CHAPTER 4
MODAL SUMMATION

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14. Laterally Varying Media

This program permits one to approximate surface-wave propagation in a laterally varying medium. It assumes that the medium parameters change only with depth and distance from the source, so that no two-dimension ray tracing is required to define the path from the source to the receiver.

Program control is through the command line:

`slat2d96`*[flags]*, where the command flags are

`-D dist` (no default) desired distance

`-C cmdfil` (no default) file with distance-eigenfunctioninformation

The cmdfil has entries of the form

```
200    '../OCEAN/sregn96.egn'
200    '../GULF/sregn96.egn'
2000   '../SHIELD/sregn96.egn'
```

where the first entry indicates the width of the region and the second entry, enclosed in single quotes for a FORTRAN read, gives the name of the eigenfunction file created by **sregn96** or **slegn96**.

The works by using the models up to the given distance. For example, if the distance is 100 km, only the ocean model is considered; if the distance is 300 km, the ocean and gulf models are used.

`-?`

`-h` this help message

The program represents the laterally varying medium by a sequence of blocks. As the surface-wave propagates, no mode conversion or reflections are permitted. Conservation of energy is assumed. The theory is given in Keilis-Borok *et al.* (1989).

Following their presentation, in a laterally homogeneous model, the Fourier spectrum for a single mode at a distance, r, and buried at a depth, z, from the source which is at depth, h, is

$$u(\omega, r, \phi, z) = e^{-i\pi/4} \cdot \frac{1}{\sqrt{2\pi kr}} \cdot e^{-i\omega r/C} \cdot \frac{W(\omega, h)}{\sqrt{2CUI_0}} \cdot \frac{\varepsilon V(\omega, z)}{\sqrt{2CUI_0}} \qquad (1)$$

ω is the angular frequency, k is the wave number, C is the phase velocity, U is the group velocity, and $\mathbf{I_0}$ is the energy integral. ε is either a real or imaginary number, depending upon the component of motion. $\mathbf{V(\omega, z)}$ is the eigenfunction sampled at the receiver, and $\mathbf{W(\omega, z)}$ is a source mechanism dependent linear combination of the eigenfunctions at the source. The energy integral is defined as

$$\int_0^\infty \rho \left(\mathbf{V^{(3)}} \right)^2 \mathbf{dz}$$

for Love waves and

$$\int_0^\infty \rho \left[\left(V^{(1)} \right)^2 + \left(V^{(2)} \right)^2 \right] dz$$

for Rayleigh waves. The V (j) are the eigenfunctions. The term

$$A = \frac{1}{2CUI_0}$$

is called the amplitude response. The four terms in equation (1) represent, respectively, the phase term from the Hankel function, the propagation phase term from the Hankel function, the sampling of the wavefield at the receiver, and the sampling of the excitation at the source.

Chapter 2 of Keilis-Borok et al (1989) discusses surface waves in media with weak lateral inhomogeneity. In section 2.2, they state (2.19,2.20) that the Love-wave amplitude at a position (x, y, z) is

$$V_\phi^{(3)}(x, y, z) = K_L(\phi) \sqrt{\frac{C}{J\, I_1}}\, V^{(3)}(z) \tag{2}$$

where J is the geometrical spreading, C is the local phase velocity at (x, y), and the energy integral I_1 is related to the group velocity through the relation $I_1 = CUI_0$. The $K_L(\phi)$ is the source input applied to the beginning of the ray.

By taking the ration of amplitudes at two positions along the same ray using (2), (1) is modified to be give the following for a receiver at a depth z from the source which is at depth h is

$$u(\omega, r, \phi, z) = \exp(-i\pi / 4) \cdot e^{-i\omega \int_L ds / C(s)} \cdot \frac{1}{\sqrt{2\pi k_r J}} \cdot \frac{W_s(\omega, h)}{\sqrt{(2CUI_0)_s}} \cdot \frac{\varepsilon V_r(\omega, z)}{\sqrt{(2CUI_0)_r}}$$

where the subscripts, s, and, r, represent evaluations of the quantities at the source and receiver, respectively. The integral gives the phase delay over the path from the source to the receiver. J is the Jacobian for geometrical spreading. In the case considered here, e.g., a model with axial symmetry, $J = r$.

