Amplitude and $Q$ of $\phi S_0$ from the Sumatra Earthquake as Recorded on Superconducting Gravimeters and Seismometers

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Online material: Testing window shifts, use of synthetic data, and tables of all frequencies, amplitudes, and $Q$ values for Figures 5, 6, and 7.

INTRODUCTION

It has been known for some time that superconducting gravimeters (SGs) can provide excellent recordings of the seismic normal modes. Studies by Banka and Crossley (1999) and van Camp (1999) established the low noise level of SGs in the seismic normal mode band, especially at periods longer than 100 s. In an extensive study of all the currently operating SGs (about 18), Rosat et al. (2003) showed that the best SGs have lower noise than the New Low Noise Model (Peterson 1993) at periods longer than 1,000 s. Also, several studies have compared the performance of SGs and broadband seismometers at long periods and shown that the correction of meteorological influence on the SGs can be more effective than on seismometers (Freybourger et al. 1997; Hinderer et al. 2002). Zürn and Widmer-Schnidrig (2006) demonstrated the need to correct seismometers for pressure effects at long periods. In the frequency domain the sensitivity of SGs reaches 1 nanogal (10^{-11} m/s^2) or less, especially at periods longer than 100 s. Widmer-Schnidrig (2003) summarized the advantages of using SGs for long-period seismology, noting that in addition to sensitivity the SGs are calibrated to DC with an amplitude accuracy of better than 0.1%. Here we combine and compare, for the first time, seismic data from the Incorporated Research Institutions for Seismology (IRIS) Data Management System and data from SGs contributing to the Global Geodynamics Project (GGP) (Crossley et al. 1999).

The 26 December 2004 Sumatra earthquake is the largest event to be recorded both by the Global Seismographic Network (GSN) and SGs, with a magnitude larger than 9.0. It therefore provides a very good opportunity to test the calibration levels of our instruments (Davis et al. 2005; Park, Song, et al. 2005). Rosat et al. (2007) recently published a study that is similar in some ways to our own. Their goal was to use the recording of the fundamental radial mode $\phi S_0$ as a means of possibly identifying upper mantle heterogeneity by accurately taking account of the contamination of $\phi S_0$ by $\phi S_2$ and noting the geographic distribution of the amplitudes recorded by SGs. We shall refer to this study later in the discussion.

Our study most closely follows that of Davis et al. (2005), which pointed out that the goal of the GSN is to publish instrument responses "to an accuracy of 1% in amplitude." They used seismic data to show that instruments from the GSN can provide constant amplitudes for $\phi S_0$, but there are many stations where the calibrations are clearly more than 1% away from the best stations. In this study we want to see how well SGs perform by comparing the results of amplitude and $Q$ from SGs and seismometers. The question is, do SG data give a better amplitude calibration than seismic data in the low frequency band?

The radial mode $\phi S_0$ has a period of 20.5 m, and its amplitude at the surface of the earth should be the same everywhere on a spherically symmetric, non-rotating, elastic, and isotropic (SNREI) earth model, with no splitting. In the frequency domain this mode has a single peak, and due to its small shear attenuation, we can easily identify its peak many months after the earthquake. Careful measurement of the amplitude of $\phi S_0$ can provide a constraint on our instrument response and give a check of the calibration (Park, Butler, et al. 2005).

DATA

Normal modes can easily be seen in seismic or gravimeter data following earthquakes of magnitude $M_w > 6.5$. The initial estimate of magnitude for the Sumatra 2004 event was 9.0, but this was later revised upward to 9.3 by Okal and Stein (2005) based on observations of long-period modes (see comment in Electronic Supplement). This makes it the largest event since the beginning of the SG network in 1997, and one of the largest moment magnitude events since Chile earthquake (1960). This guarantees that the normal mode peaks are seen with exceptional signal-to-noise ratio, even in the long-period band (right up to $\phi S_2$). The peak of $\phi S_0$ was still visible five months after the event, and reexcited by the second Sumatra earthquake ($M_w = 8.8$) in March 2005 (Zürn and Widmer-Schnidrig 2006).

