Option **1** computes the predictions and partial derivatives for the current model. The information for the receiver functions presents the time window used for comparison with the observed, whether the wave is an incident P-wave, the time delay, sample interval, ray parameter in *sec/km*, the $ alpha $ parameter of the Gaussian filter, the number of data point in the observed receiver function, whether a $2x$ length series is computed to avoid Fast Fourier Transform wrap around, the fit between the observed and predicted receiver functions, the station and file names.

The weight assigned to the particular file is that assigned using option *31*, divided by the Gaussian filter parameter value α. The division is required since larger α correspond to greater bandwidths which then affects the amplitude of the initial pulse in the receiver function. If this internal divisision by α were not done, a noisy recveivr function obstinaed using $\alpha = 5.0$ might dominate the inversion.,

The changes in the model obtained for the current model can be seen by invoking optikon *18*.

```
rftn96 18
  INVERSION FOR S-VEL
  Estimated data standard dev.:  0.0195770562
  RMS model perturbation:  0.0118706543
      DEPTH THICKNESS     S-VEL SIG DELVL RESL in H DEL (VEL)
    2.5000    5.0000    3.6548 0.881E-02 0.196E+02   -0.0099
    7.5000    5.0000    3.6175 0.728E-02 0.222E+02   -0.0120
   12.5000    5.0000    3.5894 0.624E-02 0.244E+02   -0.0157
   17.5000    5.0000    3.6772 0.587E-02 0.229E+02   -0.0186
   22.5000    5.0000    3.6809 0.591E-02 0.186E+02   -0.0176
   27.5000    5.0000    3.6928 0.606E-02 0.164E+02   -0.0192
   32.5000    5.0000    3.7408 0.633E-02 0.156E+02   -0.0249
   37.5000    5.0000    3.8260 0.674E-02 0.155E+02   -0.0252
   42.5000    5.0000    4.3141 0.717E-02 0.149E+02    0.0100
   47.5000    5.0000    4.8778 0.764E-02 0.156E+02    0.0081
   52.5000    5.0000    4.9445 0.745E-02 0.184E+02   -0.0167
   57.5000    5.0000    4.8957 0.648E-02 0.226E+02   -0.0185
   62.5000    0.0000    4.8677 0.535E-02 0.782E+01   -0.0103
```

This display shows that the shear-wave velocity of the first layer would be increased by 0.0014 km/sec for the current damping value (**32**). *At present the estimated errors in the velocity are imperfectly computed*. One can look at the model graphically using option **7** or **9**.

This script created three figures after the 5 iterations. Figure 1 shows the starting model, the current model and the observed and predicted dispersion, while Figure 2 shows the models and the observed and predicted receiver functions.

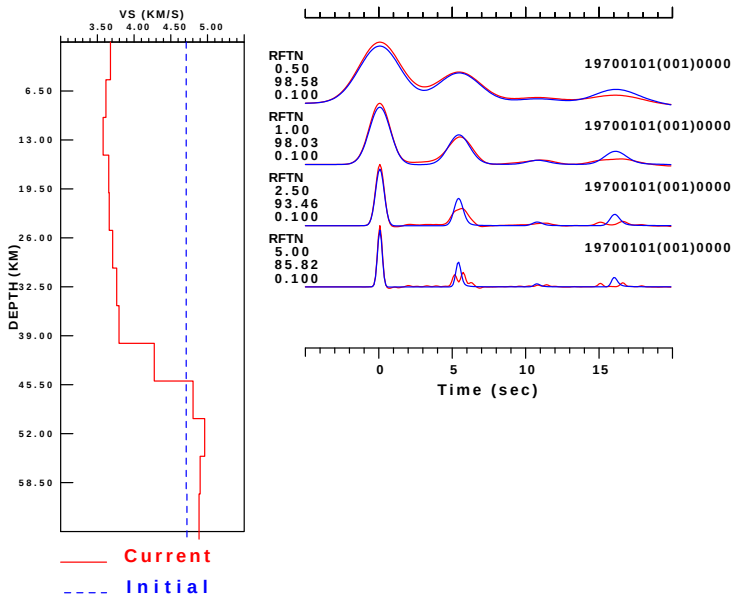

Fig. 1. Output using Option **7**. The observed and predicted RFTN's are plotted in blue and red, respectively. The receiver functions are labeled with the year/month/day (day of year) hour/minute at the upper right of each trace, and with the station name, Gaussian filter parameter, the percentage of fit, and the ray parameter (sec/km) at the left of each trace. A time scale is also provided.

The earth models created at each iteration are saved and numbered with the iteration: `tmpmod96.000`, ..., `tmpmod96.005`. Since the true model is known, Figure 3 compares the true answer with the result of each step of the inversion.

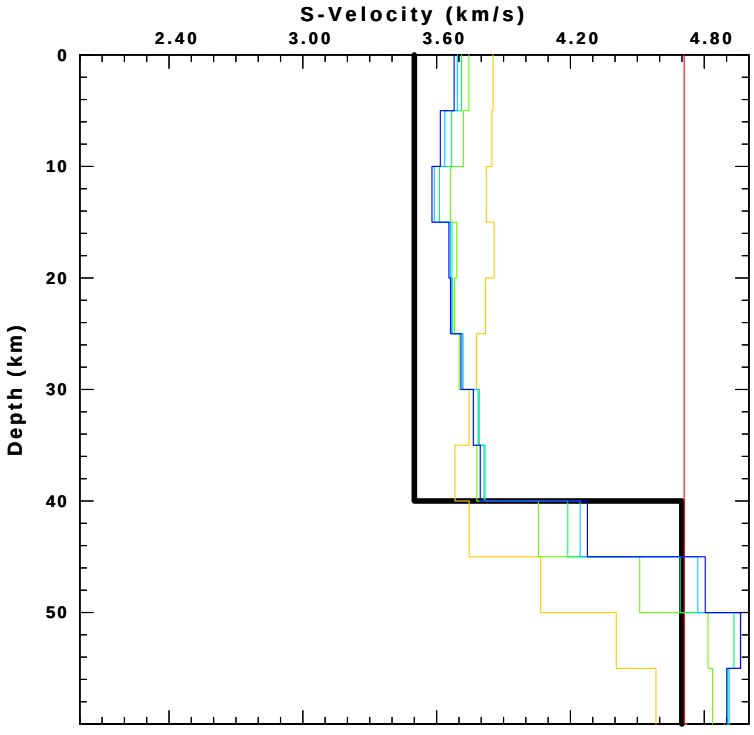

Fig. 2. True model, solid black line; iteration models, red is initial, and blue is the final model.

