

Introduction to Earthquake Seismology

Assignment 14

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Rays in a Constant Velocity Sphere

Goals:

- Understand the extension of Snell's law to a sphere
- Compute the travel-time versus distance relationship for a simple spherical model
- Understand surface reflections

Background:

As in introduction to travel times in the Earth, we will consider the simple case of a constant velocity sphere. Let the radius of the fluid sphere be a , the compressional wave velocity be c , the source be at a radius r_s , the receiver be at the free surface, the travel time be T , and the spherical angular distance of the receiver from the source be an angle Δ .

The easiest way to obtain a travel time distance plot is to compute both the travel time and distance parametrically in terms of the angle of the ray leaving the source i_s , which is 0° for a ray going downward from the source. The ray leaving the source will make an angle at the free surface, i_r , defined by the Snell's law relation for spherical wave propagation:

$$p = \frac{r_s \sin i_s}{c} = \frac{r_r \sin i_r}{c}, \quad (1)$$

where the receiver radius is defined here as $r_r = a$.

Let $N+1$ be the number of linear ray segments between the source and receiver. $N = 0$ indicates a direct path between the source and the receiver, and $N = 1$ indicates two ray segments with a single free surface reflection. The expressions for the travel time and distance are

$$T(i_s) = \frac{\left(r_s^2 + r_r^2 - 2r_s r_r \cos(\pi - i_r - i_s) \right)^{1/2}}{c} + \frac{N2a \cos i_r}{c} \quad (2)$$

$$\Delta(i_s) = \pi - i_s - i_r + N(\pi - 2i_r). \quad (3)$$

What you must do:

Create a spreadsheet starting with the following values: $C = 10.0$ km/sec, $A=6000$ km, $RS=5400$ km, $RR=6000$ km.

Create the following columns: IS, IR, Delta (degrees), p (sec/deg), Time(sec) which are created with the formulas for a given IR:

$$IR: =ASIN((RS/RR)*SIN(DEGRAD*A7))/DEGRAD$$

$$\text{Delta}=180-A8-B8+NZERO*(180-2*B8)$$

$$p=\text{DEGRAD}*RS*SIN(\text{DEGRAD}*A7)/'C'$$

$$\text{Time}=\text{SQRT}(RS*RS+RR*RR-2*RS*RR*\text{COS}(((180-A7-B7)*\text{DEGRAD}))/C \\ +NZERO*2*A*\text{COS}(\text{DEGRAD}*B7)/C$$

where $\text{DEGRAD}=3.1415927/180$

Then copy the columns and create a formula replacing NZERO with NONE=1

- Plot the two travel time curves with the X-axis Delta(degrees) from 0 to 360, and with the Y-axis (travel time sec) from 0 to 2400 sec.

for the plots if $\Delta > 180$ degrees, use $360 - \Delta$ for the plot.

What you must submit:

- A derivation of equations (2) and (3)
- A table showing the computations.
- Also why is plotting $360 - \Delta$ permissible.

The plot may look as follows:



