APPLICATION OF WAVELET TECHNIQUE TO THE EARTH TIDES OBSERVATIONS ANALYSES

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1. INTRODUCTION

Wavelet analysis is a powerful and popular tool for the analysis of non-stationary signals. The wavelet transform is a joint function of the time series of interest *x(t)* and an analysing wavelet *ψ(t)*. This transform isolates signal variability both in time *t*, and also in scale *s*, by rescalling and shifting the analysing wavelet. The wavelet itself can be said to pay the role of a lens through which a signal is observed, and therefore, it is important to understand how the wavelet transform depends upon the wavelet properties. Such understanding would permit the identification of optimal wavelets which most accurately represent signal characteristics in the properties of the transform. This paper presents the results of the diploma thesis based on the application of the wavelet transform to the analyses of the Earth tides observations recorded in the Astro-Geodetic Observatory at Jozefoslaw.

2. METHOD

Wavelet transform is derived from Fourier Transform, but it is much more flexible. The FT could not be used to the non-stationary time series, in which stochastic characteristics change in time. If we assume that non-stationary signal consists of several stationary signals the STFT (Short-Time Fourier Transform) could be applied. The signal is divided into small segments which are assumed to be stationary. The main role in such analysis plays "window", which is used to divide the signal. But in this case we act with indeterminacy. If narrow window is chosen the accurate information about time is obtained, less accurate about frequency. In case of wide window just the other way about.

Continuous Wavelet Transform (CWT) assumes that the signal is a composition of a several functions (wavelets in this case). CWT of a signal $x(t) \in L^2(\Re)$ is a sequence of projections onto rescaled and translated versions of an analysing functions of wavelets *ψ(t)* (Mallat, 1989):

$$CWT_x^{\psi}(s,\tau) = \int_{-\infty}^{\infty} x(t)\psi_{s,\tau}(t)dt \qquad (1)$$

where:

$$\psi_{s,\tau}(t) = \frac{1}{\sqrt{|s|}}\psi\left(\frac{t-\tau}{s}\right), \quad s,\tau \in R, \quad s \neq 0 \qquad (2)$$

The equation presents wavelet function, which depends on two parameters:

   s - scale coefficient,
   τ - time shift.

Using this equation we can derive the family of the functions from *ψ* - mother wavelet using scale factor and shift. The wavelet has finite length and is concentrated around *t*=0 point and its mean value is equal to zero:

$$\int_{-\infty}^{+\infty} \psi(t)dt = 0 \qquad (3)$$

The $1/\sqrt{s}$ part is also worth mentioned. It is a kind of normalization of the signal to keep the same energy for all scales.

The algorithm of CWT contains:

- comparison of the wavelet with the beginning of the signal. The factor C is calculated which could be interpreted as the correlation between wavelet and the part of the signal;
- using shift factor τ the next part of the signal is chosen and the subsequent comparison is done. This step is repeated until the whole signal is compared.
- using scale factor we extend the wavelet and make the comparison once again.

3. DATA

The data analysed in this project was collected at Astro-Geodetic Observatory in Jozefoslaw. The Observatory belongs to the Warsaw University of Technology and is placed at the suburb of Warsaw, 15 km from the city centre, but the vicinity is rather quiet. The data had been collected by the ET-26 LaCoste&Romberg gravimeter since January 2002. To these analyses the data from 2006 to 2008 were used because of the highest consistency. The data was only despiked and degapped using TSoft software (Van Camp and Vauterin, 2005).
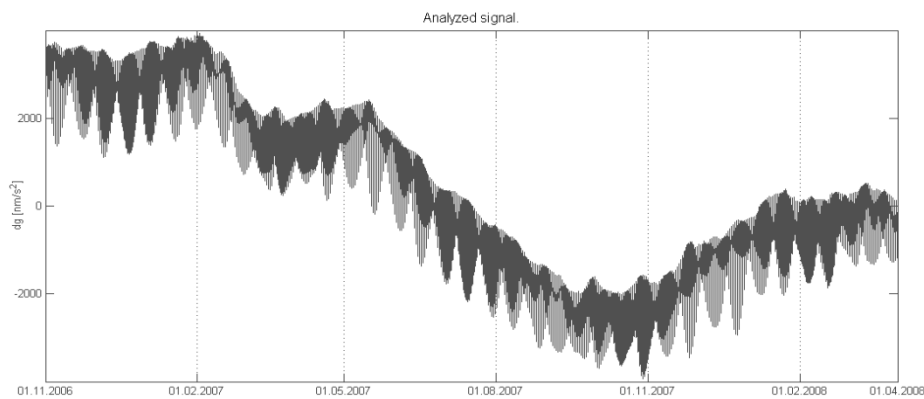

Fig. 1. Tidal data from LC&R ET-26 gravimeter.

