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ORIGINAL ARTICLE

Determination of the free core nutation period from tidal gravity observations of the GGP superconducting gravimeter network

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Abstract This study is based on 25 long time-series of tidal gravity observations recorded with superconducting gravimeters at 20 stations belonging to the Global Geodynamic Project (GGP). We investigate the diurnal waves around the liquid core resonance, i.e., K_1, ψ_1 and φ_1, to determine the free core nutation (FCN) period, and compare these experimental results with models of the Earth response to the tidal forces. For this purpose, it is necessary to compute corrected amplitude factors and phase differences by subtracting the ocean tide loading (OTL) effect. To determine this loading effect for each wave, it was thus necessary to interpolate the contribution of the smaller oceanic constituents from the four well determined diurnal waves, i.e., Q_1, O_1, P_1, K_1. It was done for 11 different ocean tide models: SCW80, CSR3.0, CSR4.0, FES95.2, FES99, FES02, TPXO2, ORI96, AG95, NAO99 and GOT00. The numerical results show that no model is decisively better than the others and that a mean tidal loading vector gives the most stable solution for a study of the liquid core resonance. We compared solutions based on the mean of the 11 ocean models to subsets of six models used in a previous study and five more recent ones. The calibration errors put a limit on the accuracy of our global results at the level of ±0.1%, although the tidal factors of O_1 and K_1 are determined with an internal precision of close to 0.05%. The results for O_1 more closely fit the DDW99 non-hydrostatic anelastic model than the elastic one. However, the observed tidal factors of K_1 and ψ_1 correspond to a shift of the observed resonance with respect to this model. The MAT01 model better fits this resonance shape. From our tidal gravity data set, we computed the FCN eigenperiod. Our best estimation is 429.7 sidereal days (SD), with a 95% confidence interval of (427.3, 432.1).

Keywords Global Geodynamics Project · Free core nutation period · Tidal gravity models

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1 Introduction

The Luni-solar tidal potential produces changes of gravity associated with deformations of the Earth. The tidal deformations can be theoretically computed from real Earth models (Wahr 1981; Dehant 1987; Matsumoto et al. 1995). Of peculiar interest in this context is the existence of a free core nutation (FCN), due to the fact that the rotation axes of the mantle and of the core do not coincide in space. In Earth-fixed coordinates, this phenomenon is called nearly diurnal free wobble (NDFW).

This resonance, located in the diurnal band, somewhere between the waves K_1 and ψ_1, modifies the diurnal tidal spectrum (Melchior 1978). In space, the associated nutations are similarly affected (Herring et al. 1986). The FCN depends strongly on the coupling mechanism at the core-mantle boundary (flattening, topography, electro-magnetic coupling...). Confrontation between theoretical modelling (Dehant et al. 1999; Mathews 2001) and experimental determination using Earth tides and nutations is thus very important.

This study is based on the tidal gravity observations performed with superconducting gravimeters (SGs) in the framework of the Global Geodynamics Project (GGP) (Crossley et al. 1999). The main goal is the computation of mean tidal factors corrected from the ocean tide loading (OTL) effects and their comparison with the theoretical tidal factors computed using recent models of the Earth's response, such as DDW99 (Dehant et al. 1999) or MAT01 (Mathews 2001). It will also be possible to experimentally determine the FCN period.

Several investigations based on the GGP results have already been performed, but generally including few stations and selected ocean tide models (Hinderer et al. 2000; Sato et al. 2002, 2004). This study is an extension of a previous global approach by Ducarme et al. (2002), Sun et al. (2003) and Xu et al. (2004).

2 The tidal gravity observations

Since July 1997, some 20 stations equipped with SGs have been operating in the framework of the GGP. We selected data sets longer than 2 years available in the GGP data bank. The 1-min sampled data are carefully pre-processed and analysed at the International Centre for Earth Tides (ICET) following standard procedures (Ducarme and Vandercoilden 2000). These data are corrected using a remove-restore technique based on the *T-soft* software (Van Camp and Vauterin 2005) and decimated to 1 h prior to the analysis by the *ETERNA* software (Wenzel 1996).

Atmospheric pressure is the only auxiliary channel available at all the stations. The details concerning these 24 GGP data sets can be found in Xu et al. (2004). At Wuhan, we also consider the records obtained between 1 January 1986 and 29 June 1994 with T004 at a station located near the city centre. Finally, we introduce the results of the renovated ASK228 gravimeter at Pecny (Broz et al. 2002), which has an RMS error on the unit weight better than many of the oldest cryogenic instruments. Altogether, we thus consider 26 data sets from 21 stations. Eleven stations are located outside Europe. For each of them, we are able to extract 13 diurnal tidal groups: $\sigma_1, Q_1, \rho_1, O_1, \mathrm{NO}_1, \pi_1, P_1, K_1, \psi_1, \varphi_1, \theta_1, J_1, \mathrm{OO}_1$, named according to their main wave.