The only problem with determining the amplitude of $\phi S_0$ is the uncertainty in calibration factor. Figure 1A shows the
They have largely replaced spring gravimeters for observatory instruments did not come online until 1997 (Crossley 2004). In Europe and China in the late 1980s, but most of the current SGs started recording at a few sites in the frequency point of 0S0. Thus the reduction in seismometer response by several orders of magnitude at the frequency of 0S0 leads to theoretical coupling to 0S0 via lateral heterogeneity, which is difficult to achieve at an accuracy better than 1%. SGs started recording at a few sites in Europe and China in the late 1980s, but most of the current instruments did not come online until 1997 (Crossley 2004). They have largely replaced spring gravimeters for observatory purposes due to their superior stability and sensitivity characteristics. Gravimeters record the ground motion as acceleration and can give a very good record at frequencies lower than 0.8 mHz (Widmer-Schnidrig 2003). Figure 1B shows the amplitude response of the antialiasing filter of a typical SG. We see that SGs differ from the seismometers in that the SG amplitude response is flat to DC in the normal mode frequency band.

We used the gravity records from 18 SG series (including some dual-sphere instruments, so there are only 15 distinct stations), which are located in Europe, Asia, and elsewhere (Figure 2). Because we wanted to compare the observed results from SGs and seismometers, we chose 52 seismic stations close to the SGs (Figure 2). The data length from GSN stations is in principle unlimited, but the raw (1- or 2-s) data from the GGP network is readily available for only 36 days following the Sumatra event. We could have used 1-m GGP data from the International Center for Earth Tides, but this had already been passed through another filter stage, so we chose to use the limited-length original SG data. Thus 36 days was taken as the common time period for all the SG and GSN stations.

We should mention here the issue of data units and conversions. The GSN data is recorded initially as counts and has to be converted to the appropriate units when the system response (Figure 1A) is removed. Because of the ultimate comparison with gravity data, we chose to process the seismic data as total acceleration, i.e., \( \omega^2 U' \), where \( \omega \) is the frequency and \( U' \) is the total equivalent seismic displacement. This is theoretically the sum of the inertial and free-air effects from a theoretical normal mode computation and should also include the perturbation in the gravity field, but there is no such additional term for 0S0 (or degree 1 modes) (see Dahlen and Tromp 1998, 238). The SG data was converted directly from voltage to acceleration using a constant scale factor determined from comparisons (usually) with absolute gravimeters to an accuracy of 0.1% or better (e.g., Amalvict et al. 2001). This is equivalent to using only the flat portion of the SG response (Figure 1B). Thus conversion between acceleration (\( \mu g \)) and displacement (micron) requires only the use of the factor \( \omega^2 \); the conversion for 0S0 is 1 \( \mu m = 0.0026201 \mu g \).

Figure 1. Nominal amplitude response of the seismometers and SGs used in this paper. (A) includes all the seismometers (CZ: Czech Regional Seismic Network; GE: GEOFON; II: Global Seismograph Network(GSN—IRIS/IDA); IU: Global Seismograph Network(GSN—IRIS/USGS); PS: Pacific21) and (B) shows the amplitude characteristics of a typical GGP filter board used for most SGs. Arrows show the frequency of 0S0; fc: corner frequency.

(METHODOLOGY)

For any singlet normal mode, its frequency \( \omega_0 \), initial amplitude \( A_0 \), phase \( \Phi_0 \), and decay rate \( Q \) are the main properties of interest for determining Earth structure and earthquake source parameters. In this research we measure the amplitude and decay rate of 0S0, which is well-isolated in the frequency band 0.7–1.0 mHz (Figure 3). The close proximity of the multiplet 0S3 leads to theoretical coupling to 0S0 via lateral heterogeneity, but there is no direct rotational or elliptical coupling between the two modes. The problem is how to measure the initial earthquake amplitude precisely. Several methods have been reported in the literature, mainly using the decay of spectral peaks (Roult et al. 2006). After starting this study, we discovered the paper of Davis et al. (2005), in which they presented two alternate methods for finding the initial amplitude of 0S0. Both methods
predetermined values of the eigenfrequency $\omega_0$ and damping $Q$. Because we wanted to also determine both the amplitude and $Q$ differences from seismometers and SGs, we chose a more direct method: using the amplitude spectrum of sliding windows of the data. The price we paid was not being able to easily estimate the initial phase of the Sumatra event (given as 65° by Davis et al. 2005; Park, Song, et al. 2005).

It is worth commenting that the phase response of SGs is not as well determined as their amplitude response. Unless a specific electronic calibration has been done on an instrument (only a handful of SGs have been calibrated on site by this method), the group phase delay due partly to the antialiasing filters (typically 8–16 s) may be known only within 10%. This can introduce uncertainties in determination of the mode phases, and this is made more problematic by the complex nature of the Sumatra source mechanism (e.g., Park, Butler, et al. 2005). This is the main reason we did not solve for phase in our study.

