Development of lunar ephemeris LE-405/406 to Poisson series

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General description

The archive LEA-406.zip contains coefficients of the Poisson series representing a new analytical development of the modern long-term numerical ephemeris of the Moon LE-405/406 (Standish 1998). The development is made by using a new modification of the spectral analysis method which gives expansion of a tabulated function directly to Poisson series (where both amplitudes and arguments of the series' terms are high-degree polynomials of time).

The complete development, LEA-406a, includes 42270 Poisson series' terms of minimal amplitude equivalent to 1 cm and is valid over 1500–2500. A simplified version of the development, LEA-406b, includes 7952 series' terms of minimal amplitude equivalent to 1 m and is valid over 3000BC–3000AD. Over 1500-2500 the maximum difference between the lunar coordinates calculated by means of the complete analytical series LEA-406a and numerical ephemeris LE-406 is 3.2 m in geocentric distance, 0."0056 in ecliptic longitude, and 0."0018 in ecliptic latitude. Over 3000BC-3000AD the maximum difference between the lunar coordinates calculated by means of the simplified analytical series LEA-406b and numerical ephemeris LE-406 is 0.20 km in geocentric distance, 0."42 in ecliptic longitude, and 0."33 in ecliptic latitude.

Expansion form of lunar coordinates

As a set of variables describing the position of the Moon in space we chose spherical coordinates of its centre: r (geocentric distance), V (ecliptic longitude reckoned along the moving ecliptic from the mean equinox of date) and U (ecliptic latitude reckoned from the moving ecliptic).

Transformation of lunar rectangular coordinates from the reference plane of DE-405/406 (defined by the mean geoequator and equinox of epoch J2000) to the reference system defined by the moving ecliptic and mean equinox of date was done with use of the following precession quantities (Simon et al. 1994)

$$\theta_A = 20042.''0207t - 42.''6566t^2 - 41.''8238t^3 - 0.''0731t^4 - 0.''0127t^6 \tag{1}$$

$$\zeta_A = 23060.''9097t + 30.''2226t^2 + 18.''0183t^3 - 0.''0583t^4 - 0.''0285t^5 - 0.''0002t^6 \tag{2}$$

$$z_A = 23060.''9097t + 109.''5270t^2 + 18.''2667t^3 - 0.''2821t^4 - 0.''0301t^5 - 0.''0001t^6 \tag{3}$$

$$\varepsilon_A = 23^\circ 26' 21.''412 - 468.''0927t - 0.''0152t^2 + 1.''9989t^3 - 0.''0051t^4 - 0.''0025t^5 \tag{4}$$

where t is hereafter TDB measured in thousands of Julian years (365250 d) from J2000 (JD2451545.0). The expressions (1)-(4) are based on the value for the general precession in longitude p_A by Williams et al. (1991)

$$p_A = 50288.''200t + 111.''2022t^2 + 0.''0773t^3 - 0.''2353t^4 - 0.''0018t^5 + 0.''0002t^6 . \tag{5}$$

Geocentric spherical coordinates of the Moon r, V, U are expanded to Poisson series of the form

$$r(t) = \sum_{k=1}^{N_r} \left\{ A_{k0}^{(r)} \cos\left[\omega_k^{(r)}(t) + \varphi_{k0}^{(r)}\right] + A_{k1}^{(r)} t \cos\left[\omega_k^{(r)}(t) + \varphi_{k1}^{(r)}\right] + A_{k2}^{(r)} t^2 \cos\left[\omega_k^{(r)}(t) + \varphi_{k2}^{(r)}\right] \right\} \tag{6}$$

$$V(t) = \overline{V}(t) + \sum_{k=1}^{N_V} \left\{ A_{k0}^{(V)} \sin\left[\omega_k^{(V)}(t) + \varphi_{k0}^{(V)}\right] + A_{k1}^{(V)} t \sin\left[\omega_k^{(V)}(t) + \varphi_{k1}^{(V)}\right] \right.$$
$$\left. + A_{k2}^{(V)} t^2 \sin\left[\omega_k^{(V)}(t) + \varphi_{k2}^{(V)}\right] \right\} \tag{7}$$

$$U(t) = \sum_{k=1}^{N_U} \left\{ A_{k0}^{(U)} \sin\left[\omega_k^{(U)}(t) + \varphi_{k0}^{(U)}\right] + A_{k1}^{(U)} t \sin\left[\omega_k^{(U)}(t) + \varphi_{k1}^{(U)}\right] \right.$$
$$\left. + A_{k2}^{(U)} t^2 \sin\left[\omega_k^{(U)}(t) + \varphi_{k2}^{(U)}\right] \right\} \tag{8}$$

where

$$\bar{V}(t) = 218^\circ.31664563 + 17325643723.''0470t - 527.''90t^2 + 6.''665t^3 - 0.5522t^4 \quad (9)$$

is the mean longitude of the Moon referred to the moving ecliptic and mean equinox of date (Simon et al. 1994).

