

Geophysical test of the universality of free-fall

Sachie Shiomi

Center for Measurement Standards, Industrial Technology Research Institute, Hsinchu, Taiwan 300, Republic of China

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We point out that the universality of free-fall can be tested by observing surface-gravity changes of the Earth. The Earth's inner core is weakly coupled to the rest part of the Earth by mainly gravitational forces. If there were a violation of the universality of free-fall, because of their different chemical compositions and/or of different mass fractions of binding energies, the inner core and the rest part of the Earth would fall at different rates towards the Sun and other sources of gravitational fields. The differential acceleration could be observed as surface-gravity effects. By assuming a simple Earth model, we discuss the expected surface-gravity effects of violations of the universality and experiments to search for such effects by using superconducting gravimeters. It is shown that the universality can be tested to a level of 10^{-9} using currently operating superconducting gravimeters. Some improvements can be expected from combinations of global measurements and applications of advanced data analyses.

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I. INTRODUCTION

The universality of free-fall, stating that every material (point mass) in a gravitational field falls at the same rate, is accepted as one of the most fundamental principles in modern physics; the Newtonian mechanics and Einstein's general relativity are based on this principle. All experiments performed so far support the universality. However, theories towards the unification of the four forces (strong, weak, electromagnetic and gravitational interactions) typically introduce new interactions and predict violations of the principle [1–6]. These theories predict new Yukawa-potential-type interactions between putative charges, such as baryon number, isospin, and electrostatic energy density, which are functions of proton and neutron numbers in elements. This prediction implies chemical composition-dependence of free-fall. Also, many alternative metric theories of gravitation predict violations of the universality of free-fall of massive bodies [7]. Therefore, testing the universality at a high sensitivity is expected to make a breakthrough in the current understanding of physics. Because there are many unknown factors in those theories, no precise prediction has been made for the magnitudes of the putative violations; the universality should be tested as precisely as possible. Also, because the characters of the putative interactions are uncertain, the universality has to be tested for various putative charges, using different kinds of test bodies, at different ranges. In addition, to confirm experimental results, it should be tested by at least two different experimental methods. Considering the importance of testing the universality and the necessity of variety in experimental approaches, we present a new method of testing the universality. So far, it has been tested by various experiments (e.g. [8]) and it is verified to a level of 10^{-13} by laboratory Eötvös-type experiments [9] and by lunar laser ranging (LLR) [10].

The merits of the proposed method may be as follows: (1) this is the first application of the superconducting

gravimeters, whose performance has been verified to be sensitive and stable in geophysical studies, to tests of the universality, (2) this is the first attempt to use Earth's interior as test bodies, and (3) unlike most of the experiments for testing the universality, this method has a potential to test the universality of free-fall of gravitational self-energy.

We present the concept of this method in Sec. II, the theory in Sec. III, and the expected sensitivity in Sec. IV.

II. THE CONCEPT

In this method, the test bodies are the solid inner core and the rest part of the Earth (the liquid outer core, the mantle, and the crust). They are in free fall in the gravitational field mainly due to the Sun. If there were a violation of the universality of free-fall, there would be differential acceleration between the inner core and the rest part of the Earth towards the Sun (see Sec. III for detail). The differential acceleration can be searched for by measuring surface-gravity changes. Relative gravity changes can be measured with high resolution by using superconducting gravimeters. They have been used to search for translational motions of Earth's inner core (the Slichter triplet [11]) (e.g. [12–16]). It is shown that they are capable of measuring long period surface-gravity effects. We attempt to apply the geophysical tool to test the theory of gravitation.

In the test bodies, there are mainly two kinds of differences that could result in the differential acceleration due to violations of the universality. One is the difference in their chemical compositions (the chemical composition-dependent effect), which implies the difference in proton and neutron numbers in the elements that compose the test bodies. The solid inner core mainly consists of iron and nickel, and its density is approximately $\rho_{ic} \approx 13\,000\text{ kg m}^{-3}$ [17], while the rest part of the Earth is mainly made of lighter elements such as silicon oxides,

except the liquid outer core [which mainly consists of heavy elements and whose density is slightly less than the inner core ($\rho_{oc} \approx 11\,000\text{ kg m}^{-3}$ [17])]. The average density of the rest part, including the outer core, is approximately 5400 kg m^{-3} .