We first wanted to check the eigenfrequencies reported by Davis et al. (2005) and Rosat et al. (2007). To do this, we used the 36-day records for an SG station and applied a Hann data window (Dahlen 1982). The series were then padded to a power of 2 (> 3 times the length of the original data) and a normalized FFT amplitude spectrum is computed. We fitted a Hann spectral window (not the usual Lorentzian function because for a $Q$ of 5400 this is narrower than the Hann spectral window) to the amplitudes in the range 0.8135–0.8155 mHz, and used the rms levels in the bands 0.8120–0.8135 and 0.8155–0.8170 mHz as a measure of the noise. Our results from the 18 SG series are shown individually in Table S1 (Electronic Supplement), and the weighted mean value shown in Table 1 is compared to the

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**Figure 2.** The location of SGs (open circles) and seismometers (open triangles) and their great circle paths from the Sumatra 2004 earthquake (star).

**Figure 3.** Normalized amplitude spectrum of three SG stations using three days of data following the Sumatra earthquake. The peak $s_{S0}$ is unsplit, as is the test mode of frequency 0.725 mHz injected with an amplitude of 0.1 microgal. The error bands are used to determine the noise levels in the weighting for the amplitude and $Q$ calculations.
studies mentioned above. We confirm the value from Rosat et al. (2007), and both results seem to add slightly more precision to the eigenfrequency than previously reported from seismic records. For the remainder of the paper we used our weighted eigenfrequency to identify the peak amplitude; note this observational value is uncorrected by coupling effects to other modes.

As noted above, most amplitude measurements require a time domain taper that spreads the frequency domain energy and reduces the true amplitude at the expected peak. To address this problem we injected a test frequency of 0.725 mHz of amplitude 0.1 µgal into each time series and then measured the relative amplitude of the test mode and $0S_0$ and scaled these to the eigenfrequency than previously reported from seismic records. For each window we used a Hanning data taper and FFT depending on the quality of the record (generally six hours for SGs). For each window we used a Hanning data taper and FFT periodogram to compute the amplitude of $0S_0$ (calibrated by the test signal) and assigned a weight based on the noise level in the spectra either side of $0S_0$ (also shown in Figure 3). The weighting is an important quantity, because the noise level varied significantly during the 36-day series.

Seismic data generally had only short gaps, and they were interpolated by a straight line. SG data were essentially gap-free except for one exception. Station TG (Tigo Concepcion) had gaps longer than three days for which the data were not included in the inversion.

Equation 3 can be solved iteratively for $Q$ and $A_0$ by computing $A_{12}$ for each window. A typical station had about 800 estimates for $A_{12}$ in the decay curve. Figure 4 is an example of the result for SGs and seismometer sites. We reanalyzed series such as Figure 4B, MA (Matsushiro), and omitted the data that were noisy and gave large errors in amplitude determination. Interestingly, the solutions for $A_0$ and $Q$ did not improve compared to using the whole record, indicating the effectiveness of the data-weighting in the inversions. Apart from TC, all SG data were used, and even very noisy data such as from WU (Wuhan) yielded believable values.

| TABLE 1 |
| Comparison of $0S_0$ Frequencies |
| Davis et al. (2005) | 0.814657* |
| Rosat et al. (2007) SGs | 0.8146566 ± 1.6 ×10⁻⁶ |
| This paper SGs | 0.8146565 ± 1.2 ×10⁻⁶ |

*as corrected by P. Davis (personal communication)

This cannot be linearized as in Equation 2. Nowroozi (1968) used Equation 3 with only two separate time intervals, but we chose to implement Equation 3 using a moving window where $t_2 - t_1$ varied between one and days. We recognize that using overlapping windows leads to a nondiagonal data covariance matrix and thus compromises the least-squares inversion for the amplitude and $Q$. After extensive trials, however, we found that a three-day window with a one-hour time shift gave consistent results, and we further tested our analysis technique on a synthetic data series, which also supported our method. These tests are reported in detail in the Electronic Supplement (see Figures S1–S5). The first window had $t_1 =$ four or six hours, depending on the quality of the record (generally six hours for SGs). For each window we used a Hanning data taper and FFT periodogram to compute the amplitude of $0S_0$ (calibrated by the test signal) and assigned a weight based on the noise level in the spectra either side of $0S_0$ (also shown in Figure 3). The weighting is an important quantity, because the noise level varied significantly during the 36-day series.