For each of the 13 diurnal wave groups, we computed the corrected tidal gravity vectors $\mathbf{A}_c(\delta_c A_{th}, \alpha_c)$ via the relation:

$$\mathbf{A}_c(\delta_c \mathbf{A}_{th}, \alpha_c) = \mathbf{A}(\delta A_{th}, \alpha) - \mathbf{L}(L, \lambda) \tag{1}$$

where $\mathbf{A}(\delta A_{th}, \alpha)$ is the observed tidal vector, $\mathbf{L}(L, \lambda)$ the computed tidal load vector for a given ocean tide model (see Sect. 3), A_{th} the theoretical amplitude of the corresponding main wave, δ and α the observed amplitude factor phase difference respectively (Melchior 1978). The corrected tidal factors (δ_c, α_c) can be directly compared with theoretical tidal factors derived from recent models of the Earth's tidal response, such as DDW99 (Dehant et al. 1999) or MAT01 (Mathews 2001).

3 The tidal loading corrections

We computed the OTL corrections using 11 different ocean tide models: AG95 (Andersen 1995), CSR3.0 (Eanes and Bettadpur 1996), CSR4.0 (Eanes and Schuler 1999), FES95.2 (Le Provost et al. 1994), FES99 (Lefèvre et al. 2002), FES02 (no paper published on this model), GOT00 (Ray 1999), NAO99 (Matsumoto et al. 2000), ORI96 (Matsumoto et al. 1995), SCW80 (Schwiderski 1980) and TPXO2 (Egbert et al. 1994).

Ocean tide models are either empirical, based on altimeter data (Topex/Poseidon or ERS), such as CSR, GOT, ORI and TPX, or hydrodynamical with assimilation of tide-gauge and satellite data, such as FES, NAO and SCW. All these models provide the four major diurnal components (Q_1, O_1, P_1, K_1). The SCW80 model was used as a working standard for more than 20 years, but its coverage is not sufficient in many areas.

The new generation of ocean tide models really emerged around 1995 with the use of satellite altimetry (Andersen et al. 1995). CSR3 and FES95.2 had been recommended by Shum et al. (1997) and tested on tidal gravity data by Melchior and Francis (1996). Most of the recent models have been intercompared and tested on tidal gravity data by Baker and Bos (2003) or Boy et al. (2003).

The tidal load vector $\mathbf{L}(L, \lambda)$, which takes into account the direct attraction of the water masses, the flexure of the ground and the associated change of gravitational potential, was evaluated by performing a convolution integral between the ocean tide models and the load Green's function computed by Farrell (1972). We used the computer code of Agnew (1997). The tidal load vectors are directly proportional to the amplitude of the wave in the exciting tidal potential, and the change of phase with changing frequency exhibits a regular behaviour, at least in the diurnal band (Ducarme et al. 2002).

In the diurnal band, it is thus reasonable to interpolate the load vectors for the minor components starting from the load vectors computed for the four major components. The load vectors have to be normalised first, dividing them by the corresponding theoretical amplitude in the tidal potential. It is necessary to take into account that the core resonance also affects the ocean tides (Wahr and Sasao 1981). Before interpolation, we thus have to correct this resonance effect for the main diurnal waves, especially K_1, then interpolate or extrapolate the weaker components and then again apply the resonance to the results.

This procedure, described in Xu et al. (2004), has been applied to the real and imaginary parts of the ocean load vectors computed using the 11 oceanic models listed earlier. We computed nine additional diurnal components: $\sigma_1, \rho_1, \mathrm{NO}_1, \pi_1, \psi_1, \varphi_1, \theta_1, J_1$, and OO_1. As some ocean tide models provide different weak components, it was possible (for the small waves ψ_1 and φ_1) to compare the OTL computed directly from the model or interpolated by our technique. From a test with FES95, the agreement is always very good concerning the phase, but the interpolated amplitude is some 10% lower. We are thus confident that our OTL interpolation, even if it is not perfect, does improve the results.

The ocean tide models never perfectly conserve the ocean mass during one tidal cycle. Algorithms have been proposed to restore the mass conservation by the repartition of the mass excess or deficiency among the different cells of the model (Melchior et al. 1980), using different mass conservation schemes. In Table 1, we give a comparison of the results obtained using the five most recent models with mass correction (MCOR) or without mass correction (NOMCOR), and show that the discrepancies among the mean corrected tidal factors are still within the error bars. It means that, statistically over the world ocean, the differences cancel each other and that for the purpose of this study, both options are certainly valid.

Finally, the load computations are affected by the discrepancies among the various ocean tide models, and the error in the OTL computations depends also on the location of the stations. In the diurnal band, the OTL is close to 0.5% of the corresponding gravity tide amplitude in Europe, while it can reach 5% in Japan. Moreover, it is difficult to precisely compute the OTL for stations located close to the coast with global ocean tides models. One should use a refined grid to model the coastal area. Zahran (2000) has shown that the mean of the ocean tide models has a better agreement with tide-gauge results than any of the individual models.

The determination of the FCN parameters is more stable when the mean of several ocean tide models is used for the correction of the OTL effect (Sun et al. 2002a). Our previous studies (Ducarme et al. 2002; Sun et al. 2003; Xu et al. 2004) used only six ocean tide models, i.e. CSR3.0, CSR4.0, FES95.2, ORI96, SCW80 and TPX02, for 22 SG time-series. Here we shall use the mean of 11 models and also compute a separate solution based on the five more recent models: CSR4.0, FES99, FES02, GOT00 and NAO99.

4 Preliminary investigation of the results

For each wave, we have a total of 286 values (26 data sets times 11 ocean tides models) for the amplitude factors and phase differences. We can compute the average load correction of the 11 charts for each station and determine so the mean corrected factors for each series m_i ($1 \leq i \leq 26$). Then we determine the global mean amplitude factor δ_c and phase difference α_c by averaging the 26 time-series and compute the corresponding standard deviation (σ_1) of m_i. Averaging the ocean models reduces the global ocean loading error due to the differences between ocean tide models. The dispersion σ_1 is thus mainly due to instrumental errors.

We simply apply the 3σ criterion for the rejection of outliers in the data sets based on the σ_1 value for the amplitude factors and phase differences. Bandung, being an equatorial station, has a very low signal-to-noise ratio for the diurnal waves and has been rejected by the 3σ criterion. Syowa is another difficult station. It is located on a small island close to the Antarctic coast and no global oceanic model seems to fit the observed residual. Thus, it was also rejected. On these grounds we kept 24 data sets for the main diurnal waves. A Pearson statistical test shows that the distribution is largely improved by the rejection of these stations.

For the small waves, i.e., waves with amplitude lower than 20 nm s^{-2} at a 45° latitude or 5% of K_1, the rejected series correspond to the noisiest ones: Brasimone, Kyoto, Wuhan (T004) and Wettzel (T103). The minimum number of series is 20 for ψ_1. The present situation is comparable with a previous study (Ducarme et al. 2002), where, for six ocean tide models, we kept respectively 21 data sets out of 22 series for the main waves and a minimum of 17 series for ψ_1.

The standard deviation σ_1 of the 24 series on the main diurnal tidal waves O_1, P_1 and K_1 is lower than 0.003 nm s^{-2} (0.3% relative error) for the amplitude factor, and 0.07° (0.1% relative error) for the phase differences. The associated RMS error on the mean tidal parameters is close to 0.0006 nm s^{-2} (0.05% relative error) for the amplitude factors, and 0.015° for the phase differences. The standard deviations increase at the very edge of the tidal spectrum, where the extrapolation of the load vectors from various ocean tide models increases the dispersion.

As the distribution is not a true normal/Gaussian one, even after rejection of some outlying stations, we still suspect systematic effects. The calibration of tidal gravimeters is not easy. The most usual procedure for SGs is to compare them in situ with absolute gravimeters operated continuously during a few days (e.g. Francis 1997; Sun et al. 2002). It is known that the calibrations are not

Table 1 Mean corrected tidal factors δ_c and α_c (deg) for the diurnal waves using five recent charts for comparison of ocean tidal loading (OTL) computation methods

Mean factors Five charts MCOR					Mean factors Five charts NOMCOR					Model DDW 99NH
Wave N	δ_c (ε_δ)	α_c (ε_α)	σ_1 ×100	Discr. ×100	Wave N	δ_c (ε_δ)	α_c (ε_α)	σ_1 ×100	Discr. ×100	δ_{th}
σ_1	1.1588	0.059	0.63	0.46	σ_1	1.1595	−0.017	0.77	0.53	1.1542
22	*± 0.0013*	*± 0.032*			*22*	*± 0.0016*	*± 0.078*			
Q_1	1.1551	0.028	0.38	0.08	Q_1	1.1553	−0.038	0.33	0.10	1.1543
24	*± 0.0008*	*± 0.027*			*24*	*± 0.0007*	*± 0.034*			
ρ_1	1.1558	0.019	0.29	0.15	ρ_1	1.1559	−0.032	0.22	0.16	1.1543
23	*± 0.0009*	*± 0.048*			*23*	*± 0.0005*	*± 0.052*			
O_1	1.1544	0.023	0.29	0.01	O_1	1.1546	−0.003	0.27	0.03	1.1543
24	*± 0.0006*	*± 0.021*			*24*	*± 0.0006*	*± 0.012*			
NO_1	1.1550	−0.003	0.42	0.11	NO_1	1.1550	0.001	0.44	0.11	1.1539
24	*± 0.0009*	*± 0.034*			*24*	*± 0.0009*	*± 0.041*			
π_1	1.1517	−0.056	0.81	0.10	π_1	1.1517	−0.058	0.79	0.10	1.1507
22	*± 0.0017*	*± 0.052*			*22*	*± 0.0017*	*± 0.050*			
P_1	1.1496	−0.006	0.27	0.05	P_1	1.1501	−0.045	0.31	0.10	1.1491
23	*± 0.0006*	*± 0.021*			*24*	*± 0.0006*	*± 0.017*			
K_1	1.1368	0.032	0.27	0.20	K_1	1.1367	0.018	0.19	0.17	1.1348
24	*± 0.0006*	*± 0.027*			*24*	*± 0.0006*	*± 0.019*			
ψ_1	1.2641	0.100	1.49	−0.71	ψ_1	1.2643	−0.017	1.58	−0.69	1.2712
20	*± 0.0033*	*± 0.181*			*20*	*± 0.0035*	*± 0.173*			
φ_1	1.1699	0.070	0.88	−0.06	φ_1	1.1703	0.044	0.89	−0.02	1.1705
24	*± 0.0018*	*± 0.081*			*24*	*± 0.0018*	*± 0.081*			
θ_1	1.1559	0.103	0.44	−0.11	θ_1	1.1557	0.005	0.56	−0.13	1.1570
21	*± 0.0010*	*± 0.093*			*22*	*± 0.0012*	*± 0.094*			
J_1	1.1568	0.050	0.46	−0.01	J_1	1.1570	−0.003	0.38	0.01	1.1569
24	*± 0.0009*	*± 0.060*			*24*	*± 0.0008*	*± 0.050*			
OO_1	1.1547	0.202	0.61	−0.16	OO_1	1.1556	0.100	0.57	−0.07	1.1563
22	*± 0.0013*	*± 0.140*			*22*	*± 0.0012*	*± 0.140*			

MCOR: with mass correction, NOMCOR: without mass correction, N: number of series, σ_1: standard deviation of the series for the corrected amplitude factors δ_c, δ_{th}: theoretical amplitude factor for the DDW99 non-hydrostatic/anelastic model, Discr.: difference $\delta_c - \delta_{th}$, ε_δ and ε_α (deg): RMS errors on the mean corrected tidal factors

very homogeneous inside the GGP network (Ducarme et al. 2002; Baker and Bos 2003). This directly affects the so-determined amplitude factors, increasing the dispersion of the results, and could introduce a general bias.

Therefore, we tried to investigate the homogeneity of the GGP network by comparing, for the main tidal constituents O_1, K_1, M_2 and S_2, the results from the 14 European series at 10 stations with those of 10 time-series outside Europe representing nine stations (Bandung and Syowa excluded; see earlier). The results show that there is a systematic difference at the 0.2% level on the amplitude factors of the diurnal and semi-diurnal waves. The bias of the global solution could thus be of the order of 0.1%.

5 Comparison of the results with models of the Earth response to the tidal forces

In Table 2, we give the mean corrected tidal factors and phase differences for 11 diurnal components and three ocean tide subsets (6, 5 and 11 models). It is found that there is no significant discrepancy between the results based on different sets of ocean tide models.

We compare the results in Table 2 with the δ_{th} computed with the DDW99, elastic or non-hydrostatic/anelastic (NH) models (Dehant et al. 1999), and MAT01 model. There are discrepancies between models at the 0.1% level. The mean value of $\delta_c(O_1) = 1.1546 \pm 0.0006$ agrees within 0.1% with the value $\delta_{th} = 1.1543$ computed from the DDW99NH model and the value $\delta_{th} = 1.1540$ given by MAT01. The DDW99 elastic model with $\delta_{th} = 1.1528$ is further away. Due to the possible bias of the global solution at the 0.1% level, we cannot refine our conclusions. The value derived from the European subset ($\delta_c(O_1) = 1.1536$) is closer to MAT01, while the result of the nine stations outside Europe ($\delta_c(O_1) = 1.1561$) is offset.

To study the shape of the observed resonance, it is very convenient to consider the ratio of the different waves with respect to O_1 (Table 3). The observations confirm the asymmetry of the tidal amplitude factors

Table 2 Mean corrected tidal factors δ_c and α_c (deg) for the diurnal waves using 6 (Ducarme et al. 2002), 11 and 5 charts (this study)

Wave N	Mean factors Six charts δ_c (ε_δ)	α_c (ε_α)	σ_1 $\times 100$	N	Mean factors Eleven charts δ_c (ε_δ)	α_c (ε_α)	σ_1 $\times 100$	Mean factors Five charts δ_c (ε_δ)	α_c (ε_α)	σ_1 $\times 100$	DDW 99NH δ_{th}	MAT 01 δ_{th}	α_{th}
σ_1	1.1550	0.164	0.47		1.1582	0.046	0.59	1.1595	−0.017	0.77	1.1542	1.1541	−0.026
19	*± 0.0011*	*± 0.043*		*22*	*± 0.0012*	*± 0.045*		*± 0.0016*	*± 0.078*				
Q_1	1.1538	0.044	0.37		1.1549	0.003	0.36	1.1553	−0.038	0.33	1.1543	1.1541	−0.026
20	*± 0.0008*	*± 0.026*		*24*	*± 0.0007*	*± 0.014*		*± 0.0007*	*± 0.034*				
ρ_1	1.1545	0.017	0.35		1.1558	−0.004	0.26	1.1559	−0.032	0.22	1.1543	1.1541	−0.026
17	*± 0.0009*	*± 0.048*		*23*	*± 0.0005*	*± 0.044*		*± 0.0005*	*± 0.052*				
O_1	1.1544	0.010	0.32		1.1546	0.002	0.27	1.1546	−0.003	0.27	1.1543	1.1540	−0.024
20	*± 0.0007*	*± 0.010*		*24*	*± 0.0006*	*± 0.014*		*± 0.0006*	*± 0.012*				
NO_1	1.1553	−0.023	0.58		1.1552	−0.006	0.42	1.1550	0.001	0.44	1.1539	1.1535	−0.021
19	*± 0.0012*	*± 0.041*		*24*	*± 0.0008*	*± 0.039*		*± 0.0009*	*± 0.041*				
π_1	1.1510	−0.054	0.67		1.1517	−0.063	0.79	1.1517	−0.058	0.79	1.1507	1.1504	−0.008
17	*± 0.0016*	*± 0.091*		*22*	*± 0.0017*	*± 0.050*		*± 0.0017*	*± 0.050*				
P_1	1.1501	−0.039	0.29		1.1501	−0.047	0.30	1.1501	−0.045	0.31	1.1491	1.1489	−0.002
21	*± 0.0006*	*± 0.016*		*24*	*± 0.0006*	*± 0.016*		*± 0.0006*	*± 0.017*				
K_1	1.1362	0.025	0.31		1.1364	0.017	0.27	1.1367	0.018	0.27	1.1348	1.1349	0.062
21	*± 0.0007*	*± 0.015*		*24*	*± 0.0005*	*± 0.016*		*± 0.0006*	*± 0.019*				
ψ_1	1.2630	0.073	1.36		1.2641	−0.018	1.59	1.2643	−0.017	1.58	1.2712	1.2655	0.022
16	*± 0.0034*	*± 0.230*		*20*	*± 0.0036*	*± 0.172*		*± 0.0035*	*± 0.173*				
φ_1	1.1691	0.061	0.79		1.1701	0.043	0.89	1.1703	0.044	0.89	1.1705	1.1693	−0.068
19	*± 0.0018*	*± 0.096*		*24*	*± 0.0018*	*± 0.083*		*± 0.0018*	*± 0.081*				
θ_1	1.1569	0.097	0.79		1.1555	0.065	0.69	1.1557	0.005	0.56	1.1570	1.1564	−0.028
18	*± 0.0019*	*± 0.132*		*22*	*± 0.0015*	*± 0.088*		*± 0.0012*	*± 0.094*				
J_1	1.1557	0.014	0.52		1.1561	0.031	0.43	1.1570	−0.003	0.38	1.1569	1.1562	−0.027
19	*± 0.0012*	*± 0.055*		*24*	*± 0.0009*	*± 0.041*		*± 0.0008*	*± 0.050*				
OO_1	1.1513	0.300	0.75		1.1536	0.187	0.67	1.1556	0.100	0.57	1.1563	1.1556	−0.024
18	*± 0.0017*	*± 0.071*		*22*	*± 0.0014*	*± 0.043*		*± 0.0012*	*± 0.140*				

