

## Introduction to Earthquake Seismology Assignment 7

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### Distances and Azimuths on a Sphere

**Goals:**

- Given the latitude and longitude coordinates of a station and epicenter, compute the epicentral distance and the epicenter-to-station and station-to-epicenter azimuths.

**Background:**

Let  $\phi$  and  $\lambda$  represent latitude and longitude, respectively. In addition represent the epicenter, **E**, coordinates with an e subscript and similarly use the **S** and **s** to represent the station.

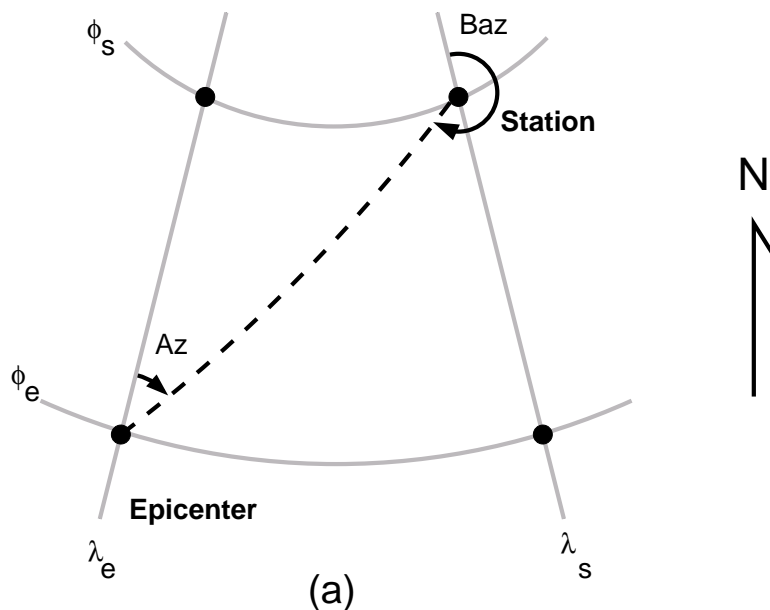


Fig. 1. Sketch of station and epicenter location on a spherical Earth

Computation of the epicentral distance  $\Delta$  is easily understood using vector analysis, such that the coordinates of the station and epicenter on a sphere with unit radius are

$$\mathbf{e} = (\cos \lambda_e \cos \phi_e, \sin \lambda_e \cos \phi_e, \sin \phi_e) = (e_1, e_2, e_3)$$

and

$$\mathbf{s} = (\cos \lambda_s \cos \phi_s, \sin \lambda_s \cos \phi_s, \sin \phi_s) = (s_1, s_2, s_3)$$

and the vector elements point in the latitude-longitude directions of (0,0), (0,90) and (90,0).

The great-circle arc distance between  $\mathbf{e}$  and  $\mathbf{s}$  is just

$$\cos \Delta = \mathbf{e} \cdot \mathbf{s} = e_1 s_1 + e_2 s_2 + e_3 s_3 \quad (1)$$

Although mathematically correct, this equation may yield numerically incorrect values of  $\Delta$  when the right side is  $\approx 1$ . Such cases occur when the vectors  $\mathbf{e}$  and  $\mathbf{s}$  are parallel. Figure 2 illustrates this case.

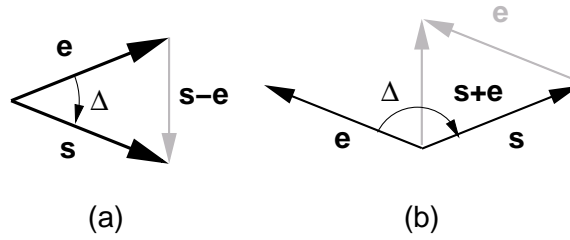


Fig. 2. Illustration of derivation of alternate formulas to determine  $\Delta$ .

Figure 2a illustrates the case when  $\Delta \approx 0$ . Since  $\mathbf{e}$  and  $\mathbf{s}$  are unit length vectors, the figure shows an isosceles triangle. Drawing a horizontal bisector through the angle, it follows that

$$\sin \frac{\Delta}{2} = \frac{|\mathbf{s}-\mathbf{e}|}{2} = \frac{1}{2} \sqrt{(s_1 - e_1)^2 + (s_2 - e_2)^2 + (s_3 - e_3)^2} \quad (2)$$

where the symbol  $||$  indicates the length of the vector. For  $\Delta \approx \pi$ , Figure 2b indicates that we should use the formula

$$\cos \frac{\Delta}{2} = \frac{|\mathbf{s}+\mathbf{e}|}{2} = \frac{1}{2} \sqrt{(s_1 + e_1)^2 + (s_2 + e_2)^2 + (s_3 + e_3)^2} \quad (3)$$

Note that (2) and (3) can be combined to form  $\sin \Delta = 2 \sin \frac{\Delta}{2} \cos \frac{\Delta}{2}$  used with (1) to give

$$\Delta = \tan^{-1}(\sin \Delta, \cos \Delta) \quad (4)$$

where  $\cos \Delta$  is given by (1), and

$$\sin(\Delta) = \frac{1}{2} |\mathbf{s} + \mathbf{e}| |\mathbf{s} - \mathbf{e}|$$

To compute the azimuth from the epicenter to the station, we will use spherical trigonometry to define  $\sin Az$  and  $\cos Az$ . Both are computed so that we can resolve the proper angle over a  $360^\circ$  range. Figure 3 shows the location of the epicenter and station. Since we have just determined  $\Delta$ , we know the lengths of three sides of the spherical triangle and one of the angles.