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**RESULTS AND DISCUSSION**

**Gravity Processing**

Because SGs have a flat amplitude transfer function to DC, shown in Figure 1B, they naturally record the solid Earth and ocean tides at periods of four hours and longer. Even though there is little interaction between normal modes and tides, it is a tradition in gravity processing to first remove the tides and deal with the residual signal. Typical tidal amplitudes reach 50 µgal at a frequency of 0.03 mHz (M2 tide), and the issue is how much of this will leak into the long-period seismic band due to windowing effects. We compared $0S_0$ amplitude estimated from the original and tide-corrected time series. For $0S_0$ the tide removal has only a small effect, but at frequencies less than 0.08 mHz, i.e., the undetected Slichter modes, the signal leakage becomes significant. Because we wanted to get the most accurate result for amplitude, we subtracted a theoretical tidal model with nominal elastic parameters.

As discussed by many authors, e.g., Zürn and Widmer-Schnidrig (1995), it is also important to remove atmospheric
pressure effects at frequencies less than 1 mHz because of the increasing atmosphere-generated noise at these longer periods. This is not possible at all seismic stations because of the lack of a barometer, and even when a correction is made, the result is not as successful as with SGs. This is one reason why an SG performs well in the long-period seismic band. For all the SG stations we removed pressure using the standard admittance of $-0.3 \, \mu\text{gal/hPa}$.

Table S2 (in the Electronic Supplement) shows the complete results for the amplitude and $Q$ of $g_{0S0}$ from the 18 SG series. It can be seen that the mean weighted amplitude is $0.1582 \pm 0.0054 \, \mu\text{gal}$, and for $Q$ we find $5400.9 \pm 22.5$. We compared the results from Equation 2, the linear method, with Equation 3 and found some differences in both amplitude and $Q$. As these are probably due to noise in the data, we took the values computed from the more accurate Equation 3 for the rest of the paper.

**Seismic Processing**

As previously indicated, we processed the seismic data from IRIS by removing the instrument response and converting the counts into acceleration. We did not remove the tides, but otherwise processed the data as for the SG series. In all cases we obtained a clear peak for $g_{0S0}$. The final amplitudes and $Q$s are shown in Table S3 (Electronic Supplement) for all 52 seismic stations. It can be seen that the mean amplitude is $0.1425 \pm 0.0335$, and for $Q$ we find $5383.8 \pm 575.8$.

**DISCUSSION**

To compare the amplitude results we constructed a histogram of the amplitudes from both data sets and plotted the combined results in Figure 5. It can be seen that the SG amplitudes are better clustered that those from the seismometers, and the mean value is significantly higher. A formal estimate for the difference of the means (the $t$-test) yields a small probability (<0.01), indi-
cating the means may be statistically different. But the $F$-test for significance of the variances yields a value of 164, indicating the distributions have significantly different shape (this is evident in Figure 5), and thus the $t$-test may not be meaningful. We conclude only that the SG amplitudes may be biased slightly higher than the seismometer values.

A similar comparison for the $Q$ values is shown in Figure 6. In this case the population means are similar, but the spread of $Q$s from seismometers is again higher than for SGs. Note that the SG mean value for $Q$ (5400) is identical with the fixed value used in Davis et al. (2005), whereas the mean $Q$ from our seismometer stations is a little higher (5496).

We compare our results with those of Davis et al. (2005) by showing in Table 2 the initial amplitude of $\delta S_0$ in micron from Davis et al. (2005), along with our results. We chose only common stations and used only the seismometer results. Most stations have similar amplitudes but different error estimates. The differences may be due to the longer records (72 days) used in the Davis et al. (2005) study, but this seems unlikely. A scatter plot (Figure 7) confirms the similarity of amplitudes and verifies that our method for the determination of amplitudes agrees with the results from the two methods used by Davis et al. (2005) and in the later paper of Davis and Berger (2007).