4. TOOL

For the calculations Matlab software was used with help of additional library - Wavelet Toolbox and complex Morlet wavelet (Goupillaud et al.,1984):

$$\psi(x) = \frac{1}{\sqrt{\pi \cdot f_b}} e^{2i\pi f_c x} e^{-\frac{x^2}{f_b}} \qquad (4)$$

In Matlab this wavelet is described as cmor"f_b-f_c" and depends on two parameters:

f_b - bandwidth parameter;
f_c - centre frequency.

and different modifications of Morlet complex wavelet are possible, presented in figure 2 (solid and dashed lines represent real and imaginary part respectively).
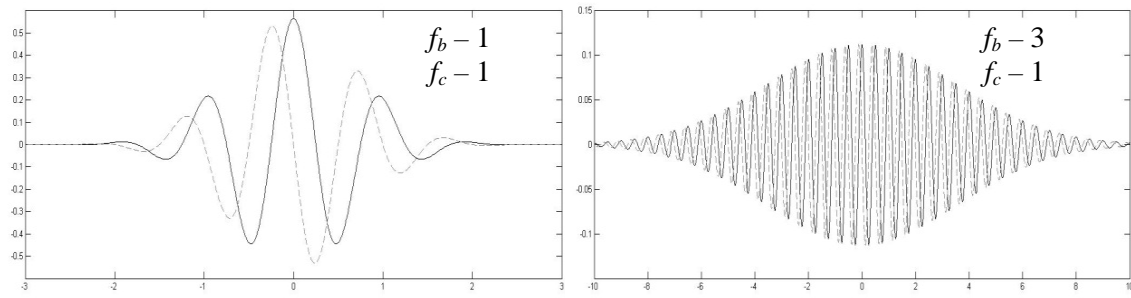
Fig. 2. Modification of complex Morlet wavelet.

Base on Nyquist rule Matlab allows to determine wavelet coefficients C for periods from 0 to f_n, where f_n is equal to half of the signal's length. On these conditions maximum determinable scale is:

$$S = \frac{1}{2} \cdot \left(2^n\right)$$

(5)

where n is the highest power of 2 to be comprised in the original signal's length.

5. RESULTS

The analyses have been started with complex Morlet wavelet $f_b=3$ and $f_c=1$ (cmor3-1) obtaining spectrogram describing power spectrum (C-coefficients) in the particular frequencies occurred in the original gravity signal (fig. 3).
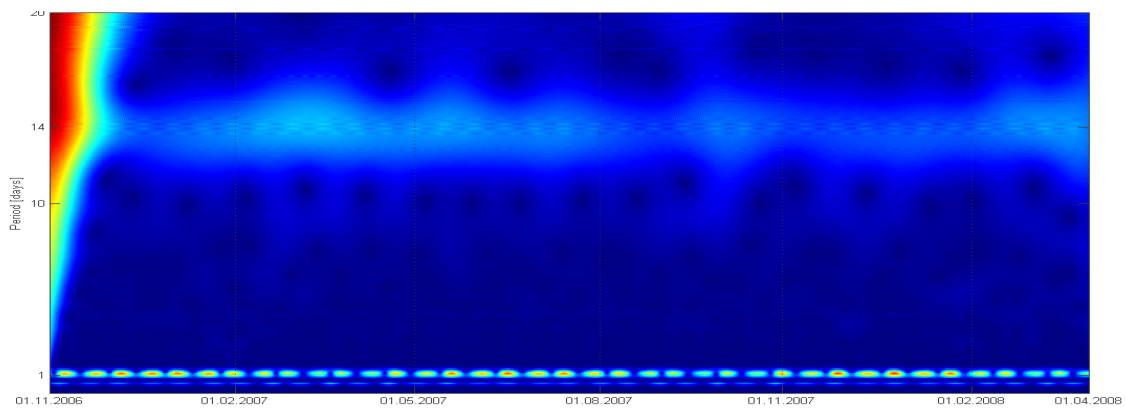

Fig. 3. Morlet Wavelet Spectrum, cmor3-1.