The command **`rftn96 9`** plots the resolution kernels after the sequence **`rftn96 1`** and **`rftn96 2`** have been fun to compute the partial derivatives and perform the singular value decomposition.

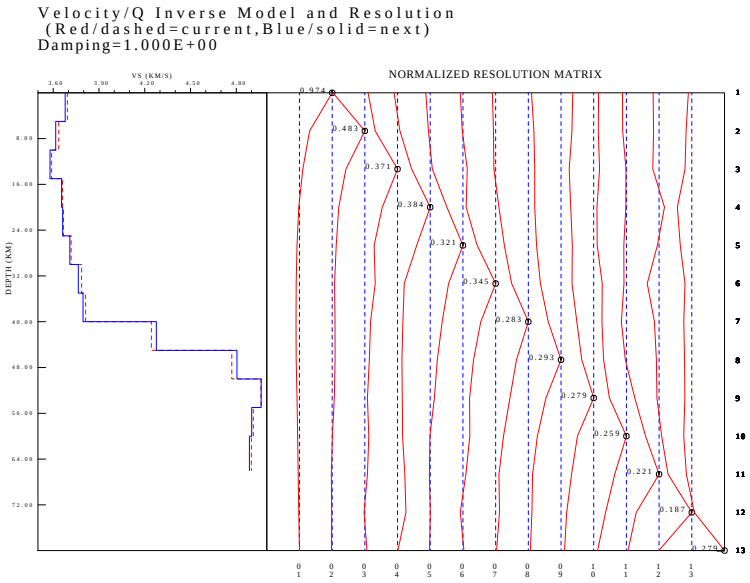

Fig. 3. Resolution kernels

6. Discussion

CHAPTER 5
JOINT INVERSION DISPERSION

1. Introduction

Having discussed the separate inversion of receiver functions and surface-wave dispersion for earth structure, we now present the joint inversion of these two data sets.

2. Joint Inversion Mathematics

The joint inversion attempts to simultaneously invert two different set of observations that are sensitive to different aspects of earth structure. Defining the following parameters:

O_{r_i}	Observed receiver function at time t_i
P_{r_i}	Predicted receiver function at time t_i
σ_{r_i}	Standard error of observation at t_i
O_{s_j}	j'th Observed surface-wave dispersion
P_{s_j}	j'th Predicted surface-wave dispersion point
σ_{s_j}	Standard error of j'th surface-wave observation
N_r	Total number of receiver function points
N_s	Total number of surface-wave dispersion points
p	Influence factor, $0 \le p \le 1$.

We seek and earth model that minimizes the functional

$$S = \frac{(1-p)}{N_r} \sum_{i=0}^{N_r} \left(\frac{O_{r_i} - P_{r_i}}{\sigma_{r_i}} \right)^2 + \frac{p}{N_s} \sum_{j=0}^{N_s} \left(\frac{O_{s_j} - P_{s_j}}{\sigma_{r_j}} \right)^2 \qquad (1)$$

We see that the parameter p changes the influence of either data set on the minimization procedure. A $p = 0$ forces a receiver function only solution, which a $p = 1$ forces a solution based solely ont he surface wave dispersion. An advantage of the statistical weighting in the terms within the large parentheses is that dividing by the observed standard error, corrects for the different physical units of the receiver function (sec^{-1}) and the dispersion (km/sec). The presence of the $1/N_r$ and $1/N_s$ before the summation sign serves to

avoid one data set dominating the other.

From the point of view of estimating model error, equation (1) is not in any of the forms introduced in Chapter 2 because for a large data set with the correct data variances, the expected minimum value of S is

$$E(S) = 1$$

To force this to be in the form of the weighted inverse discussed in §2.4, we could attempt to minimize the quantity

$$S = \left((1-p)N_r + pN_s\right)\left(\frac{(1-p)}{N_r}\sum_{i=0}^{N_r}\left(\frac{O_{r_i} - P_{r_i}}{\sigma_{r_i}}\right)^2 + \frac{p}{N_s}\sum_{j=0}^{N_s}\left(\frac{O_{s_j} - P_{s_j}}{\sigma_{r_j}}\right)^2\right) \qquad (2)$$

This functional form has leads to the correct error statistics for the end member cases $p = 0$ and $p = 1$. *This is not done, however, because the mapping of lack of fit into the Earth model error is imperfect in this non-linear problem.* Instead a number of goodness of fit parameters are output, and it is up to the user to find a realistic model that fits the observations well. This is because the best numerical solution to the problem may not be the best geophysically plausible solution.

Because the forward problem is non-linear in terms of the model parameters, an iterative sequence of linearized inversions is computed. To do this the residual corresponding to the current model is modeled as a linear combination of changes to the current model. This means that the error statistics discussed in Chapter 2 apply to the changes in the model parameters instead of the model parameters themselves. Once the solution has converged to a minimum, not necessary the global minimums, the confidence in the changes may be attributed to the model itself.

A major problem is the estimation of the σ_r's and σ_s's.If repeated observations are made, and the observations averaged, the σ's must be the standard errors of the mean. This estimation is possible if surface wave observations are averages, or if receiver functions are stacked.

At present, the inversion program permits input of the surface-wave sigma's in the **`surf96`** *dispersion format, but does not permit receiver function error traces to be input. The program does attempt to estimate these standard error values.*

To apply the weighting scheme, the program uses the standard error of model fit to the receiver functions, s_r, and in the case that the dispersion data has no corresponding σ_s estimate, the s_s in the following manner.

When the forward predictions are computed, the standard error of model prediction to the observations, s is computed. Initially this will be a large number since the observations as poorly fit. As iteration progresses and the model fit becomes better, the computed s is compared to a user provided least bound. The default receiver function value is 0.05 sec (Option 42) and the default surface wave velocity error is 0.05 km/sec (Option 40). The value of σ used is

$$\max[s, \text{user\_bound}]$$

The slight disadvantage of this is that initially one data set may be emphasized more if its computed s is nearer to the true σ. As iteration progresses, the initial goal of controlled

the influence of each data set is met.