The program **slat2d96** will take as input a list of block widths, and corresponding eigenfunction file names generated by **srgn96** or **slegn96**. The desired receiver distance is entered, and the program that can be used by **spulse96** or **sdpegn96**. The only thing different, is that the amplitude response is adjusted to represent the difference in the two formulas, the wavenumber (and spatial anelastic coefficient γ and group velocity) are adjusted to provide and effective value for propagation, e.g.,

$$\mathbf{k_{eff}} = \frac{\int_L \left(\frac{\omega \mathbf{ds}}{\mathbf{C}}\right)}{\int_L \mathbf{ds}}$$

REFERENCES

Keilis-Borok, V. I., A. L. Levshin, T. B. Yanovskaya, A. V. Lander, B. G. Bukchin, M. P. Barmin, L. I. Ratnikova, and E. N. Its (1989). *Seismic surface waves in a laterally inhomogeneous earth*, Kluwer Academic Publishers, Dordrecht.

As an example, consider a source at a depth of 20 km in the ocean and a receiver on the continent. In a crude attempt to model offshore Oregon earthquakes as recorded on the North American continent, an ocean and a shield model will be considered. For each model the surface-wave eigenfunctions are required. To make this work simply the following directory structure is created with the following scripts and model files:

```
OCEAN-CONTINENT
|-- SHIELD
|    |-- DOIT
|    |-- shield.mod
|-- OCEAN
|    |-- DOIT
|    |-- ocean.mod
|-- NEWSHIELD
|    |-- DORECSEC
|-- NEWOCEAN-SHIELD
     |-- DORECSEC
```

The directories OCEAN and SHIELD compute the eigenfunctions for the synthetics for a source depth of 10.0 km for the OCEAN and SHIELD models, which are taken from Harkrider (1970). The models are compared in Figure 18. Note that the ocean model has some low velocity sediments which will influence the Love waves.

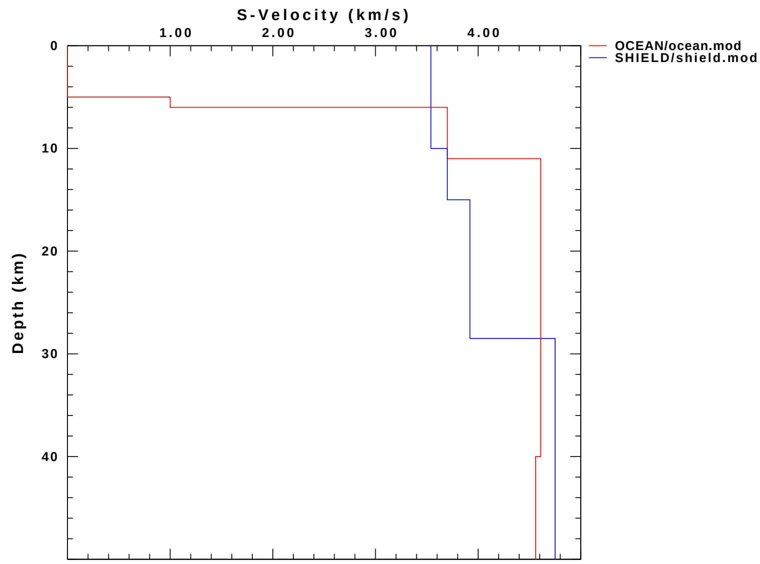

Fig. 18. Comparison of the upper 50 km of the ocean and shield models.

