Introduction to Earthquake Seismology
Assignment 16

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Geiger's Method

Goals:

- Introduce Geiger's method for earthquake location
- Solve a simple location problem
- Appreciate possibles instabilities

Background:

Geiger (1910, 1912) introduced an iterative least-squares technqiue for earthquake location. This is a simple technique to understnad and to implement. However, the answer depends upon the spatial distribution of the observations. One must appreciate this before accepting the resulting numbers.

The earthquake location problem is high non-linear in the sense that there is no simple linear relationship between the observed arrival times and the desired spatial and temporal coordinates of the source. This non-linearity arises from the determination of the distances and azimuths for the source to each observationand also from the model predicted travel time as a function of distance and depth.

One approach is to linearize the problem, by focusing on slight changes in the source coordinates. To simplify the presentation, we will consider a local earthquake problem, for which a cartesian coordinate system is used. This might be useful if one is considering small earthquakes in a mine.

Least Squares

Let the coordinates of the earthquake be (X, Y, Z, T), where X, Y and Z are the spatial coordinates and T is the origin time. (The three spatial coordinates define the hypocenter whicle the two surface coordinates define the epicenter). Let the coordinate of the j'th station be $(x_j, y_j, 0)$, where we assume the station is at the surface. The corresponding arrival time is t_j.

For the given spatial coordinates, the difference between the observed and predicted arrival time for observation j is

$$res_j = t_j - t_{pred_j}$$

where

$$t_{pred_j} = T - TT(x_j, y_j, 0, X, Y, Z)$$

where TT is the predicted travel time as a function of the particular phase and the station and hypocenter coordinates. A non-zero value of the residual may arise from noisy arrival time observations, an incorrect velocity model for predicting the travel time or from an incorrect choice of the earthquake source parameters.

Linearization of this problem starts with the assumption that the residual is due to an incorrect (X_0, Y_0, Z_0, T_0), where the 0 indicates the initial value. If we wish to predict the travel time for a new location, $X_0 + \Delta X, Y_0 + \Delta Y, Z_0 + \Delta Z, T_0 + \Delta T$), the estimated predicted arrival time is

$$T_0 + \Delta T + TT(x_j, y_j, 0, X, Y, Z) + \frac{\partial TT_j}{\partial X} \Delta X + \frac{\partial TT_j}{\partial Y} \Delta Y + \frac{\partial TT_j}{\partial Z} \Delta Z$$

where the partial derivatives are evaluated at the current coordinates ($X_{0,} Y_0$, Z_0). We want the change in coordinates to account for the residual, or

$$res_j = \Delta T + \frac{\partial TT_j}{\partial X} \Delta X + \frac{\partial TT_j}{\partial Y} \Delta Y + \frac{\partial TT_j}{\partial Z} \Delta Z$$

You will now see that the changes from the initial coordinates just appear as linear terms.

One way to approach this problem is to select the changes such that the minimize the following expression (hence least squares):

$$R = \sum res_j^2$$

The conditions to forece this to be an extremum (hopefully a minimum) are $\frac{\partial R}{\partial \Delta T} = 0$, $\frac{\partial R}{\partial \Delta X} = 0$, $\frac{\partial R}{\partial \Delta Y} = 0$, and $\frac{\partial R}{\partial \Delta Z} = 0$

With little effort, this can be placed in a matrix form as

$$\begin{bmatrix} \sum_j TT_X TT_X & \sum_j TT_Y TT_X & \sum_j TT_Z TT_X & \sum_j TT_X \\ \sum_j TT_X TT_Y & \sum_j TT_Y TT_Y & \sum_j TT_Z TT_Y & \sum_j TT_Y \\ \sum_j TT_X TT_Z & \sum_j TT_Y TT_Z & \sum_j TT_Z TT_Z & \sum_j TT_Z \\ \sum_j TT_X & \sum_j TT_Y & \sum_j TT_Z & N \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \Delta T \end{bmatrix} = \begin{bmatrix} \sum_j res_j TT_X \\ \sum_j res_j TT_Y \\ \sum_j res_j TT_Z \\ \sum_j res_j \end{bmatrix} \qquad (1)$$

where the shorthand notation $TT_X = \frac{\partial TT_j}{\partial X}$ We also use that fact that $\frac{\partial TT_j}{\partial T} = 1$.

The solution of this system of linear equations provides the change in source coordinate, such that the new estimates of the source coordinates are:

$$X_1 = X_0 + \Delta X$$

$$Y_1 = Y_0 + \Delta Y$$

$$Z_1 = Z_0 + \Delta Z$$

and

$$T_1 = T_0 + \Delta T$$

This process is repeated again to compute $X_2 = X_1 + \Delta X$, $Y_2 = \cdots$, $Z_2 = \cdots$ and $T_2 = \cdots$.

This iterative process continues until the changes are less than a predefined amount to define convergence.

Matrix Algebra

In setting up (1), we assume that all observations have equal importance. It is not difficult to weight the observations. However, we must now focus on the task of solving (1). Equation (1) is of the form $\mathbf{Ax} = \mathbf{b}$. A formal mathmatical solution is to determine the inversio of the matrix **A** to write

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{y}$$

References

Geiger, L. (1910). Herbsetimmung bei Erdbeben aus den Ankunfzeiten, *K. Gessell. Wiss. Goett.* **4**, 331-349

Geiger, L. (1912). Probability method for the determination of earthquake epicenters from the arrival time only, *Bull. St. Louis Univ.* **8**, 60-71.

What you must do:

What you must submit: