First Arrivals, Slowness and Depth

Goals:
• this extends Assignment 12, but now requiring you to plot the slowness and depth derivative of the first arrival as a function of depth for a simple model.

Background:
For a single layer over a halfspace velocity model, travel time computations are simply for a source within the layer. Assuming a layer with thickness $H$ and velocity $V_1$ overlying a halfspace with velocity $V_2$ and a source depth of $h$ measured from the surface, the equations of the arrivals of interest are as follow:

**Direct:**

$$ t_{direct} = \frac{\sqrt{x^2 + (h)^2}}{V_1} $$

(1)

**Refraction**

$$ t_{refr} = \frac{(2H - h) \cos i_{c12} + x}{V_1} + \frac{x}{V_2} $$

(2)

where the critical angle is defined by the relation based on Snell’s law:

$$ \sin i_{c12} = \frac{V_1}{V_2} $$

(3)

These equations are simply derived and agree with those of Assignment 10 in the limiting case of
\( h- > 0. \)

**What you must do:**

1. For the direct arrival and the refracted arrival, derive the following derivatives: \( dt/dx \) and \( dt/dh \).

For the following model:

<table>
<thead>
<tr>
<th>H (km)</th>
<th>V_p (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>6.0</td>
</tr>
<tr>
<td>-</td>
<td>8.0</td>
</tr>
</tbody>
</table>

For source depths of 0, 10, 20, 30 and 40 km,

a) Compute the and plot the two derivatives for the first arrival.

**What you must submit:**

a) Plot the \( dT/dx \) for the direct and refracted arrivals using a different color/line type for each source depth and a different line type/color for the direct and refracted arrivals for the distance range 0 - 300 km

b) Write a paragraph telling me how source depth affect these derivatives.

c) At large distances, is there any sensitivity to source depth in the \( dT/dh \) quantity?

d) Considering the \( dT/dh \) values, at which distances must you have data in order to have a sensitivity with source depth changes? A location program will require data at different distances to see different different values of \( dT/dh \).

**NOTE:**

If you use EXCEL, or oocalc, to do the work, you can use a formula to create a column of first arrivals. For example if Column A2 is distance, Column B2 is the time of the direct arrival, and Column C2 is the time of the refracted arrival for a given source depthm then you can use a formula to create column D2, the first arrival time, e.g., the entry in D2 would be

\[
=\min(B2:C2)
\]