Introduction to Earthquake Seismology
Assignment 11

Department of Earth and Atmospheric Sciences **EASA-462**
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Simple Crustal Model - Source Depth

Goals:

- Compute and plot the P-wave first arrival travel times for a simple crustal model for different source depths within the crust

Background:

For a single layer over a halfspace velocity model, travel time computations are simply for a source within the layer. Assuming a layer with thickness H and velocity V_1 overlying a halfspace with velocity V_2 and a source depth of h measured from the surface, the equations of the arrivals of interest are as follow:

Direct:

$$t_{direct} = \frac{\sqrt{x^2 + (h)^2}}{V_1} \qquad (1)$$

Refraction

$$t_{refr} = \frac{(2H - h)\cos i_{c_{12}}}{V_1} + \frac{x}{V_2} \qquad (2)$$

where the critical angle is defined by the relation based on Snell's law:

$$\sin i_{c_{12}} = \frac{V_1}{V_2} \qquad (3)$$

These equations are simply derived and agree with those of Assignment 10 in the limiting case of

$h -> 0$.

What you must do:

For the following model:

Simple crustal model (SCM)	
H (km)	V_P (km/s)
40	6.0
-	8.0

For source depths of 0, 10, 20, 30 and 40 km,

a) Compute the direct arrival
b) Compute the refracted arrival
c) Plot the first arrival time (use the MIN function of EXCEL)

What you must submit:

a) Plot all first arrival times on the same figure using a different color/line type for each source depth for the distance range 0 - 300 km
b) Write a paragraph telling me how source depth affects the travel time curves.

Extra Material

Check these equations and text for errors.

If there is more than one layer, there will be no simple relation for the upward propagating direct arrival. A parametric form must be used. Assume that there are N layers in the medium and that the source depth is at the boundary of the k and $k+1$ layers (Fig.1) [placing the source at a layer boundary simplifies the computations, but ultimately requires the complexity of using the source depth to introduce additional layers into the model).]

The direct arrival is given as a function of the ray parameter p by the equations

$$X(p) = \sum_{i=1}^{k} \frac{H_i p V_i}{\sqrt{1 - p^2 V_i^2}} \tag{4}$$

$$T(p) = \sum_{i=1}^{k} \frac{H_i}{V_i} \frac{1}{\sqrt{1 - p^2 V_i^2}} \tag{5}$$

The refracted arrivals arise from a signal going down from the source, encountering a refracting layer and then propagate up to the surface. If the refracting layers if given by index $j \le N$, then the travel time - distance relation is

$$T = \sum_{i=k+1}^{j-1} \frac{H_i \sqrt{1 - p^2 V_i^2}}{V_i} + \sum_{i=1}^{j-1} \frac{H_i \sqrt{1 - p^2 V_i^2}}{V_i} + px \tag{6}$$

where $p = \dfrac{1}{V_j}$. Note that in this expression the order of the terms represents the downward, upward and horizontally propagating path contributions. *Also note that a refraction is not possible from a layer if any of the arguments of the square-roots is negative.*

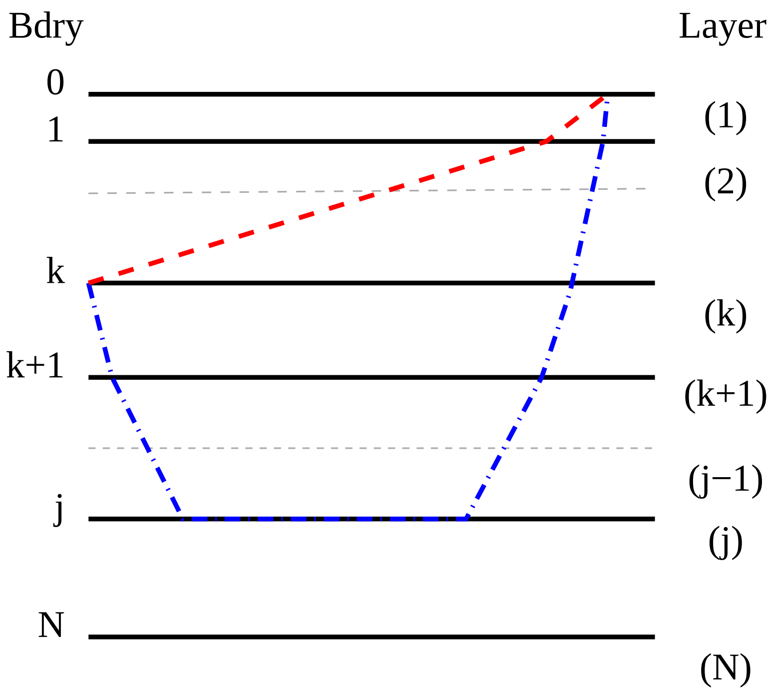

Fig.1. Multilayered medium showing the direct arrival, equations (4) and (5) (red-dashed) and refracted arrival, equation (6) (blue- dot-dashed) paths