Equation (1) is minimized by applying singular value decomposition to estimate the changes in the model, Δm:

$$w_r \frac{\partial r}{\partial m_1} \Delta m_1 + \ldots + w_r \frac{\partial r}{\partial m_N} \Delta m_N = w_r res$$

$$\ldots$$

$$w_s \frac{\partial s}{\partial m_1} \Delta m_1 + \ldots + w_s \frac{\partial r}{\partial m_N} \Delta m_N = w_s res$$

where the weights

$$w_r = \left(\frac{(1-p)}{N_r \sigma_r^2} \right)^{1/2}$$

$$w_s = \left(\frac{p}{N_s \sigma_s^2} \right)^{1/2}$$

are used for each receiver function and dispersion observation, respectively. The partial derivatives of the receiver function, $\frac{\partial r}{\partial m}$ and surface-wave dispersion $\frac{\partial s}{\partial m}$ are computed as in **`rftn96`** and **`surf96`**.

3. joint96

The operation of **`joint96`** is similar to that of **`surf96`** and **`rftn96`**. Operation of the program requires the existence of the `jobs.d` control file, a surface-wave dispersion file, and the list of the receiver functions to be inverted. If none of these exist, the program will permit the user to interactive create the files.

3.1. Data preparation

The data preparation was discussed in detail in Chapter 3 for surface waves and Chapter 4 for receiver functions.

The surface-wave dispersion data must be in `surf96` format.

The receiver functions must be prepared as described in Chapter 4. It is essential that the Gaussian filter parameter, α be placed in USER0, the ray parameter in USER4 and the receiver function delay be mirrored in the B value of the SAC header for each receiver

function.

3.2. Interactive control file setup

The following example shows the interactive dialog for the case that neither the dispersion file, initial earth model file nor the list of receiver functions exist. The user input is indicated by the *italic* font.

```
joint96
 Enter h,dcl,dcr
    h = fraction change in period to get group vel
        (0.005 is reasonable)
    dcl, dcr are phase velocity increment in root
        search for Love and Rayl respectively
0.005 0.005 0.005
  Enter 1 if variance based on residual or
        0 if variance based on obs std err
1
  Enter maximum number of Love gamma modes to process
       0 means DO NO PROCESS LOVE gamma data
0
  Enter maximum number of Love Phvel modes to process
       0 means DO NO PROCESS LOVE phase vel data
2
  Enter maximum number of Love Gpvel modes to process
       0 means DO NO PROCESS LOVE group vel data
2
  Enter maximum number of Rayl gamma modes to process
       0 means DO NO PROCESS RAYL gamma data
0
  Enter maximum number of Rayl Phvel modes to process
       0 means DO NO PROCESS RAYL phase vel data
2
  Enter maximum number of Rayl Gpvel modes to process
       0 means DO NO PROCESS RAYL group vel data
0
  Enter inversion technique
     0   invert for Vs :Va,rho fixed
     1 : invert for Vs :Poisson fixed, rho from Vp
1
  Enter name of model file
modl.d
  Interactively setting up initial model file:
 modl.d
 Is model flat (0) or spherical (1)
0
 Enter descriptive title for this model
Test model
  Enter d,a,b,rho,qa,qb
   d=0.0 or EOF  indicates halfspace and end of input
40 6 3.5 2.5 100 100
0 8 4.7 3.3 1000 1000
  Enter name of dispersion file
disp.d
 Enter ilvry,iporg,imode,per,val,dval
 ilvry=1(Love)
    =2(Rayleigh)
 iporg=1 (phase velocity km/s)
    =2 (group velocity km/s)
    =3 (gamma 1/km)
 imode (mode number) e.g., 0=fundamental, 1=first
```

```
 per=the period
   val=dispersion value, velocity or gamma
  dval=error in dispersion value
     (Enter 1.0 if stderr from residuals)
  NOTE: Enter all zeros or negative to terminate input
1 1 0 10.0 3.5 0.01
1 2 0 15.0 3.6 0.01
2 2 0 20.0 3.0 0.01
0 0 0 0 0 0
   Enter name of receiver function  file list
rftn.lst
 Interactively setting up receiver function file list:
rftn.lst
Enter receiver function SAC binary file name, EOF to end
../MKRFTN/05.rfn
../MKRFTN/10.rfn
CTRL D        (for UNIX/LINUX, CTRL Z for DOS)
```

If this is successful, then the initial program menu will be presented. Before presenting this, the requested input in the initial setup must be described.

```
Enter h,dcl,dcr
```

Determination of surface wave dispersion requires a search in the frequency - phase velocity space of the surface-wave period equation. Is is know that all surface wave modes are bounded at the low end by some fraction of the smallest shear-wave velocity, or in the case of a surface fluid layer, the lowest compressional-wave velocity. The upper bound of the dispersion for a fixed period, is the halfspace shear-wave velocity. **dcl** and **dcr** are the search increments to find the roots of the period equation. If these numbers are too large, modes may be missed by jumping past them. If the numbers are too small, computations take too long. The value of *0.005* km/sec are acceptable for crustal studies, but could be made smaller when studying dispersion in low velocity sediments.

The parameter **h** is used to compute group-velocity partial derivatives. For example, $\partial \mathbf{U}/\partial \mathbf{V_S}$ can be computed by using $\partial \mathbf{c}/\partial \mathbf{V_S(f)}$ and $\partial \mathbf{c}/\partial \mathbf{V_S((1+h)f)}$. The recommended value of *0.005* seems adequate.

```
Enter 1 if variance based on residual or
      0 if variance based on obs std err
```

This relates to the computation of confidence limits in the earth model and also the weighting of the individual data. *Currently* only the surface wave data permits a tabulation of the confidence of any observation.

```
Enter maximum number of Love gamma modes to process
     0 means DO NO PROCESS LOVE gamma data
```

This sequence of 6 questions permits the user to invert subsets of the data contained within the dispersion file. *Currently* the anelastic attenuation coefficients are used in the joint inversion to determine the Q structure.

```
Enter inversion technique
   0   invert for Vs :Va,rho fixed
   1 : invert for Vs :Poisson fixed, rho from Vp
```

The first option, *0* was introduced for the special case of investigating receiver function in deep soil layers for which the P-velocity was known. The second case, *1*, iteratively inverts for the S-wave velocity and then updates the P-velocity using the Vp/Vs ratio of the initial model; the new density is computed from the new Vp using the Nafe-Drake relation.

After this the names of the earth model file, dispersion and receiver function list are requested. If the files do not exist, then they are interactively constructed. Once this is done interactively, the format of each is correctly prototyped and the user may use an editor to modify the files faster than using this interactive dialog.