N: number of series effectively included (same numbers of series for the 11 and 5 charts solutions), σ_1: standard deviation of the series for the corrected amplitude factors δ_c, ε_δ and ε_α (deg): RMS errors on the mean corrected tidal factors, δ_{th} and α_{th} (deg): theoretical amplitude factor and phase difference for the DDW99 non-hydrostatic/anelastic and MAT01 models

on each side of the resonance with larger values for J_1 than for Q_1. However, there are some contradictions. The ratio $\delta_c(K_1)/\delta_c(O_1)$ is 0.1% lower than the value reported by DDW99NH. For ψ_1 the observed ratio is 0.5% lower than the DDW99NH value. It corresponds to a shift of the observed resonance with respect to the value of 431 SD assumed in DDW99NH. This point will be illustrated in the next section.

MAT01 differs from DDW99NH not only by a slight offset of −0.025% at O_1 frequency (Table 2) but also in the shape of the resonance (Table 3). The decrease of K_1 is less pronounced, as well as the amplification of ψ_1 and φ_1. It is closer to the observed resonance shape. Concerning the phase advance of K_1 with respect to O_1 forecasted by Mathews (2001) (α_{th} in Table 2), no firm conclusion is possible. We observe a 0°.015 phase advance for K_1 with respect to O_1, but it is of the order of the associated RMS errors. To minimize the discrepancies between observations and modelling it is thus interesting to build experimental models by fitting the observed amplitude factors and phase differences of O_1, P_1, K_1, ψ_1 and ϕ_1 to a resonance model.

6 Determination of the FCN eigenperiod and associated models

The resonance of the liquid core affects the nutation spectrum as well as the diurnal tides. The accurate determination of the FCN parameters using astronomical observations is easier, as the most resonant tidal wave ψ_1 is small, while the associated annual nutation is large. Moreover, the corrected tidal parameters depend strongly on the OTL corrections. Up to now, the more reliable results, especially concerning the quality factor of the resonance, are derived from very long baseline interferometry (VLBI) observations.

Previous attempts to determine the FCN parameters using tidal gravity observations prior to the beginning of the GGP observation campaign are given by

Table 3 Ratio between the tidal amplitude factor δ_c of the different waves and $\delta_c(O_1)$

	Eleven charts $\delta_c/\delta_c(O_1)$ (ε)	Six charts $\delta_c/\delta_c(O_1)$ (ε)	Five charts $\delta_c/\delta_c(O_1)$ (ε)	DDW 99NH $\delta_{th}/\delta_{th}(O_1)$	MAT 01 $\delta_{th}/\delta_{th}(O_1)$	DSX11 C $\delta_{th}/\delta_{th}(O_1)$	DSX06 C $\delta_{th}/\delta_{th}(O_1)$	DSX05 C $\delta_{th}/\delta_{th}(O_1)$
Q_1	1.0003 ± *0.0009*	0.9995 ± *0.0010*	1.0006 ± *0.0009*	1.0000	1.0001	1.0002	1.0002	1.0002
ρ_1	1.0010 ± *0.0008*	1.0001 ± *0.0011*	1.0011 ± *0.0008*	1.0000	1.0001	1.0002	1.0002	1.0002
•O_1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
NO_1	1.0005 ± *0.0010*	1.0008 ± *0.0013*	1.0003 ± *0.0011*	0.9997	0.9996	0.9996	0.9995	0.9996
π_1	0.9975 ± *0.0018*	0.9971 ± *0.0017*	0.9975 ± *0.0018*	0.9969	0.9971	0.9969	0.9969	0.9970
P_1	0.9961 ± *0.0009*	0.9963 ± *0.0009*	0.9961 ± *0.0009*	0.9955	0.9956	0.9956	0.9957	0.9957
K_1	0.9842 ± *0.0008*	0.9842 ± *0.0009*	0.9845 ± *0.0009*	0.9831	0.9835	0.9841	0.9843	0.9844
ψ_1	1.0949 ± *0.0037*	1.0941 ± *0.0035*	1.0950 ± *0.0036*	1.1017	1.0966	1.0949	1.0941	1.0954
φ_1	1.0134 ± *0.0019*	1.0127 ± *0.0019*	1.0136 ± *0.0019*	1.0141	1.0133	1.0127	1.0125	1.0126
θ_1	1.0008 ± *0.0016*	1.0022 ± *0.0020*	1.0010 ± *0.0013*	1.0024	1.0021	1.0018	1.0018	1.0018
J_1	1.0013 ± *0.0011*	1.0011 ± *0.0013*	1.0021 ± *0.0010*	1.0023	1.0019	1.0016	1.0016	1.0016

ε: RMS errors on the ratio, • reference wave. DSXnn: experimental models derived from Table 4 (case C, nn: number of ocean tides models)

Neuberg et al. (1987), Defraigne et al. (1994) and Imanishi and Segawa (1998). There are many recent determinations of the FCN period since the beginning of the GGP (Hinderer et al. 2000; Sato et al. 2002, 2004; Ducarme et al. 2002; Sun et al. 2002a).