The NEWSHIELD directory uses **slat2d96** to compute synthetics for a pure shield model, while the NEWOCEAN-SHIELD used **slat2d96** to compute synthetics for a mixed path of 200 km of ocean and then a shield path. Synthetics are computed for an epicentral distance of 1000 km and then multiple filter analysis (MFT) (**do_mft**/ **sacmft96**) is applied to estimate the spectral amplitudes.The actual and MFT spectral amplitudes are compared as well as the observed and MFT estimated group velocities. MFT is used at this stage since this is the preferred technique used to select spectral amplitudes for source inversion. For use in actual source inversion, we need to know the appropriate value of the Gaussian filter α value as a function of length of the water path for good spectral amplitude estimates (α much be such that the Gaussian filter impulse response is greater than the underlying signal). We also need to know the robust range of periods that can be used for the source inversion so that lack of detailed knowledge of the oceanic path has minimal effect.

DOIT scripts: These differ only in velocity model used. Here is the script for the OCEAN model:

```
#!/bin/sh

MODEL=ocean.mod

sprep96 -L -R -M ${MODEL} -HS 10 -HR 0 -NMOD 1 -NPTS 8192 -DT 0.5 -FACL 20 -FACR 20
sdisp96
sregn96
slegn96
```

The *DORECSEC* script for making the synthetic for the laterally varying ocean-continent path is

```
#!/bin/sh

#####
#               copy the source model here
#####
cp ../OCEAN/ocean.mod .

MYPWD=`pwd`

for DIST in 1000
do

echo $DIST

rm -f *.[ZRT]EX *.[ZRT]DD *.[ZRT]SS *.[ZRT]DS

if [ ! -d ${DIST} ]
then
               mkdir $DIST
fi

#####
#               note that for synchronization it is necessary t hat the
#               defile be identical with the DISTANT used in the
#               slat2d96 (actually the slat2d96 only needs the path proportions
#####

cat > cmdfil << EOF
200             '../OCEAN/sregn96.egn'
2000            '../SHIELD/sregn96.egn'
EOF
slat2d96 -D ${DIST} -C cmdfil

cat > cmdfil << EOF
200             '../OCEAN/slegn96.egn'
2000            '../SHIELD/slegn96.egn'
EOF
slat2d96 -D ${DIST} -C cmdfil

cat > dfile << EOF
${DIST} 1 8192  0 0
EOF

spulse96 -FUND -EQEX -V -p -l 1  -d dfile -LAT -2 > file96
cat file96 | f96tosac -B -T
mv *.[ZRT]EX *.[ZRT]DD *.[ZRT]SS *.[ZRT]DS $DIST
mv *.egn $DIST
cd $DIST
gsac << EOF
r *.[ZRT]EX *.[ZRT]DD *.[ZRT]SS *.[ZRT]DS
rtr
int
w
q
EOF
#####
#               NOW DO MFT FOR THE Z COMPONENT, edit the MFT96CMP to handle the lategn
#####
sacmft96 -f 0${DIST}*.ZSS -PMIN 4.000000 -PMAX 100.000000 -a0 50.000000 -A \
       -VMIN 0.100000 -VMAX 5.000000 -U cm    -R -S -A
mv MFT96CMP MFT96CMPZSS.LAT
ed MFT96CMPZSS.LAT << EOF
/sdpegn/
s/YLIN/YLIN -LAT/
w
```

```
q
EOF
sh MFT96CMPZSS.LAT
cat MFT96.PLT SREGNU.PLT SACSPC96.PLT > ZSS.PLT

#####
#              NOW DO MFT FOR THE T COMPONENT, edit the MFT96CMP to handle the lategn
#####
sacmft96 -f 0${DIST}*.TSS -PMIN 4.000000 -PMAX 100.000000 -a0 50.000000 -A \
       -VMIN 0.100000 -VMAX 5.000000 -U cm  -L -S -A
mv MFT96CMP MFT96CMPTSS.LAT
#####
#    edit the MFT96CMOP script to work with the slat2d eigenfunctions
#####
ed MFT96CMPTSS.LAT << EOF
/sdpegn/
s/YLIN/YLIN -LAT/
w
q
EOF
sh MFT96CMPTSS.LAT
cat MFT96.PLT SLEGNU.PLT SACSPC96.PLT > TSS.PLT

cd ${MYPWD}

done

rm -fr file96 dfile cmdfil
```