3.3. Main menu

```
                                JOINT96 MENU
 0- Display menu                      30- (0) Fix Vp,(1) Fix Vp/Vs
 1- Run Dispersion                    31- Change dd(i), enter i,dd(i)
 2- Run Velocity Inversion            32- Enter Damping Factor (lam)
 3- Run Q(beta) Inversion             33- Enter Tmin for RFTN (default  -5 s)
 4- Run Simultaneous Veloc-Q Inversion 34- Enter Tmax for RFTN (default  20 s)
 5- Set Thick(0)/Velocity(1) Inversion 35- Inversion: (0) Non-Causal (default)
 6- Update Model (need lam)                          (1) Decoupled Causal
 7- Plot RFTN/Dispersion/Velocity Model              (2) Coupled Causal
 8- Plot Gamma / Qb inverse  Model    36- Smoothing: (0) Global reset none
 9- Plot Resolution Kernel                           (1) Global reset diff
10- List Singular Values              37- Reset Number of Iterations
11- Gamma Data Partial Derivatives    38- Temporary End
12- Gamma Data Dispersion             39- Permanent End
13- List Q Model (need lam)           40- Enter Sigv minimum
14- Q(beta) Resolving Kernel (need lam) 41- Enter Sigg minimum
16- Velocity Data Partial Derivatives  42- Enter Sigr minimum
17- Velocity Data Dispersion           43- Joint Weighting: 0=RFTN <--> 1=SRFW
18- List Velocity Model (need lam)     44- 2x RFTN computation (0) no, (1) yes
19- Velocity Resolving Kernels(need lam)45- Show Velocity Weights
22- ASCII Gamma Dispersion (file name) 46- Show Qinv     Weights
23- ASCII Q(beta) File (file name,lam) 47- Show Inversion Controls
24- ASCII Q Resolving  (file name,lam) 48- Modify Individual Layer Smoothing
27- ASCII Veloc Dispersion (file name) 49- Show RFTN information and weight
28- ASCII Model File  (file name, lam) 50- Change individual RFTN weight
29- ASCII Vel Resolving(file name,lam)
Enter Command at READY Prompt
```

This menu appears once the control file *jobs.d* exists. If the dispersion, initial model and file listing receiver functions do not exist, then error messages will appear.

This menu is hopefully self-explanatory. Entering one of these numbers at the **`ready`** prompt causes an action. Actions **45**, **46** and **47** present other menus.

If **45** is entered, and the no smoothing option was chosen, e.g., **36** with option **0**, then the velocity weights shown are

```
V  Model Weighting Parameters: Large value forces change at boundary or layer
  I   DD(I) Inv S     depth/range          I   DD(I) Inv S     depth/range
-------------------------------------------------------------------------------
  1    1.00 Lyr 1      0.00-      5.00  |  8    1.00 Lyr 1     35.00-     40.00
  2    1.00 Lyr 1      5.00-     10.00  |  9    1.00 Lyr 1     40.00-     45.00
  3    1.00 Lyr 1     10.00-     15.00  | 10    1.00 Lyr 1     45.00-     50.00
  4    1.00 Lyr 1     15.00-     20.00  | 11    1.00 Lyr 1     50.00-     55.00
  5    1.00 Lyr 1     20.00-     25.00  | 12    1.00 Lyr 1     55.00-     60.00
  6    1.00 Lyr 1     25.00-     30.00  | 13    1.00 Lyr 1     60.00-   9999.00
  7    1.00 Lyr 1     30.00-     35.00  |
-------------------------------------------------------------------------------
 Bdy - get velocity change at boundary
 Lyr - get velocity in layer
 S =0 Vp fixed, S=1 Vp/Vs fixed in layer
 Use option 30 to change how Vp obtained
 Use option 31 to change layer weight
 Use option 48 to change layer smoothing
 Use option 45 to redisplay this menu
```

Here the weight, DD, is indicated together with the depth range in the model that it applies to. To permit significant change in the model velocities in the 45-50 km depth range, one would just enter from the command line

joint96 31 10 100

or interactively

ready
31
13 layers: 1-13 for Vs 13-26 for Qbinv
Enter i
10
Current dd(10)= 1.
Enter New dd(10)
100
ready

If differential smoothing is used,**36** with option **1**, then one would see

```
V  Model Weighting Parameters: Large value forces change at boundary or layer
  I   DD(I) Inv S     depth/range          I   DD(I) Inv S     depth/range
-------------------------------------------------------------------------------
  1    1.00 Bdy 1      5.00              |  8    1.00 Bdy 1     40.00
  2    1.00 Bdy 1     10.00              |  9    1.00 Bdy 1     45.00
  3    1.00 Bdy 1     15.00              | 10    1.00 Bdy 1     50.00
  4    1.00 Bdy 1     20.00              | 11    1.00 Bdy 1     55.00
  5    1.00 Bdy 1     25.00              | 12    1.00 Bdy 1     60.00
  6    1.00 Bdy 1     30.00              | 13    1.00 Lyr 1     60.00-   9999.00
  7    1.00 Bdy 1     35.00              |
-------------------------------------------------------------------------------
 Bdy - get velocity change at boundary
 Lyr - get velocity in layer
 S =0 Vp fixed, S=1 Vp/Vs fixed in layer
 Use option 30 to change how Vp obtained
 Use option 31 to change layer weight
 Use option 48 to change layer smoothing
 Use option 45 to redisplay this menu
```

The display is slightly different, since the DD(I) controls the change in velocity across a boundary. This display shows the depth of that boundary. To emphasize a Moho at a depth of 35 km, one would use

joint96 31 7 100

The menu produced using option **47** summarizes the current processing parameters.

```
Inversion controls for joint96
Cmd   Value   Description
        1       1 Variance based residual of fit
                0 Variance based on observed std observation
        2       Maximum number of Love     modes to use
        2       Maximum number of Rayleigh modes to use
        5       Current iteration
        4       Number of receiver functions to be inverted
        2       2 last inversion for Vs
                3 last inversion for Q inverse
                4 last inversion for Vs-Q inverse
  5     1       0 Layer thickness  inversion
                1 Layer velocity/Q inversion
 32   1.000     Damping value    (default value 1.0)
 33  -5.000     Minimum window for RFTN (default -5.0 s)
 34  20.000     Maximum window for RFTN (default 20.0 s)
 35     0       0 non-causal Vs - Q relation (default)
                1 Decoupled causal
                2 Fully coupled causal
 36     1       0 No smoothing constraint
                1 Differential smoothing constraint
 40   0.0500   Std error of fit floor for velocity disp
 42   0.0005   Std error of fit floor for RFTN
 43   0.500     Joint inversion influence parameter
                RFTN = 0 <= p <= 1 Surface Wave
 44     0       0 Match observed RFTN window
                1 Use 2x times series to compute RFTN
--------------------------------------------------------------------------------
 Use menu command cmd to change value
```

A few control parameters listed cannot be changed interactively. They are defined during the initial creation of the `jobs.d` control file. Other parameters can be changed during the iterative inversion. The most interesting parameter is the "Joint inversion influence parameter," option **43** which is used to change the relative importance of the receiver function and surface wave dispersion data sets for determining the "best model"

Version 3.16 introduced a new choice for option **36** and changes the meaning of this option slightly. When the program is initially interactively started, differential smoothing is assumed. At any time a **36** with choice **0** will force a no-smoothing inversion on all layers, a **36** with choice **1** will force a differential smoothing on all layers except the bottom layer.