The FCN parameters are estimated by fitting the observed complex-numbered tidal admittances to a damped harmonic oscillator modelling the resonance. The solutions are computed using the amplitude factors, phase differences and associated RMS errors estimated for five waves: O_1, P_1, K_1, ψ_1 and φ_1. The complex amplitude factor of a diurnal tidal wave with frequency σ can be theoretically computed as (Defraigne et al. 1994; Sun et al. 2003)

$$\delta_{th}(\sigma) = \delta_0 + \bar{A}/(\sigma - \bar{\sigma}_{FCN}), \tag{2}$$

where δ_0 is the amplitude factor computed from the static Love numbers h_0 and k_0.

The resonance is expressed by the eigenfrequency of the FCN $\bar{\sigma}_{FCN}$and the resonance strength$\bar{A}$, both depending on the geometric shape of the Earth and the rheological properties of the Earth's mantle. For an anelastic Earth model, $\bar{A}$ and $\bar{\sigma}_{FCN}$ should be described as a complex number

$$\bar{A} = A_r + \mathrm{i}A_i\,, \quad \bar{\sigma}_{FCN} = \sigma_r + \mathrm{i}\sigma_i. \tag{3}$$

These quantities are the adjusted parameters, from which one can derive the eigenperiod of the FCN

$$T_{FCN} = \Omega/(\sigma_r + \Omega) \tag{4}$$

where Ω is the angular velocity of rotation of the Earth, and its quality factor

$$Q = \sigma_r/(2\sigma_i) \tag{5}$$

Considering O_1 as quasi-static, we can subtract its contribution from both sides of Eq. (2), according to a preliminary resonance model

$$\delta - \delta(O_1) = \left(\frac{\bar{A}}{\sigma - \bar{\sigma}_{FCN}} - \frac{\bar{A}(O_1)}{\sigma(O_1) - \bar{\sigma}_{FCN}} \right) \tag{6}$$

This will eliminate systematic errors such as a general bias in the tidal factors. The Bayesian approach (Florsch and Hinderer 2000; Sato et al. 2004) is an improvement of this technique, allowing a precise determination of Q, but it is not implemented here for reasons of simplicity.

On this basis, we computed different solutions for the eigenperiod deduced from our tidal gravity data. According to the number of ocean tide models considered, we obtained the three solutions listed in Case A of Table 4. The three solutions are close to the value deduced from VLBI observations: 429.5 SD. The slightly different result obtained with the five recent oceanic models is well inside the 95% confidence interval of the other ones.

In previous studies (Ducarme et al. 2002; Sun et al. 2003), based on six oceanic models and 20 tidal gravity series, we obtained periods ranging between 429.1 SD and 429.9 SD, based on different weighting or data rejection techniques. However, the confidence interval with six charts is too large compared with the other solutions. From the experimental results in Table 3, it was suspected that it was due to the low value of $\delta_c(\psi_1)/\delta_c(O_1)$. This is why we put a weight proportional to the amplitude of the waves (Case B, Table 4).

A weight inversely proportional to the associated variances would eliminate any contribution of the smaller constituents ψ_1 and ϕ_1 from the solution. The confidence interval decreases drastically in the case of six ocean models. The solution remains stable with 11 charts, while it is degraded using only five maps. This last point is associated with the fact that the ratio $\delta_c(K_1)/\delta_c(O_1)$ is too large in Table 3 for five ocean models. It is striking that minute differences in the tidal amplitude factors can produce noticeable effects on the determination of the FCN period.

Negative Q values are physically impossible, and result from numerical instabilities due to anomalous values of the phases (Sato et al. 2004). These phase errors are mainly due to the OTL modelling, since the determination of the transfer function of the SGs is now very precise and the time-lag corrections are homogeneous at the level of a few seconds (Van Camp et al. 2000).

