Introduction to Earthquake Seismology
Assignment 14

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Rays in a Constant Velocity Sphere

Goals:

- Understand the extension of Snell's law to a sphere
- Compute the travel-time versus distance relationship for a simple spherical model
- Understand surface reflections

Background:

As in introduction to travel times in the Earth, we will consider the simple case of a constant velocity sphere. Let the radius of the fluid sphere be a, the compressional wave velocity be c, the source be at a radius r_s, the receiver be at the free surface, the travel time be T, and the spherical angular distance of the receiver from the source be an angle Δ.

The easiest way to obtain a travel time distance plot is to compute both the travel time and distance parametrically in terms of the angle of the ray leaving the source i_s, which is 0° for a ray going downward from the source. The ray leaving the source will make an angle at the free surface, i_r, defined by the Snell's law relation for spherical wave propagation:

$$p = \frac{r_s \sin i_s}{c} = \frac{r_r \sin i_r}{c}, \tag{1}$$

where the receiver radius is defined here as $r_r = a$.

Let $N+1$ be the number of linear ray segments between the source and receiver. $N = 0$ indicates a direct path between the source and the receiver, and $N = 1$ indicates two ray segments with a single free surface reflection. The expressions for the travel time and distance are

$$T(i_s) = \frac{\left(r_s^2 + r_r^2 - 2 r_s r_r \cos(\pi - i_r - i_s)\right)^{1/2}}{c} + \frac{N 2 a \cos i_r}{c} \tag{2}$$

$$\Delta(i_s) = \pi - i_s - i_r + N(\pi - 2 i_r). \tag{3}$$

What you must do:

Create a spreadsheet starting with the following values: C = 10.0 km/sec, A=6000 km, RS=5400 km, RR=6000 km.

Create the following columns: IS, IR, Delta (degrees), p (sec/deg), Time(sec) which are created with the formulas for a given IR:

IR: =ASIN((RS/RR)*SIN(DEGRAD*A7))/DEGRAD

Delta=180-A8-B8+NZERO*(180-2*B8)

p=DEGRAD*RS*SIN(DEGRAD*A7)/'C'

Time=SQRT(RS*RS+RR*RR-2*RS*RR*COS((180-A7-B7)*DEGRAD))/C
+NZERO*2*A*COS(DEGRAD*B7)/C

where DEGRAD=3.1415927/180

Then copy the columns and create a formula replacing NZERO with NONE=1

• Plot the two travel time curves with the X-axis Delta(degrees) from 0 to 360, and with the Y-axis (travel time sec) from 0 to 2400 sec.

for the plots if $\Delta > 180$ degrees, use $360 - \Delta$ for the plot.

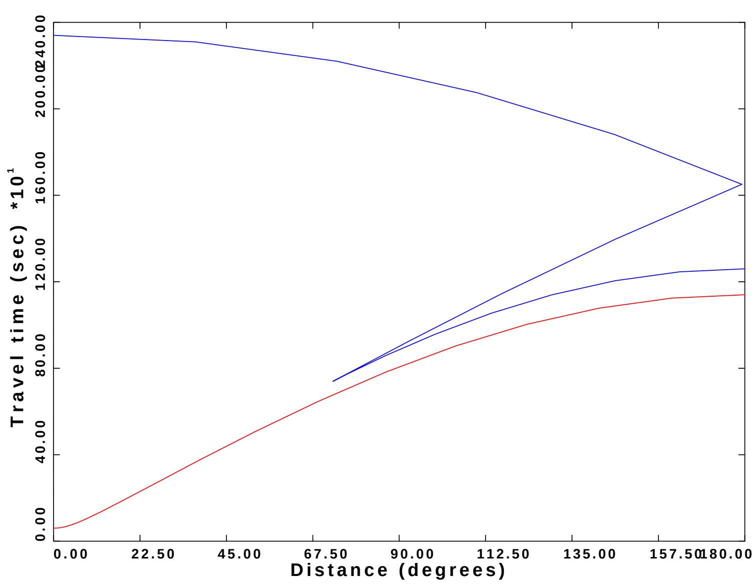
What you must submit:

• A derivation of equations (2) and (3)

• A table showing the computations.

• Also why is plotting $360 - \Delta$ permissible.

The plot may look as follows:

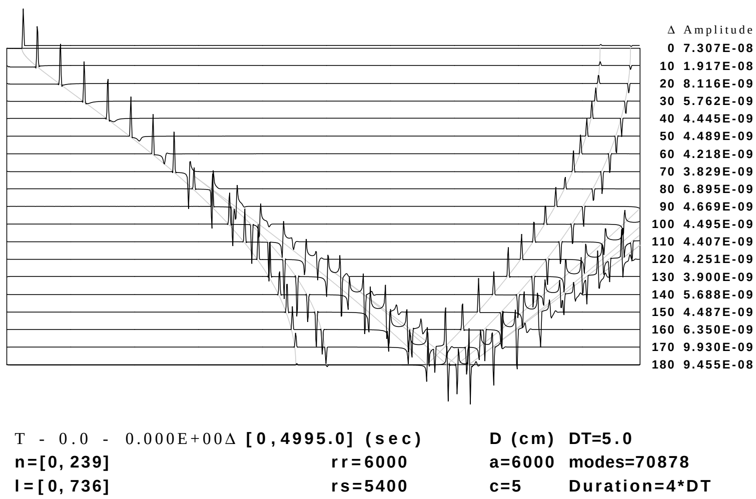


A paper by Herrmann in Seismological Research Letters (Herrmann, R. B. (1992). A Student's introduction to wave propagation in a homogeneous fluid sphere, Seism. Res. Letters 63, 161-167) and available at

http://www.eas.slu.edu/People/RBHerrmann/VITA/PUB/SRL_63_2_161-167.pdf

created synthetics for a homogeneous fluid sphere having a velocity of c=5 km/s. Thus the travel times of your problem and those of the paper differ by a factor of two. However the radius of the sphers and the source depths are the same.

This following figure from that paper shows the first 4990 seconds of the time history. The vertical axis is the epicentral distance in degrees.



The following figure shows the first 19990 seconds of the response. Note how the sphere keeps revererating due to the many arrivals. (Note that this is a reproduction of the figure in the paper and thus not perfect).