A new option **48** - *Modify Individual Layer Smoothing* was also introduced. The purpose if this is to fine tune the type of smoothing for a given layer or boundary by switching between smoothed and non-smoothed layers/boundaries. With this option one can fix a layer velocity and still permit smoothing else where.

3.4. Program operation

The program can be run interactively or from the command line. For repeated runs or to reproduce a run running from a command line may be easier, especially if the sequence of commands is placed in a shell script (batch file)For repeated runs or to reproduce a run running from a command line may be easier, especially if the sequence of commands is

placed in a shell script (batch file). The only slight difference is that creation of the output file in interactive mode, Option **28** asks the user for a command text to be placed in the **model96** header.

4. Example

This example uses the same dispersion information presented in Chapter 3 and the same receiver functions presented in Chapter 4. These synthetic data sets were created for the single layer over a halfspace model

```
MODEL
TEST MODEL
ISOTROPIC
KGS
FLAT EARTH
1-D
CONSTANT VELOCITY
LINE08
LINE09
LINE10
LINE11
HR      VP       VS  RHO QP  QS  ETAP ETAS FREFP FREFS
40           63.5 2.5 200.0 100.0 0.0  0.0  1.0   1.0
 0           84.7 3.3 900.0 500.0 0.0  0.0  1.0   1.0
```

To demonstrate operation of the program, the following shell script was run. It is assumed that the control file `jobs.d` and the initial earth model, dispersion and receiver function files exist in the current directory. The contents of `jobs.d` are

```
    0.00499999989  0.00499999989  0.  0.00499999989  0.
      1    2    2    2    2    2    2    0    1    0
  modl.d
  ../MKSURF/disp.d
  rftn.lst
```

The initial model, `modl.d` consists of a halfspace -

```
MODEL
TEST MODEL
ISOTROPIC
KGS
FLAT EARTH
1-D
CONSTANT VELOCITY
LINE08
LINE09
LINE10
LINE11
HR      VP       VS  RHO QP  QS  ETAP ETAS FREFP FREFS
 5           84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5           84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5           84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5           84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5           84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5           84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5           84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5           84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
```

```
 5             84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5             84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5             84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 5             84.71 3.3 0.0 0.0 0.0  0.0  1.0   1.0
 0             84.72 3.3 0.0 0.0 0.0  0.0  1.0   1.0
```

```
#!/bin/sh

#####
#              clean up
#####
joint96 39
#              define damping
#####
joint96 32 1.
#####
#              Select differential smoothing
#####
joint96 36 1
#####
#              set joint weighting between the two data sets
#####
joint96 43 0.5
#####
#              set up repeated run for 5 iterations
#####
joint96 1 2 6 1 2 6 1 2 6  1 2 6 1 2 6 1 2
#####
#              save last model
#####
joint96 28 modl.out
#####
#              plot the model and show the data fit after 5 iterations
#####
srfphv96
plotnps -EPS -K -F7 -W10 <  SRFPHV96.PLT > figjnt1.eps
rftnpv96
plotnps -EPS -K -F7 -W10 <  RFTNPV96.PLT > figjnt2.eps
#####
#              compare the individual models from the inversion
#              to the true model
#####
shwmod96 -K 1 -W 0.05 model.true
mv SHWMOD96.PLT T.PLT

shwmod96 -K -1  tmpmod96.???
mv SHWMOD96.PLT I.PLT

cat T.PLT I.PLT > IT.PLT
plotnps -EPS -K -F7 -W10 < IT.PLT > figjnt3.eps
```

