Introduction to Earthquake Seismology
Assignment 15

Department of Earth and Atmospheric Sciences
Instructor: Robert B. Herrmann
Office: O’Neil Hall 203
Tel: 314 977 3120
EASA-462
Office Hours: By appointment
Email: rbh@eas.slu.edu

Ray amplitudes for a constant velocity sphere

Goals:

- To compute amplitudes as a function of great cricle arc
- To develop the correction curve for magnitude determination

Background:

Geometrical optics can be used to determine ray amplitudes on a sphere. Consider Figure 1. The ray tube connects the source to the receiver. As the signal propagates from one end of the ray to the other, the total energy passing through a particular cross-sectional area, which is normal to the ray tube, is the same because of conservation of energy. This energy is distributed over the cross-sectional area. If we define A_1 and A_2 as the amplitudes of the signal at each cross-section, then the conservation of energy requires that

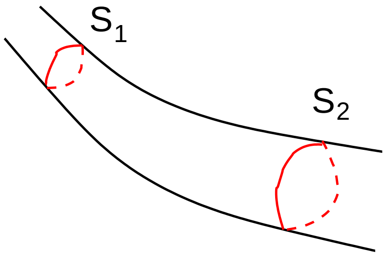

Fig. 1. Ray tube with differing cross-sectional areas.

$$A_1^2 S_1 = A_2^2 S_2$$

since the individual partical energy is proportional to the square of the amplitude.

To apply this to the Earth, consider the ray diagram in Figure 2. Here the epicentral distance is Δ, the ray connection the source at S and the observation point at R leaves the source with an angle of incidence i_s and is incident at the free surface with angle i_r. We will procede by considering the energy per unit area passing through a sector at the source and arriving at a sector at the receiver.

Recall from solid geometry that the surface-area of a sphere of radius r is $4\pi r^2$. The surface area of a small sector bounded by angles i and $i + di$ is $2\pi r^2 \sin i di$. So, at the source we have the

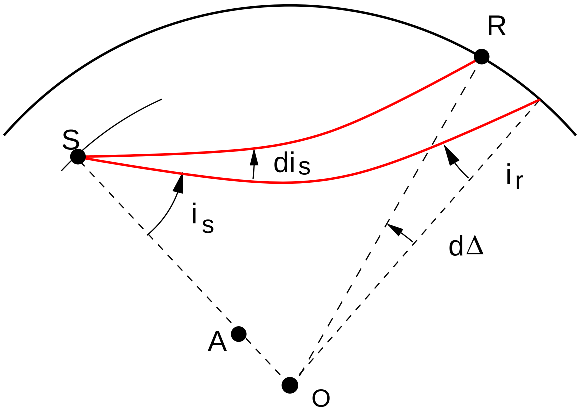

Fig. 2. Geometry for propagation in the Earth.

sector area

$$2\pi a \sin i_s di_s$$

where a is the radius of a small ellipse about the source.

At the receiver the rays leaving the source between angles i and $i + di$ create a sector on the surface of the sphere with area

$$2\pi r_o^2 \sin \Delta d\Delta$$

where r_o is the radius of the sphere and $r_o \sin \Delta$ is the length of the line AR in Figure 2. Since the conservation of energy relation requires us to consider surfaces normal to the ray, the corresponding area at the receiver is

$$2\pi r_o^2 \cos i_r \sin \Delta d\Delta$$

Since the square of the ratio of amplitudes is inversely proporational to the ratio of the area, we have

$$\left(\frac{A_R}{A_S}\right)^2 = \frac{2\pi a^2 \sin i_s di_s}{2\pi r_0^2 \sin \Delta \cos i_r d\Delta}$$

For small changes in di_s, the ratio of differentials in this expression becomes the derivative $di_i/d\Delta$.

If we use the spherical ray parameter definition, e.g.,

$$p = \frac{r_s}{v_s} \sin i_s = \frac{r_r}{v_r} \sin i_r = \frac{dT}{d\Delta}$$

we quickly see that

$$\frac{di_s}{d\Delta} = \frac{v_s}{r_s} \frac{1}{\cos i_r} \frac{d^2T}{d\Delta^2}$$

by applying a $d/d\Delta$ operator to the ray parameter equation.

Thus,

$$A_R = A_S \left(\frac{a^2 \sin i_s v_s}{r_o^2 \sin\Delta \cos i_r r_s \cos i_s} \left| \frac{d^2T}{d\Delta^2} \right| \right)^{\frac{1}{2}}$$

The || absolute values are taken because the second derivative can be negative.

This is an interesting equation. The travel-time actually contains information not only about the ray parameter, through its slope, but also the amplitude, because of its second derivative. Secondly, we can expect large amplitudes whenever the second derivative equals zero, which was the case for surface reflection in Assignment 13.

Constant velocity sphere.

For a simple problem, consider a uniform sphere with constant velocity. Also assume that the source and receiver are at the surface. For this sphere, the travel time (Assignment 13) is

$$T = \frac{2r_0}{v} \sin(\Delta/2)$$

$$p = \frac{dT}{d\Delta} = \frac{r_0}{v} \cos(\Delta/2)$$

and

$$\frac{d^2T}{d\Delta^2} = -\frac{r_0}{2v} \sin(\Delta/2)$$

Using this, the fact that $i_s = i_r$ in this case, we have

$$A_R = \frac{a}{r_0} \frac{A_S}{2\sin(\Delta/2)} \qquad (1)$$

What you must do:

- Plot the relation $1/(2\sin(\Delta/2))$ for angles between 1° and 180 °.
- Plot the relation $1/(2\sin(\Delta/2))$ for angles between 1° and 1 80 °. on a log y - lin x plot
- Plot the relation $2\sin(\Delta/2)$ for angles between 1° and 1 80 °. on a log y - lin x plot. This particular plot would be the distance correction for magnitude. **What you must submit:**

The plots and a table of $2\sin(\Delta/2)$ as a function of Δ.