For a continuous velocity versus depth model, the summations become integrals and we would have (Fig. 2)

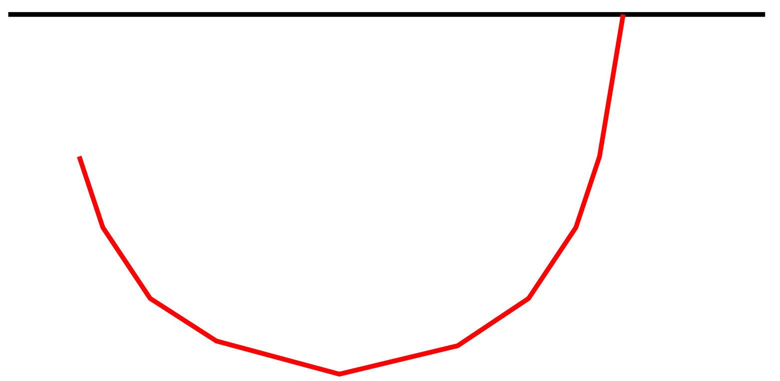

Fig. 2. Ray path for a model with a continuous increase of velocity with depth. Note that there is no horizontal ray segment in this case.

$$X(p) = \int_h^{Z_{\max}} \frac{pV}{\sqrt{1-p^2V^2}}\,dz + \int_0^{Z_{\max}} \frac{pV}{\sqrt{1-p^2V^2}}\,dz \tag{7}$$

$$T(p) = \int_h^{Z_{\max}} \frac{1}{V\sqrt{1-p^2V^2}}\,dz + \int_0^{Z_{\max}} \frac{1}{V\sqrt{1-p^2V^2}}\,dz \tag{8}$$

where $Z_{\max}$ is the maximum penetration depth of the ray and $p = \dfrac{1}{V(Z_{\max})}$.

If there is a sharp discontinuity in the velocity at $Z_{\max}$, Fig. 3., then we would have

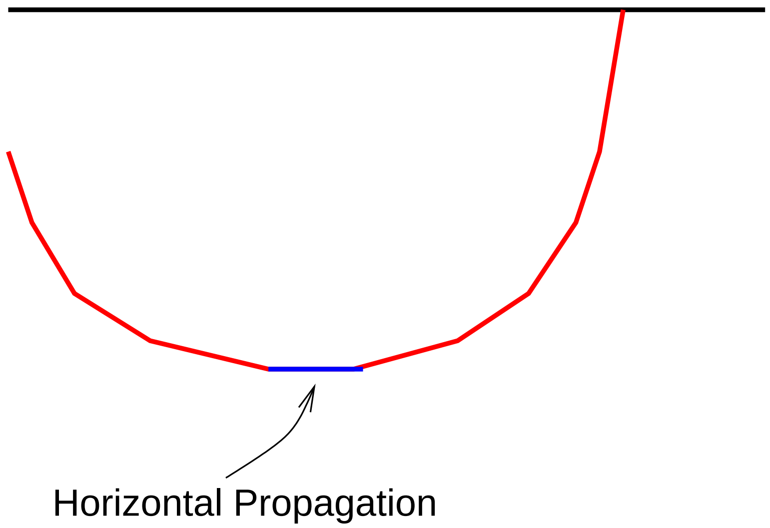

Fig. 3. Ray path for a model with a continuous increase of velocity with depth down to a sharp increase in velocity. Note that there is now a horizontal ray segment in this case.

$$T(p) = \int_h^{Z_{\max}} \frac{pV}{V\sqrt{1 - p^2V^2}}\, dz + \int_0^{Z_{\max}} \frac{pV}{V\sqrt{1 - p^2V^2}}\, dz + pX \tag{9}$$

where $p = \dfrac{1}{V(Z_{\max})}$.