The *DORECSEC* script in the NEWSHIELD directory is identical, except for the *cmdfil* which has entries of the form

```
cat > cmdfil << EOF
3000    '../SHIELD/slegn96.egn'
EOF
```

The next two figures compare the dispersion and amplitude spectra for an epicentral distance of 1000 km. The figures will have the ocean/shield model results at the top and the pure shield results at the bottom. The purpose is to understand the limitations of the shield model. For this trial, the vertical component fundamental mode spectra do not seem sensitive to the difference of the models. The only real difference is in the position of the spectral hole, which is at a shorter period in the shield model, and a slight difference in long period amplitudes near 25 sec. This result indicates that the shield model can be used to obtain source parameters for an oceanic event. However, the shield model will require a greater depth to explain the same spectra.

There are significant differences in the Love wave spectra shown in Figure 20 at all periods. The differences may be related to a factor of 6 difference in crustal thickness, as well as to the low velocity sediment layer. The conclusion here is that the use of a pure shield model for a composite path may lead to s significant effect on the source inversion.

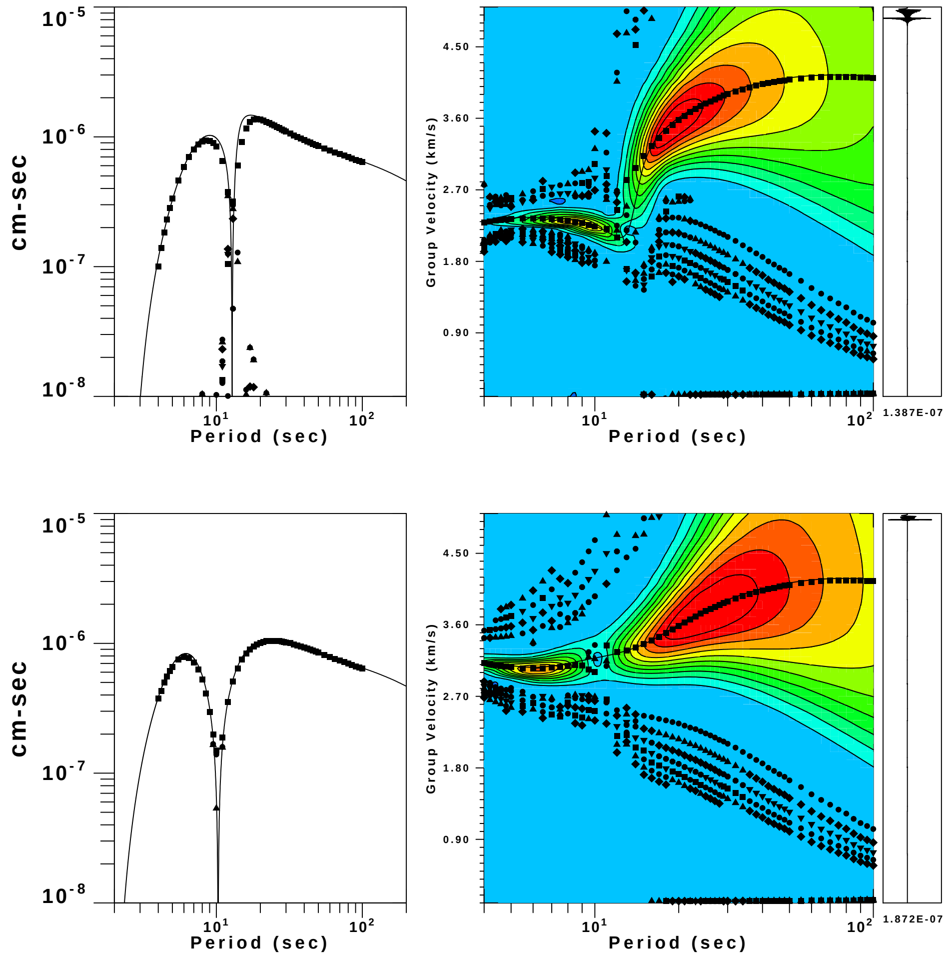

Fig. 19. Comparison of MFT output for ocean/shield path (top) and pure shield path (bottom) for the vertical component motion recorded from a vertical strike-slip source at a depth of 10 km.