Turning to the results of Rosat et al. (2007), we can make some comparisons with the amplitudes they computed. First, they show (in their Figure 1) the expected theoretical amplitude distribution of $\delta S_0$ from a comprehensive calculation of the coupling expected with a laterally heterogeneous mantle model. It can be seen that their surface pattern is similar to $\delta S_0$, which they claim leads to the strong coupling with this mode. Note their ratios are only 1.03 at the maximum, i.e., a 3% variation from the mean. To see if this could be picked up by the seismic data, we contoured our initial amplitudes of $\delta S_0$ from the 52 seismic stations where the coverage is much better than for the SGs. We also tried to contour the results for the 70 stations of Davis et al. (2005), but neither plot showed the pattern in Rosat et al. (2007), probably because of the still-large uncertainties in the seismometer calibrations.
TABLE 2
The Mean Amplitude and $Q$ Results from SGs, Seismometers

<table>
<thead>
<tr>
<th></th>
<th>Amplitude</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SGs</td>
<td>µgal</td>
<td>0.1582 ± 0.0054</td>
</tr>
<tr>
<td></td>
<td>micron</td>
<td>60.38 ± 2.06</td>
</tr>
<tr>
<td>Seismometers</td>
<td>µgal</td>
<td>0.1425 ± 0.0335</td>
</tr>
<tr>
<td></td>
<td>micron</td>
<td>54.39 ± 12.79</td>
</tr>
<tr>
<td>Davis et al. 2005</td>
<td>micron</td>
<td>57.7</td>
</tr>
</tbody>
</table>

**Figure 7.** Comparison between 16 seismic stations with $\phi S_0$ amplitudes from Davis et al. (2005) on the $x$ axis, and our results on the $y$ axis. Station KMBO was an outlier (see Table S3) and omitted from the comparison. The weighted regression line is shown, but is biased due to the errors in INCN and LSV.

**Figure 8.** Latitude variation of our amplitudes (circles) with the theoretical predictions of Rosat, Watada, and Sato 2007 (triangles). The latitude variation of an SNREI model is shown as squares.
We also compared another calculation done by Rosat et al. (2007), to check the latitude dependence of the SG amplitude values. The total observed acceleration is

\[ \omega^2 U = \omega^2 U + 1 + \frac{2g_0}{\omega R} \]  

(Dahlen and Tromp 1998, 238). The variation of \(g_0\) and \(R\) over the ellipsoid does indeed yield about 0.5% variation from pole to equator as plotted in Rosat et al. (2007, Figure 4A). Yet the free-air term is only about 0.1 of the inertial acceleration, so this reduces the role of the latitude variation on observed amplitudes to about 0.05%, a factor 0.1 less. We used the predicted theoretical variation of \(g_0\) for the 3-D model shown in Rosat, Watada, and Sato (2007, Figure 1) for all our SG stations (S. Rosat, personal communication) and plotted our SG amplitudes against these values in Figure 8.

Clearly the latitude variation of total displacement (including free-air correction) derived from an SNREI earth model is negligible compared to the amplitude variations from coupling to other modes by rotation, ellipticity, and lateral inhomogeneity in the mantle. The predicted variations at the current SG stations barely exceed 1%, and our observed amplitude variations agree at only half the stations, but this is insufficient to provide a satisfactory confirmation of theory. Clearly we would need a much denser array of SGs, comparable to the GSN, to pick up the larger variations (up to 3% in central Africa) that might discriminate between models of lateral heterogeneity.

CONCLUSIONS

We have demonstrated that SGs are a resource for studying normal modes of the Earth by focusing on the \(vS_0\) mode. The SG data should be considered as a new resource that can supplement previously used seismometer data. Specific conclusions of this study are the following:

From 18 SG recordings of the normal modes from the Sumatra earthquake, we find with small errors the eigenfrequency, initial amplitude, and \(Q\) of the radial mode \(vS_0\) as 0.8146565 ± 1.2° × 10^{-6} (mHz), 0.1582 ± 0.0054 (μgal), and 5400 ± 22 respectively.

The amplitude and \(Q\) from SGs have a significantly smaller variance than from seismometers, thus demonstrating the quality of SG data in long-period normal mode studies.

The mean weighted ground displacement from 18 SG series (60.38 ± 2.06 micron) compares well with that from 52 seismometers (54.39 ± 12.79 micron) and a previous study (57.7 micron) by Davis et al. (2005).

We could not verify that the geographical distribution of observed SG amplitudes confirms the theoretical predictions proposed by Rosat et al. (2007).

Further studies and the amplitudes of radial overtones and long-period modes using the Sumatra data set from SGs would be interesting.

ACKNOWLEDGMENTS

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