The other is the difference in the fractions of gravitational self-energy in the test bodies (about -3.7×10^{-11} and -4.2×10^{-10} for the inner core and the rest part, respectively). Testing the universality of free-fall of gravitational self-energy can be viewed as a test of the strong equivalence principle.

The chemical composition-dependent effect can be tested using laboratory-size test masses. The effect of gravitational self-energy appears significant only in massive test bodies.

III. EQUATION OF MOTION OF THE INNER CORE

We assume a simple configuration as shown in Fig. 1. The Earth is revolving around the Sun (with angular frequency ω_R) in a circular orbit with radius r . The Earth's rotation axis is perpendicular to the ecliptic plane, namely, the inclination of the rotation axis by about 23.3° is not considered. The x - y plane is set on the Earth's equatorial plane with its origin at the center of figure of the rest part of the Earth.

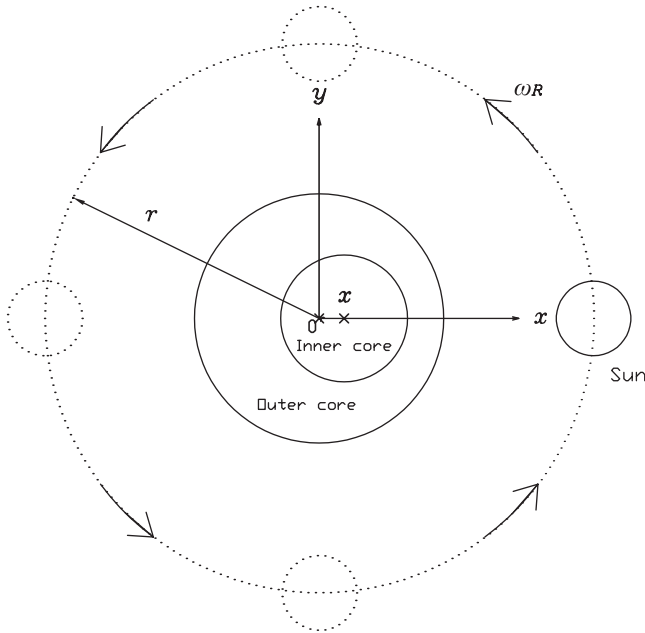


FIG. 1. A schematic cross section of the assumed configuration (not drawn to scale). The Earth is fixed and the Sun goes around the Earth in the circular orbit (radius r) at the angular frequency $\omega_R \approx 2\pi(24\text{ h})^{-1}$. The inclination of Earth's rotation axis to ecliptic is assumed to be 0° for simplicity. The x - y plane is set on the Earth's equatorial plane. The Earth's inner core oscillates along the x -axis. The gravitational influence from Earth's mantle and crust (not drawn) is negligible because of Newton's shell theorem (see text).

The Earth's interior can be classified into four parts: the solid inner core, the liquid outer core, the mantle, and the crust. We assume that the solid inner core is a homogeneous sphere (with density ρ_{ic} and radius $r_{ic} \approx 1.22 \times 10^6\text{ m}$ [17]) and it is enclosed in the spherical liquid outer core (with the average density ρ_{oc} and outer radius r_{oc}). We do not consider any deformations (such as tidal or rotational deformations). We assume that the mantle and the crust are spherical shells with uniform densities, and their centers of figures are coincident; their gravitational influence on the inner core is negligible due to Newton's shell theorem. The Coriolis acceleration splits oscillations of the inner core into a triplet of periods (the Slichter triplet [11]). However, for simplicity, we assume that the inner core oscillates only along the x -axis (with angular frequency ω_0) and we ignore the Coriolis acceleration. We do not consider any electromagnetic effects.

The effective viscosity of the outer core η is not well determined and estimates from various methods vary from $\sim 10^{-3}$ to $\sim 10^{12}\text{ Pa s}$ [18]. The Reynolds number ($\equiv \rho_{oc} v r_{ic} / \eta$, where v is the velocity of the inner core) is less than unity when η is larger than $\sim 10^7\text{ Pa s}$ and the amplitude of oscillations $X (\equiv v / \omega_0)$ is nominally 1 m. We assume that the friction between the inner core and the outer core is proportional to the velocity of the inner core.