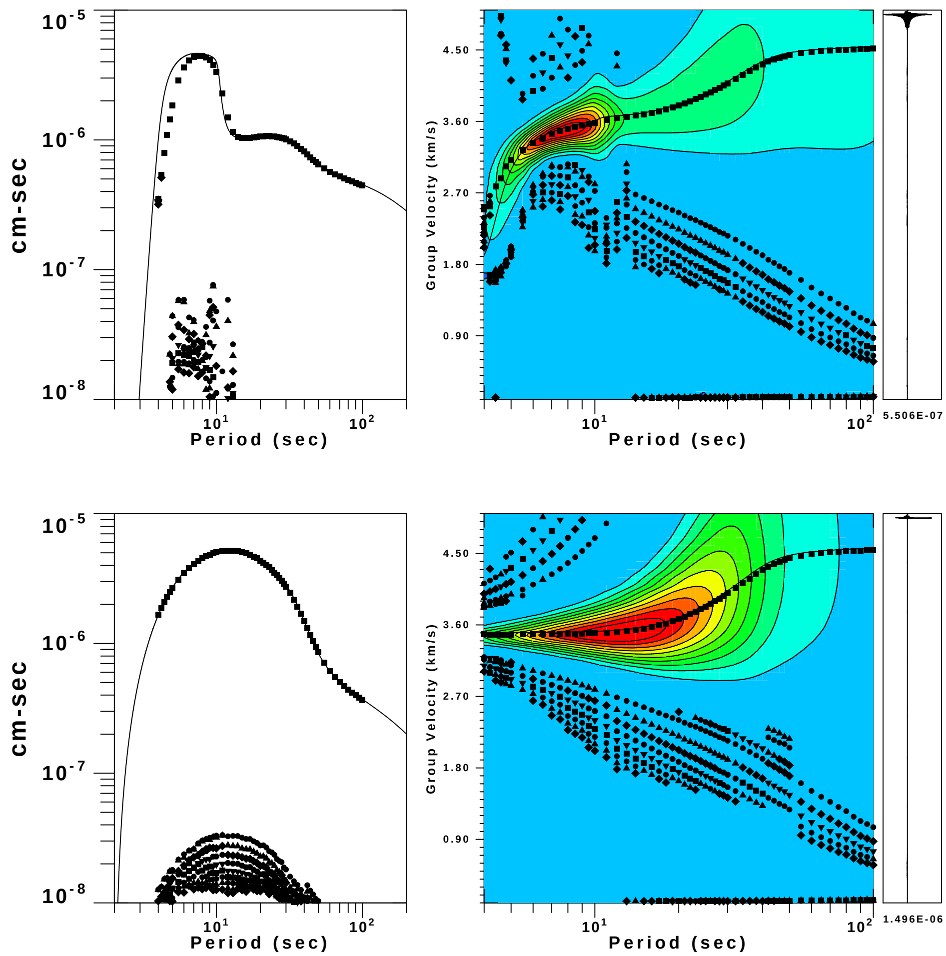

Fig. 20. Comparison of MFT output for ocean/shield path (top) and pure shield path (bottom) for the transverse component motion recorded from a vertical strike-slip source at a depth of 10 km.

TODO

To apply this with source inversion we need an *slat2d96der* program to create the depth dependent eigenfunctions used by the programs **sdprad96** and **srfgrd96**.