A STUDY OF EARTHQUAKE MAGNITUDES AND THEIR
RELATION TO THE LAW OF SEISMIC SPECTRUM SCALING

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It is known that short-period body-wave magnitude, $m_b$, is not a reliable indicator of the strength of very large earthquakes. For these earthquakes the surface-wave magnitude, $M_S$, appears to be a more reliable index of the strength. A possible solution to the body-wave magnitude problem would be to determine the magnitude from the lower-frequency part of the P-wave spectrum, such as would be obtained from the amplitude of $P$ as recorded by long-period seismographs. This P-wave magnitude is called $m_B$.

Predicted $m_b-M_S$, $m_B-M_S$ and $m_b-m_B$ curves were constructed from spectra obtained from some of the source models in current vogue; (Aki's $\omega^{-2}$, A and B Models; Archambeau's $V_r/V_p = \frac{1}{2}$, $R_s/R_o = 2$ Model; Berckhemer and Jacob's Model; and Brune's Partial Stress Drop Models $\varepsilon = 0.1$ and $0.01$). These models all predict a slope near 1 for the $m_b$ versus $M_S$ and $m_B$ versus $M_S$ curves at large magnitude. However, the different models predict widely varying values for the absolute difference between $m_B$ and $m_b$; this difference is a function of the slope of the spectra at intermediate frequencies.

Magnitude data obtained using the first few cycles of $P$ motion did not support the prediction of the models at high
magnitude, that is both $m_{b-M_S}$ and $m_{B-M_S}$ plots of observed data flatten out at the high magnitude end, although this is not predicted by the mathematical models. However, as the seismograms were analyzed, it was noted that a high percentage of the larger earthquakes had later $P$ phases which were consistently larger than the first arrival at all distances and azimuths. When these "multiple" earthquakes were identified, it was noted that they separated out (smaller $m_{b}$ and $m_{B}$ values) from the remaining shocks on the $m_{b-M_S}$ and $m_{B-M_S}$ plots and that the remaining earthquakes supported the predicted curves. Body-wave magnitudes, $m_{b}$ and $m_{B}$, were recomputed from the amplitude and period of the later "main event" for "multiple" earthquakes. On $m_{b-M_S}$ and $m_{B-M_S}$ plots the "simple" and "multiple" earthquakes became compatible, the scatter in the plots was reduced, and the plots supported the predicted curves.

The observed absolute difference between $m_{b}$ and $m_{B}$ was found to be independent of the nature of the earthquake ("simple" or "multiple") and supported predicted results of only Aki's Model B and Brune's Partial Stress Drop models.

$P$-wave spectra were computed for earthquakes over an $M_S$ range of 4.9 to 8.4. These spectra were characterized by two corner frequencies, a slope of -3 for the migration of the
low frequency corner with increasing displacement amplitude, a slope of -3 for the migration of the high frequency corner below $M_S = 7.1$, and a vertical migration of the high frequency corner above $M_S = 7.1$.

Predicted $m_b - M_S$, $m_B - M_S$ and $m_b - m_B$ curves were constructed from the observed spectra. The $m_b - M_S$ and $m_B - M_S$ curves obtained from the observed spectra do not support the Hanks and Wyss (1972) assumption that the S-wave spectrum is shifted by an amount $a/b$ to lower frequency with respect to the P-wave spectrum. The data support Savage's (1972) prediction that the spectra should be shifted the other way.

Conventional $m_b$ is considered to be a questionable parameter for all shallow earthquakes of $M_S$ greater than $5\frac{1}{2}$. Conventional $m_B$ has little relation to the strength of earthquakes of $M_S$ greater than $6\frac{1}{2}$. Conventional $m_B$ fares somewhat better in maintaining a relation to the size of the earthquake (at least $m_B$ increases when $M_S$ increases) for about another order of magnitude.