The rules of spherical trigonometry give the following relations:

$$\frac{\sin Az}{\sin(90 - \phi_s)} = \frac{\sin(\lambda_s - \lambda_e)}{\sin \Delta} = \frac{\sin(360 - Baz)}{\sin(90 - \phi_e)} \quad (5a)$$

and

$$\cos(90 - \phi_s) = \cos \Delta \cos(90 - \phi_e) + \sin \Delta \sin(90 - \phi_e) \cos Az \quad (5b)$$

These equations can be rewritten as

$$\cos \phi_e \sin \Delta \sin Az = \cos \phi_s \cos \phi_e \sin(\lambda_s - \lambda_e) \quad (6a)$$

$$\cos \phi_e \sin \Delta \cos Az = \sin \phi_s - \cos \Delta \sin \phi_e \quad (6b)$$

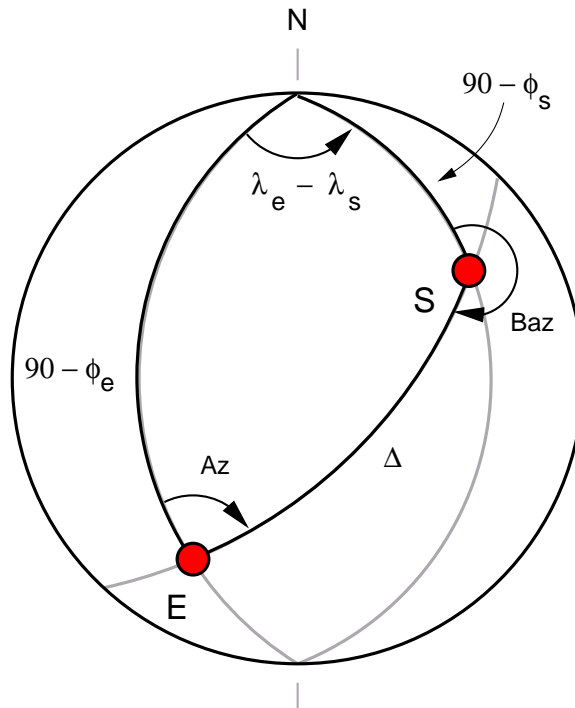


Fig. 3. Sketch of station and epicenter location on a spherical Earth. The epicenter is given by the symbol **E** with coordinates  $(\phi_e, \lambda_e)$  and the station is given by the symbol **S** with coordinates  $(\phi_s, \lambda_s)$ . The epicenter-to-station azimuth and station-to-epicenter back-azimuth are indicated.

Thus the azimuth in radians is

$$\text{atan2}(\cos \phi_s \cos \phi_e \sin(\lambda_s - \lambda_e), \sin \phi_s - \cos \Delta \sin \phi_e) \quad (7a)$$

in C or FORTRAN, or

$$\text{ATAN2}(\sin \phi_s - \cos \Delta \sin \phi_e ; \cos \phi_s \cos \phi_e \sin(\lambda_s - \lambda_e)) \quad (7b)$$

in an EXCEL or OpenOffice spreadsheet formula.

The reason for rewriting (5a) and (5b) as (6a) and (6b) was to avoid dividing by zero in some cases. In addition we realize that the angle is the same if we compute  $\tan^{-1}(y/x)$  or  $\tan^{-1}(ay/ax)$ , where  $a$  is a non-zero constant.

To convert from great circle arc in degrees to distance in kilometers, for the Earth we use  $111.195 \text{ km/sec}$ .

### What you must do:

Develop a formula for the back-azimuth  $Baz$ .

Determine the epicentral distance, azimuth from epicenter to station, and back azimuth for the following epicenter and station coordinates:

$$\phi_e = 40 \quad \lambda_e = 0 \quad \phi_s = 50 \quad \lambda_s = 10 \quad \text{Dist}(km) = \underline{\hspace{2cm}} \quad \text{Az}(deg) = \underline{\hspace{2cm}} \quad \text{Baz}(deg) = \underline{\hspace{2cm}}$$

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$$\phi_e = 60 \quad \lambda_e = 0 \quad \phi_s = 70 \quad \lambda_s = 10 \quad \text{Dist}(km) = \underline{\hspace{2cm}} \quad \text{Az}(deg) = \underline{\hspace{2cm}} \quad \text{Baz}(deg) = \underline{\hspace{2cm}}$$

$$\phi_e = 0 \quad \lambda_e = -30 \quad \phi_s = 60 \quad \lambda_s = 60 \quad \text{Dist}(km) = \underline{\hspace{2cm}} \quad \text{Az}(deg) = \underline{\hspace{2cm}} \quad \text{Baz}(deg) = \underline{\hspace{2cm}}$$

**What you must submit:**

Compare the difference between these computations and those done using the technique of Assignment 6.