Following the suggestion by Florsch and Hinderer (2000), we introduced a nonnegative function into the computation model in order to suppress this problem (Case C, Table 4)

$$Q = \frac{\sigma_r}{2\sigma_i} = 10^x \tag{7}$$

Meanwhile, Q is constrained to be no greater than 10^6 in inversion. For the FCN period, the results become homogeneous with the 95% confidence interval between 426 SD and 432 SD. The anomaly observed for the five ocean models solution is largely reduced. The FCN period is found at 429.7 SD, 429.5 SD and 428.5 SD with the 11, six and five ocean models respectively, in agreement with the values deduced from VLBI and tidal gravity observations in previous studies (Defraigne et al. 1994; Ducarme et al. 2002; Sato et al. 2002, 2004; Sun et al. 2002a, 2003).

It should be pointed out that the MAT01 model has a FCN period of 430.04 (*429.93–430.48*) SD (Mathews et al. 2002). In the DDW99NH model, the FCN period was forced to 431.37 SD, which was the value given by the VLBI observations at that time. It explains the different resonance shape pointed out in Sect. 5.

We finally built the experimental models DSX11, DSX6 and DSX5 corresponding to the results of the three different ocean tides models in Case C (Table 4). In Table 3, we see that these three models are very close. Even for ψ_1, the difference is lower than the experimental error. These models closely fit the observations on the resonant waves K_1, ψ_1 and φ_1. For K_1 and ϕ_1, the differences are only at the level of the third decimal.

7 Discussion and conclusions

We have analysed the results of 26 SG tidal gravity observation time-series at 21 globally distributed stations in order to compare the experimental results with models of the Earth's response to the tidal forces and to determine the FCN eigenperiod. To take full advantage of the unprecedented precision of these tidal gravity data, it was necessary to compute OTL corrections not only for four main diurnal waves (Q_1, O_1, P_1, K_1), but also for the minor components. Thus, we had to interpolate or extrapolate the existing load vectors to neighbouring frequencies, taking into account the effect of the liquid core resonance on the diurnal ocean tides.

We computed nine additional components: σ_1, ρ_1, $NO_1, \pi_1, \psi_1, \varphi_1, \theta_1, J_1$ and OO_1. We used 11 different ocean tide models: SCW80, CSR3.0, CSR4.0, FES95.2, FES99, FES02, TPXO2, ORI96, AG95, NAO99 and GOT00, subdivided in two subsets: six models used in a previous study and the five more recent ones. No significant differences appear in the mean corrected amplitude factors and phase differences between the different sets of ocean tide models and the different computing procedures, with or without mass correction.

We get a mean amplitude factor for $O_1 (\delta_{O1} = 1.1546)$ close to the value $\delta_{th} = 1.1543$ predicted by the DDW99 non-hydrostatic/anelastic model and to the value $\delta_{th} = 1.1540$ of the MAT01 model, but the accuracy is not better than ±0.1%, because we noticed a bias between the European GGP stations and the rest of the world. We observe a slight phase advance of K_1 with respect to O_1, as suggested by MAT01, but it is not significant as the RMS error on the phases is still too large.

We used different approaches for the computation of the FCN eigenperiod from our tidal gravity observations, using 5, 6 or 11 ocean tide models, but the results are not very sensitive to the computation scheme. With the optimal solution (C, Table 4), we obtain peri-

Table 4 Period T of the FCN resonance with 95% confidence interval

Ocean tide models	T (sidereal days) *(95% confidence interval)* Case A	T (sidereal days) *(95% confidence interval)* Case B	T (sidereal days) *(95% confidence interval)* Case C
11	429.5 *(427.3, 431.8)*	429.2 *(426.5, 432.0)*	429.7 *(427.3, 432.1)*
6	429.5 *(413.5, 446.7)*	429.3 *(426.1, 432.5)*	429.5 *(426.9, 432.0)*
5	428.7 *(426.3, 431.1)*	427.9 *(410.6, 446.7)*	428.5 *(426.1, 430.9)*

Case A: no weight, Case B: weights proportional to the amplitude of the waves, Case C: weights proportional to the amplitude of the waves + Eq. (7)

ods between 428.5 SD and 429.7 SD, close to the value 429.5 SD deduced from VLBI observations and to other independent evaluations such as 429.7 ± 1.4 SD by Sato et al. (2004). The 95% confidence interval is between 426 SD and 432 SD. The experimental models, derived from these resonance parameters, only show discrepancies that are below the RMS errors of the observations. The most recent theoretical models of the liquid core resonance are based on slightly higher FCN periods (431 SD for DDW99NH and 430 SD for MAT01) and do not recover the exact resonance shape.

Minute variations of the mean corrected tidal factors of the resonant waves can still produce noticeable variations of the FCN eigenperiod. To improve the determination of the FCN parameters from tidal gravity observations involving SGs, it may be necessary to implement a Bayesian approach so as to concentrate on stations where OTL is weak in the diurnal band, e.g. in Europe, and to carefully select the best ocean models for this region.

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