The command sequence

joint96 1 2 6 1 2 6 1 2 6 1 2 6 1 2 6 1 2

sets up five complete iterations consisting of **1** computing the partial derivatives, **2** performing the singular value decomposition, and **6** updating the model. The output for these iterations consists fo the following:

```
Processing RFTN Partials for Layer Velocity - Window [     -5.00,     20.00]
IncP Delay    Dt     Rayp   Gauss  Npts 2x   Fit(%) Sta   Weight File
 T   10.00   0.10   0.100    0.50   512  F  36.8486 RFTN    2.00 ../MKRFTN/05.rfn
 T   10.00   0.10   0.100    1.00   512  F  34.8793 RFTN    1.00 ../MKRFTN/10.rfn
 T   10.00   0.10   0.100    2.50   512  F  36.4601 RFTN    0.40 ../MKRFTN/25.rfn
 T   10.00   0.10   0.100    5.00   512  F  38.1616 RFTN    0.20 ../MKRFTN/50.rfn
Receiver function fit         std err  :    0.010452
Dispersion  fit (vel)         std err  :    0.3089 (km/s)
Dispersion  fit (vel)    mean residual :   -1.0676 (km/s)
Dispersion  fit (vel)    avg |residual|:    1.0676 (km/s)
Percent of Signal Power Fit (RFTN)     :  36.32523% for    4 RFTNs  1004 points
Percent of Signal Power Fit (Disp)     :  89.05252% for   68 SW Obs
RMS change in S-wave velocity model    :   1.2049 km/sec
ITERATION     1 done: UPDATING V
------------------------------------------------------------------------
Processing RFTN Partials for Layer Velocity - Window [     -5.00,     20.00]
IncP Delay    Dt     Rayp   Gauss  Npts 2x   Fit(%) Sta   Weight File
 T   10.00   0.10   0.100    0.50   512  F  90.0604 RFTN    2.00 ../MKRFTN/05.rfn
 T   10.00   0.10   0.100    1.00   512  F  83.9401 RFTN    1.00 ../MKRFTN/10.rfn
 T   10.00   0.10   0.100    2.50   512  F  80.2502 RFTN    0.40 ../MKRFTN/25.rfn
 T   10.00   0.10   0.100    5.00   512  F  80.9305 RFTN    0.20 ../MKRFTN/50.rfn
Receiver function fit         std err  :    0.002381
Dispersion  fit (vel)         std err  :    0.2257 (km/s)
Dispersion  fit (vel)    mean residual :    0.2867 (km/s)
Dispersion  fit (vel)    avg |residual|:    0.2908 (km/s)
Percent of Signal Power Fit (RFTN)     :  86.89568% for    4 RFTNs  1004 points
Percent of Signal Power Fit (Disp)     :  99.01604% for  160 SW Obs
RMS change in S-wave velocity model    :   0.3340 km/sec
ITERATION     2 done: UPDATING V
------------------------------------------------------------------------
Processing RFTN Partials for Layer Velocity - Window [     -5.00,     20.00]
IncP Delay    Dt     Rayp   Gauss  Npts 2x   Fit(%) Sta   Weight File
 T   10.00   0.10   0.100    0.50   512  F  95.6311 RFTN    2.00 ../MKRFTN/05.rfn
 T   10.00   0.10   0.100    1.00   512  F  91.4207 RFTN    1.00 ../MKRFTN/10.rfn
 T   10.00   0.10   0.100    2.50   512  F  86.0853 RFTN    0.40 ../MKRFTN/25.rfn
 T   10.00   0.10   0.100    5.00   512  F  84.6284 RFTN    0.20 ../MKRFTN/50.rfn
Receiver function fit         std err  :    0.001621
Dispersion  fit (vel)         std err  :    0.0976 (km/s)
Dispersion  fit (vel)    mean residual :    0.0135 (km/s)
Dispersion  fit (vel)    avg |residual|:    0.0726 (km/s)
Percent of Signal Power Fit (RFTN)     :  92.93363% for    4 RFTNs  1004 points
Percent of Signal Power Fit (Disp)     :  99.92731% for  156 SW Obs
RMS change in S-wave velocity model    :   0.0880 km/sec
ITERATION     3 done: UPDATING V
------------------------------------------------------------------------
Processing RFTN Partials for Layer Velocity - Window [     -5.00,     20.00]
IncP Delay    Dt     Rayp   Gauss  Npts 2x   Fit(%) Sta   Weight File
 T   10.00   0.10   0.100    0.50   512  F  96.9917 RFTN    2.00 ../MKRFTN/05.rfn
 T   10.00   0.10   0.100    1.00   512  F  93.4524 RFTN    1.00 ../MKRFTN/10.rfn
 T   10.00   0.10   0.100    2.50   512  F  87.6105 RFTN    0.40 ../MKRFTN/25.rfn
 T   10.00   0.10   0.100    5.00   512  F  85.5576 RFTN    0.20 ../MKRFTN/50.rfn
Receiver function fit         std err  :    0.001413
Dispersion  fit (vel)         std err  :    0.0673 (km/s)
Dispersion  fit (vel)    mean residual :    0.0086 (km/s)
Dispersion  fit (vel)    avg |residual|:    0.0492 (km/s)
Percent of Signal Power Fit (RFTN)     :  94.47678% for    4 RFTNs  1004 points
Percent of Signal Power Fit (Disp)     :  99.96575% for  158 SW Obs
RMS change in S-wave velocity model    :   0.0436 km/sec
ITERATION     4 done: UPDATING V
------------------------------------------------------------------------
Processing RFTN Partials for Layer Velocity - Window [     -5.00,     20.00]
IncP Delay    Dt     Rayp   Gauss  Npts 2x   Fit(%) Sta   Weight File
 T   10.00   0.10   0.100    0.50   512  F  97.6121 RFTN    2.00 ../MKRFTN/05.rfn
 T   10.00   0.10   0.100    1.00   512  F  94.5526 RFTN    1.00 ../MKRFTN/10.rfn
```

```
 T   10.00   0.10   0.100    2.50   512  F  88.5034 RFTN    0.40 ../MKRFTN/25.rfn
 T   10.00   0.10   0.100    5.00   512  F  86.1384 RFTN    0.20 ../MKRFTN/50.rfn
Receiver function fit         std err  :    0.001296
Dispersion  fit (vel)         std err  :    0.0510 (km/s)
Dispersion  fit (vel)    mean residual :    0.0017 (km/s)
Dispersion  fit (vel)    avg |residual|:    0.0370 (km/s)
Percent of Signal Power Fit (RFTN)     :  95.25683% for    4 RFTNs  1004 points
Percent of Signal Power Fit (Disp)     :  99.98046% for  156 SW Obs
RMS change in S-wave velocity model    :   0.0269 km/sec
ITERATION     5 done: UPDATING V
------------------------------------------------------------------------
Processing RFTN Partials for Layer Velocity - Window [     -5.00,     20.00]
IncP Delay    Dt     Rayp   Gauss  Npts 2x   Fit(%) Sta   Weight File
 T   10.00   0.10   0.100    0.50   512  F  98.0505 RFTN    2.00 ../MKRFTN/05.rfn
 T   10.00   0.10   0.100    1.00   512  F  95.3041 RFTN    1.00 ../MKRFTN/10.rfn
 T   10.00   0.10   0.100    2.50   512  F  89.1835 RFTN    0.40 ../MKRFTN/25.rfn
 T   10.00   0.10   0.100    5.00   512  F  86.7527 RFTN    0.20 ../MKRFTN/50.rfn
Receiver function fit         std err  :    0.001202
Dispersion  fit (vel)         std err  :    0.0425 (km/s)
Dispersion  fit (vel)    mean residual :    0.0052 (km/s)
Dispersion  fit (vel)    avg |residual|:    0.0309 (km/s)
Percent of Signal Power Fit (RFTN)     :  95.81573% for    4 RFTNs  1004 points
Percent of Signal Power Fit (Disp)     :  99.98636% for  158 SW Obs
```

Option **1** computes the predictions and partial derivatives for the current model. The information for the receiver functions presents the time window used for comparison with the observed, whether the wave in an incident P-wave, the time delay, sample interval, ray parameter in *sec/km*, the α parameter of the Gaussian filter, the number of data point in the observed receiver function, whether a *2x* length series is computed to avoid Fast Fourier Transform wrap around, the fit between the observed and predicted receiver functions, the station and file names.

The dispersion information is summarized by given the mean difference between observed and predicted dispersion, the standard error of fit and the L1 norm of the fit.