Under these assumptions, the x -component of the equation of motion of the inner core (mass $m_{ic} = 4\pi r_{ic}^3 \rho_{ic} / 3 \approx 1.0 \times 10^{23}\text{ kg}$) can be written approximately in a form of so-called damped forced oscillation:

$$\ddot{x} \approx -\omega_0^2 x - 2k\dot{x} + \Delta a \cos \omega_R t, \quad (1)$$

where

$$\begin{aligned} \omega_0^2 &\approx \frac{4}{3} \pi G \frac{\rho_{ic} - \rho_{oc}}{\rho_{ic}} \rho_{oc} \approx 4.7 \times 10^{-7} \text{ s}^{-2} \\ &\approx \{2\pi(2.5\text{ h})^{-1}\}^2, \end{aligned} \quad (2)$$

$$2k \equiv \frac{6\pi r_{ic} \eta}{m_{ic}} \approx 2.3 \times 10^{-16} \eta \text{ s}^{-1}. \quad (3)$$

The stiffness ω_0^2 is mainly due to the gravitational pull by the outer core and, as seen in Eq. (2), determined by the density difference in the core with the gravitational constant, $G = 6.67 \times 10^{-11}\text{ N m}^2 \text{ kg}^{-2}$. The stiffness due to Sun's tidal force that acts to enlarge any displacements of the inner core from the center of the Earth twice per its revolution is about 8 orders of magnitude smaller than the gravitational stiffness and ignored. The stiffness due to the Moon's tidal force is about twice larger than the one due to the Sun's tidal force and ignored. The stiffness due to the centrifugal force caused by Earth's revolution ($\approx \omega_R^2$) is about 2 orders of magnitude smaller than the gravitational stiffness and ignored.

Δa in Eq. (1) is the magnitude of the putative differential acceleration due to a violation of the universality of free-fall in the Sun's gravitational field. The signals of the

violation are expected to have the frequency of once per revolution of the Earth: $\omega_R \approx 2\pi(24 \text{ h})^{-1} \approx 7.3 \times 10^{-5} \text{ s}^{-1}$. At this frequency, the major obstacle would be the tides (the body tides and the ocean tides) which have the same frequency as the violation signals. The tidal effects and other known effects can be largely removed from the gravity data by applying appropriate models and data analysis methods (e.g. [19]).

IV. EXPECTED SENSITIVITY

Assuming that $k < \omega_0$ (or $\eta < 5.9 \times 10^{12} \text{ Pa s}$), the solution of Eq. (1) can be written by a sum of two terms that express damped oscillations and forced oscillations. The damped oscillations decay exponentially with time t (e^{-kt} ; k^{-1} is about 24 min for $\eta = 5.9 \times 10^{12} \text{ Pa s}$ or when $k = \omega_0$), while the forced oscillations remain:

$$x = A \sin(\omega_R t - \delta), \quad (4)$$

where

$$A = \frac{\Delta a}{\sqrt{(\omega_0^2 - \omega_R^2)^2 + 4k^2 \omega_R^2}}, \quad (5)$$

$$\tan \delta = \frac{2k\omega_R}{\omega_0^2 - \omega_R^2}. \quad (6)$$

When the damping coefficient is sufficiently small (i.e. $4k^2 \ll \omega_0^4/\omega_R^2 \sim 4 \times 10^{-5} \text{ s}^{-2}$ or $\eta \ll 2.8 \times 10^{13} \text{ Pa s}$), we obtain

$$A_{\max} \approx \frac{\Delta a}{\omega_0^2} \quad (7)$$

and $\tan \delta \approx 0$. When the inner core is shifted from the center of the Earth by A_{\max} , along the x -axis, this displacement will change the gravitational field at a surface point on the x -axis by

$$\Delta g \approx \frac{2Gm_{ic}}{r_e^3} \frac{\Delta a}{\omega_0^2} \sim \frac{\Delta a}{10}, \quad (8)$$

where $r_e = 6.4 \times 10^6 \text{ m}$ is the average radius of the Earth.

The noise level of superconducting gravimeters at quiet sites is a few $10^{-11} \text{ m s}^{-2}$ at the signal frequency [15]. Assuming a somewhat better sensitivity of $10^{-12} \text{ m s}^{-2}$ at the signal frequency, we could estimate the universality of free-fall to

$$\frac{\Delta a}{a_{\text{sun}}} \sim 1.6 \times 10^{-9}, \quad (9)$$

where $a_{\text{sun}} \approx 5.9 \times 10^{-3} \text{ m s}^{-2}$ is the Sun's gravitational acceleration that acts on the Earth. Because of the inclination of the Earth's rotation axis, the maximum violation signals can be expected at observatories located on the equator in Spring and Autumnal equinox points, and on

Tropic of Cancer or Capricorn in Summer and Winter solstices.