The argument of every series' term is a time polynomial obtained as a linear combination of multipliers of the fourth-degree polynomial expressions for the mean longitude of the ascending node of the Moon Ω (referred to the mean ecliptic and equinox of J2000), for Delaunay variables D, l', l, F (mean elongation of the Moon from the Sun, mean anomaly of the Sun, mean anomaly of the Moon, and mean longitude of the Moon subtracted by Ω, respectively), and for mean longitudes of eight major planets referred to the mean ecliptic and equinox of J2000 (Simon et al. 1994):

$$\Omega = 125.^\circ04455501 - 69679193.''631t + 636.''02t^2 + 7.''625t^3 - 0.3586t^4 \quad (10)$$

$$D = 297.^\circ85019547 + 16029616012.''090t - 637.''06t^2 + 6.''593t^3 - 0.3169t^4 \quad (11)$$

$$l' = 357.^\circ52910918 + 1295965810.''481t - 55.''32t^2 + 0.''136t^3 - 0.1149t^4 \quad (12)$$

$$l = 134.^\circ96340251 + 17179159232.''178t + 3187.''92t^2 + 51.''635t^3 - 2.4470t^4 \quad (13)$$

$$F = 93.^\circ27209062 + 17395272628.''478t - 1275.''12t^2 - 1.''037t^3 + 0.0417t^4 \quad (14)$$

$$\lambda_{Me} = 252.^\circ25090552 + 5381016286.''88982t - 1.''92789t^2 + 0.''00639t^3 \quad (15)$$

$$\lambda_{Ve} = 181.^\circ97980085 + 2106641364.''33548t + 0.''59381t^2 - 0.''00627t^3 \quad (16)$$

$$\lambda_{Ea} = 100.^\circ46645683 + 1295977422.''83429t - 2.''04411t^2 - 0.''00523t^3 \quad (17)$$

$$\lambda_{Ma} = 355.^\circ43299958 + 689050774.''93988t + 0.''94264t^2 - 0.''01043t^3 \quad (18)$$

$$\lambda_{Ju} = 34.^\circ35151874 + 109256603.''77991t - 30.''60378t^2 + 0.''05706t^3 + 0.04667t^4 \quad (19)$$

$$\lambda_{Sa} = 50.^\circ07744430 + 43996098.''55732t + 75.''61614t^2 - 0.''16618t^3 - 0.11484t^4 \quad (20)$$

$$\lambda_{Ur} = 314.^\circ05500511 + 15424811.''93933t - 1.''75083t^2 + 0.''02156t^3 \quad (21)$$

$$\lambda_{Ne} = 304.^\circ34866548 + 7865503.''20744t + 0.''21103t^2 - 0.''00895t^3 \quad (22)$$

Data format

Coefficients of Poisson series for every coordinate are given in the following files:

Folder LEA-406a (complete solution):

file LEA-406a-R.txt contains coefficients for the coordinate *r*;

file LEA-406a-V.txt contains coefficients for the coordinate *V*;

file LEA-406a-U.txt contains coefficients for the coordinate *U*;

Folder LEA-406b (simplified solution):

file LEA-406b-R.txt contains coefficients for the coordinate *r*;

file LEA-406b-V.txt contains coefficients for the coordinate *V*;

file LEA-406b-U.txt contains coefficients for the coordinate *U*.

Every file has the following structure:

Lines 1-8 give a short description of the data included to the file.

Every subsequent line contains coefficients of one (let's call it "the k^{th}") term in the relevant expansion [(6), (7) or (8)], namely:

NN, *l*, *l'*, *F*, *D*, Ω, *Me*, *Ve*, *Ea*, *Ma*, *Ju*, *Sa*, *Ur*, *Ne*, p_A, A_{k0}, A_{k1}, A_{k2}, φ_{k0}, φ_{k1}, φ_{k2},

where

- NN is the sequential number of the term;
- *l*, *l'*, *F*, *D*, Ω, *Me*, *Ve*, *Ea*, *Ma*, *Ju*, *Sa*, *Ur*, *Ne*, p_A are multipliers at the relevant values [given by the *complete* expressions (10)-(22) and (5), incl. the free term]. The respective linear combination yields a fourth-degree time polynomial argument $\omega_k(t)$;
- A_{k0}, φ_{k0} are the amplitude (in [km] for the coordinate *r* и in [arcsec] for the coordinates *V* и *U*) and phase (in [deg]) of the purely trigonometric part of the k^{th} term;
- A_{k1}, φ_{k1} are the amplitude (in [km/365250d] for the coordinate *r* and in [arcsec/365250d] for the coordinates *V* and *U*) and phase (in [deg]) of the 1[st] order Poisson part of the k^{th} term;

- A_{k2}, φ_{k2} are the amplitude (in [km/(365250d)^2] for the coordinate r and in [arcsec/(365250d)^2] for the coordinates V and U) and phase (in [deg]) of the 2nd order Poisson part of the k^{th} term;

The format of such a line is: I6,2X,5I3,1X,8I3,1X,I3,F16.7,2F11.6,3F19.12.

The total (over all three coordinates) number of non-zero trigonometric and Poisson terms is 42270 in the complete solution LEA-406a and 7952 in the simplified solution LEA-406b.

References

1. *Standish E.M.* - JPL IOM 312.F-98-048, 1998, Pasadena
2. *Simon J.L., Bretagnon P., Chapront J. et al.* - Astron. Astrophys., 1994, v.282, pp.663-683
3. *Williams J.G., Newhall XX, Dickey J.O.* - Astron. J., 1991, v.241, pp.L9-L12