The changes in the model obtained for the current model can be seen by invoking

```
joint96 18
  INVERSION FOR S-VEL
  Estimated data standard dev.:  0.0204858948
      DEPTH THICKNESS     S-VEL SIG DELVL RESL in H DEL (VEL)
  RMS Velocity model perturbation   :  0.0211627949
    2.5000    5.0000    3.5128 0.241E-02 0.107E+02    0.0046
    7.5000    5.0000    3.4995 0.200E-02 0.123E+02    0.0079
   12.5000    5.0000    3.4593 0.160E-02 0.170E+02    0.0075
   17.5000    5.0000    3.4339 0.145E-02 0.226E+02    0.0003
   22.5000    5.0000    3.4199 0.154E-02 0.256E+02   -0.0118
   27.5000    5.0000    3.4700 0.174E-02 0.272E+02   -0.0193
   32.5000    5.0000    3.6238 0.193E-02 0.280E+02   -0.0144
   37.5000    5.0000    3.9229 0.211E-02 0.270E+02    0.0029
   42.5000    5.0000    4.2981 0.224E-02 0.246E+02    0.0242
   47.5000    5.0000    4.5746 0.224E-02 0.215E+02    0.0374
   52.5000    5.0000    4.7159 0.208E-02 0.238E+02    0.0398
   57.5000    5.0000    4.7167 0.181E-02 0.201E+02    0.0326
   62.5000    0.0000    4.6566 0.169E-02 0.535E+01    0.0181
```

This display shows that the shear-wave velocity of the first layer would be increased by 0.0046 km/sec for the current damping value (**32**). *At present the estimated errors in the velocity are imperfectly computed*. One can look at the model graphically using option **7**

or **9**.

This script created three figures after the 5 iterations. Figure 1 shows the starting model, the current model and the observed and predicted dispersion, while Figure 2 shows the models and the observed and predicted receiver functions.

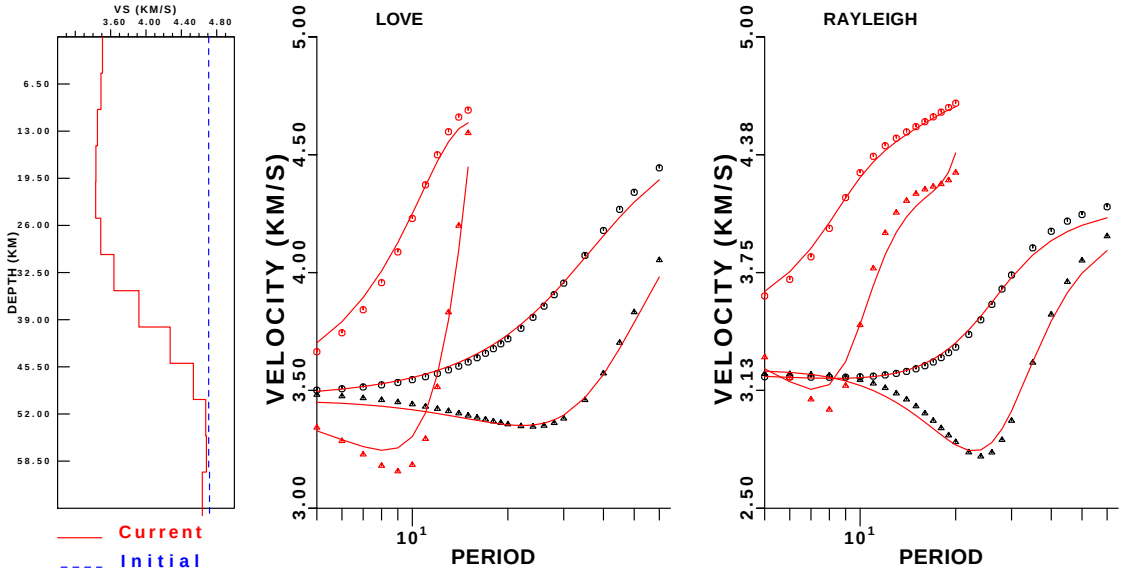

Fig. 1. Output using Option **7**. Dispersion data are indicated symbols. Predictions by the solid curves.

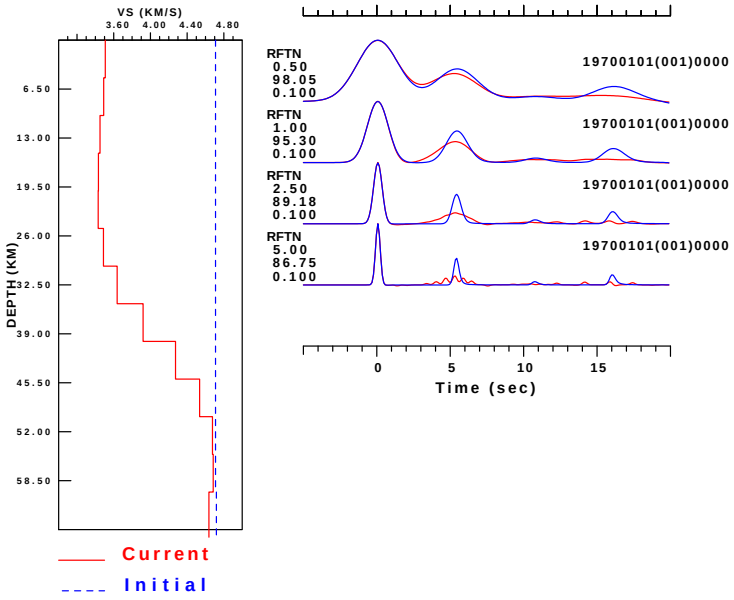

Fig. 2. Output using Option **7** (second page). The observed and predicted RFTN's are plotted in blue and red, respectively. The receiver functions are labeled with the year/month/day (day of year) hour/minute at the upper right of each trace, and with the station name, Gaussian filter parameter, the percentage of fit, and the ray parameter (sec/km) at the left of each trace. A time scale is also provided.

The earth models created at each iteration are saved and numbered with the iteration: `tmpmod96.000`, ..., `tmpmod96.005`. Since the true model is known, Figure 3 compares the true answer with the result of each step of the inversion.