Equations (7) and (8) indicate that the relative displacement of the inner core can be measured to $\sim 20 \mu\text{m}$. For this amplitude, the Reynolds number is unity when $\eta \approx 200 \text{ Pa s}$. As a result, the range of the effective viscosity, discussed above, is $200 \text{ Pa s} \leq \eta < 6 \times 10^{12} \text{ Pa s}$. This range is consistent with the estimates summarized in [18].

V. DISCUSSION

This work is based on the simple Earth model and configuration. Elaborate modelings have to be applied for more accurate analyses of expected violation signals and of the expected sensitivity. Especially, the value of ω_0 , which affects the estimates of the expected signals and sensitivity, is highly model dependent. The latest theoretical studies with elaborate Earth models predict smaller values of ω_0 than the value we have used [Eq. (2)], between $\sim 2\pi(6 \text{ h})^{-1}$ and $\sim 2\pi(4 \text{ h})^{-1}$ [20,21]. Because the expected sensitivity is proportional to $\sim \omega_0^{-2}$, the expected sensitivity could be better by ~ 3 to 6 times by applying elaborate Earth models. However, we have assumed the rather optimistic sensitivity of $10^{-12} \text{ m s}^{-2}$. Therefore, the expected sensitivity would remain to be on the order of 10^{-9} .

The current limits on violations of the universality of free-fall are on the order of 10^{-13} . In order to get comparable results, a sensitivity better than a few $10^{-15} \text{ m s}^{-2}$ (0.1 picogal) at the signal frequency is required. Presently, superconducting gravimeters are the most sensitive instruments for measurements in the frequency range. Though it seems difficult to achieve this sensitivity, there are several possibilities to improve the sensitivity. One way may be applications of more sophisticated data analyses than the usual Fourier analyses, as discussed in [22] to search for the Slichter triplet. Another way may be to carry out coincidence measurements with two superconducting gravimeters located ideally opposite sides of the Earth near the equator. If there were a violation towards the Sun, the expected magnitude of the violation signal at the two superconducting gravimeters is the same but the sign should be opposite. By combining such coincidence signals, we could double the magnitude of the expected signals and the sensitivity would be improved by a factor of 2. Candidate observatories for such coincidence measurements may be the one at Hsinchu in Taiwan (25°N 121°E), where we are currently setting up two new superconducting gravimeters, and the one at Concepcion in Chile (37°S 73°W). If the noise level of data from the observatories were high, it might be better to use data from low noise sites considering the degrees of signal compensation depending on the latitude and longitude of the sites. Such global observations would be possible through the Global Geodynamics Project network (GGP [23]). Further studies are necessary to figure out the optimal schemes for

global observations and noise reduction. Currently, we are checking the performance of the new superconducting gravimeters, installed in Hsinchu, Taiwan, in March 2006. We plan to carry out a preliminary test of the universality in the near future.

As described in Sec. II, the difference in the mass fraction due to gravitational self-energy is $\alpha_{\text{grav}} \sim 4 \times 10^{-10}$. Therefore, from Eq. (9) and the above discussion, the geophysical test with current superconducting gravimeters would be only sensitive to the chemical composition-dependent effect, but not sensitive enough to test the universality of gravitational self-energy. If the sensitivity were improved to ~ 0.1 picogal, we could test the universality of free-fall of gravitational self-energy to the same level as the LLR experiment [10,24,25].

VI. CONCLUSIONS

We have considered a new method of testing the universality of free-fall. In this method, differential acceleration between the Earth's inner core and the rest part of the Earth is to be searched for by measuring surface-gravity effects. Based on a simple model, we have shown that the univer-

sality would be tested to a level of 10^{-9} with current sensitive superconducting gravimeters. Some improvements can be expected from combinations of global measurements and developments of methods of data analysis. We plan to carry out a preliminary test of the universality using superconducting gravimeters in Hsinchu, Taiwan. To get a comparable result with the LLR experiment, the sensitivity has to be improved by about 4 orders of magnitude at the signal frequency (once per day). A breakthrough in developments of gravity measurements is necessary to achieve this sensitivity.

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