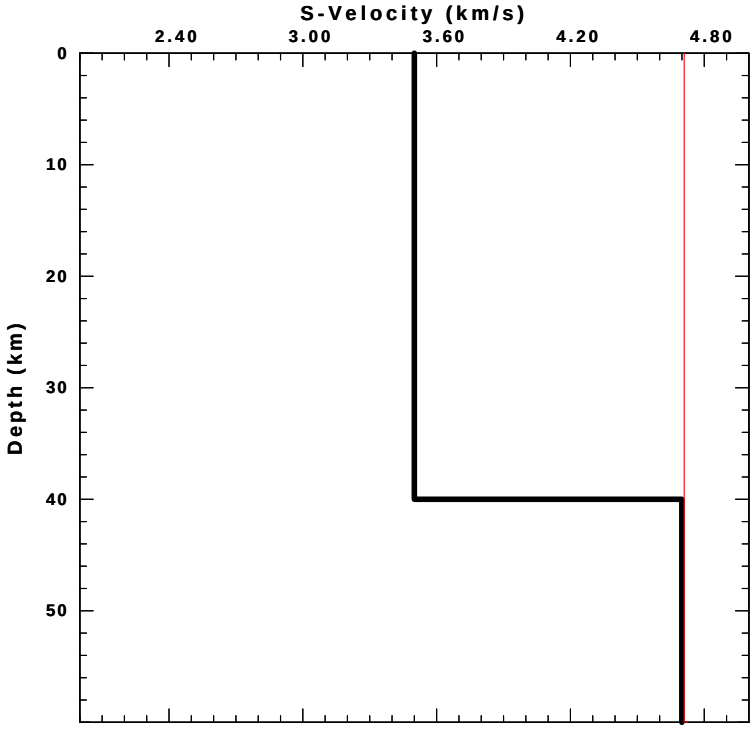

Fig. 3. True model, solid black line; iteration models, red is initial, and blue is the final model.

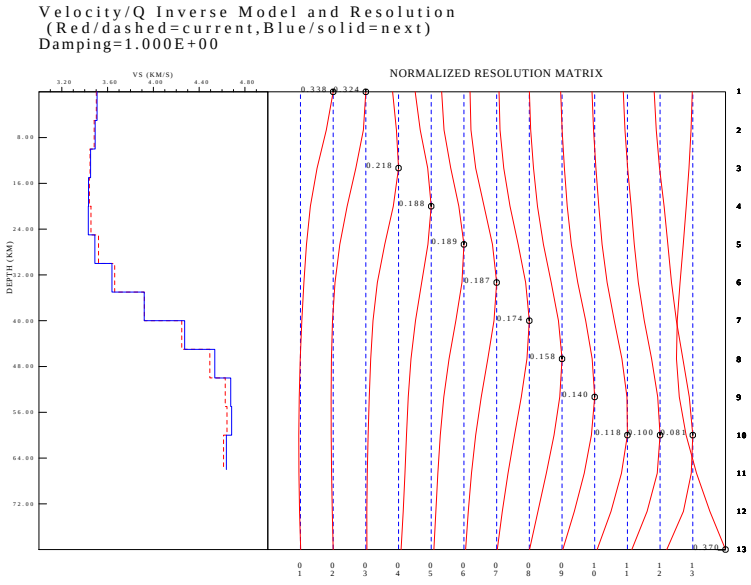

Fig. 4. Resolution kernels obtained using **`joint96 9`**.

5. Discussion

The example shows that the surface wave dispersion data are fit well by the model despite the fact that this is a smoothed version of the true single layer over a halfspace model. Smoothing is required for stable inversion of a many-layered model with the penalty that sharp discontinuities are not permitted.

The same model fits the receiver function data fairly well for the smallest value of the filter parameter, $\alpha = 0.5$. The smooth model has difficulty fitting the converted phase at t = 6 sec for higher α especially the later phases at t = 16 sec.

The receiver function can be fit better by continuing the iterations with a lower value of damping (Option 32) which relaxes the smoothing condition. This many introduce some problems though, in that a receiver function forced discontinuity may be at the wrong depth and the surface wave dispersion may be slightly misfit.

As mentioned in the discussion of **`surf96`**, the fit to the surface-wave data may be biased toward one period range. Since the depth of penetration of a surface wave is proportional to its period, a data set such as the one used here, will place more weight on the shallow part of the structure because or the greater proportion of data at the shorter periods.

In practice neither the receiver function nor the surface-wave dispersion is well defined, and fits to small "bumps" in the receiver function may not be significant.

APPENDIX A
INSTALLATION

1. Tailoring Shell Scripts

All required programs are compiled and installed as part of the normal installation procedure. Some fine tuning is required for the following shell scripts which are located in **`PROGRAMS.325/bin`**:

`rftnvp.bat`
`srfgph.bat`
`srfrph.bat`
`srfvph.bat`

Consider the script **`rftnvp.bat`**.

```
#!/bin/sh
case $TERM in
vt100|vt100n) clear
              mgotek
              rftnpv96 -
              plot4014 < RFTNPV96.PLT
              rm RFTNPV96.PLT
              sleep 10
              mrttek ;;
4014|tek)
              clear
              rftnpv96
              plot4014 < RFTNPV96.PLT
              rm RFTNPV96.PLT
              sleep 10;;
xterm|ddterm|sun-cmd)
              rftnpv96
              plotxvig < RFTNPV96.PLT
              rm RFTNPV96.PLT
              ;;
*) echo 'TERMINAL UNKNOWN USE TEKTRONIX' ;;
esac
```

Since the programs **`surf96`**, **`rftn96`** and **`joint96`** can be run interactively or from the command line from a wide variety of terminals, the program itself does not know which graphics device to use for output. Instead it examines the **`TERM`** environment parameter to guess the type of graphics device. For example, the terminal emulation program `Teraterm` supports various DEC VT-??? termminal emulations but also emulates a Tektronics srotage terminal. If the **`TERM`** is set to `vt100`, then the program turns on the Tektronix emulation, `mgotek`, runs the program to compute the graphics output, `rftnpv96`, and then converts the binary CALPLOT file, `RFTNPV96.PLT` to drive the Tektronix graphics.

If the **TERM** parameter is `xterm` or SUN's `dtterm` or SUN's `sun-cmd`, then X11 graphics is used. The `|` symbol in the case choices is a logical `or`.

2. Hard Copy

Since the shell scripts remove the CALPLOT file, how does one get a nice plot for ooptions `7`, , `8` or `9`? Just do the following from within any of the three programs:

```
38

rbh> rftnpv96
rbh> plotnps < RFTNPV96.PLT > rftn.ps
```

Just temporarily leave the program without deleting any of the control files, `38`. At the shell prompt type the name fo the approprite program. And then use the CALPLOT programs to convert the output to the suitable format, e.g., `plotnps -EPS -K`.