Application of cmor3-1 wavelet did not allowed to separate particular diurnal and semi-diurnal tidal waves (left figure below). Better solution was obtained using cmor25-8 wavelet (right figure below).
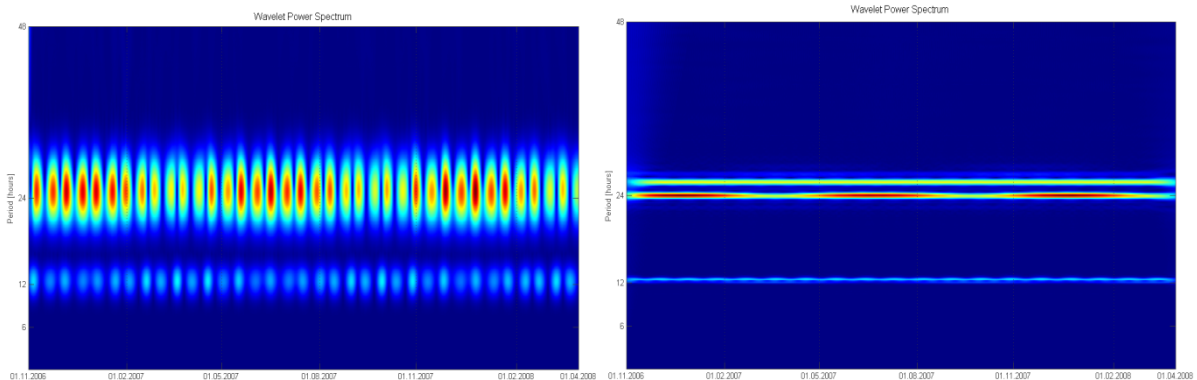

Fig. 4. Comparison of Morlet Wavelet Spectrum (cmor3-1 – left, cmor25-8 – right).

As it was mentioned before the results of the CWT is the matrix of *C*-coefficients, which are the amounts of the energy in particular periods. To recalculate it into amplitude the linear relationship was used (Kalarus, 2007):

$$A = \frac{1}{C_n} \cdot C \tag{6}$$

where:

      A is the amplitude,

      C - wavelet coefficient,

      C_n - integral from the envelope of the wavelet function used for calculations.

In practice, C_n is calculated by making wavelet transform of the artificial signal of amplitude 1 and period determined by the transform of the original signal. The C_n coefficients obtained by this method are different for different frequencies (Fig. 5).
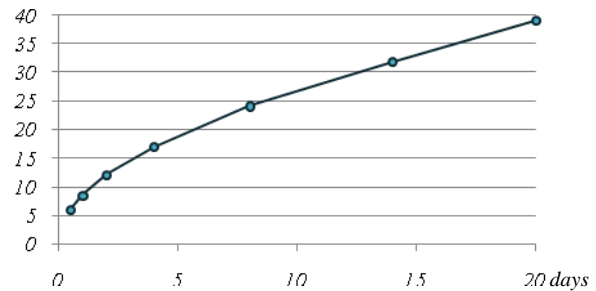

Fig. 5. Calculated values of C_n factors.

6. COMPARISON

The amplitudes obtained by this method were compared to those determined using classical least square manner (Chojnicki, 1977) calculated using Eterna 3.4 (Wenzel, 1996) with the same original signal of gravity changes (see table 1).

Table 1. Frequencies of the tidal waves.

Frequency [cycle/day]		Name	Amplitude [nm/s^2]	Std. dev. [nm/s^2]	Frequency [cycle/day]		Name	Amplitude [nm/s^2]	Std. dev. [nm/s^2]
from	to				from	to			
0.501370	0.842147	SGQ1	2,76	0,143	1.035251	1.054820	J1	27,45	0,124
0.842148	0.860293	2Q1	8,83	0,135	1.054821	1.071833	SO1	4,61	0,128
0.860294	0.878674	SGM1	10,45	0,137	1.071834	1.090052	OO1	14,85	0,089
0.878675	0.896968	Q1	66,09	0,127	1.090053	1.470243	NU1	2,85	0,087
0.896969	0.911390	RO1	12,53	0,131	1.470244	1.845944	EPS2	2,43	0,058
0.911391	0.931206	O1	346,55	0,124	1.845945	1.863026	2N2	8,44	0,061
0.931207	0.949286	TAU1	4,61	0,165	1.863027	1.880264	MU2	10,21	0,067
0.949287	0.967660	M1	27,19	0,109	1.880265	1.897351	N2	64,11	0,065
0.967661	0.981854	CHI1	5,37	0,122	1.897352	1.915114	NU2	12,23	0,068
0.981855	0.996055	PI1	9,15	0,149	1.915115	1.950493	M2	335,38	0,068
0.996056	0.998631	P1	161,06	0,156	1.950493	1.970390	L2	9,60	0,102
0.998632	1.001369	S1	3,49	0,227	1.970391	1.998996	T2	9,15	0,065
1.001370	1.004107	K1	480,85	0,140	1.998997	2.001678	S2	155,54	0,066
1.004108	1.006845	PSI1	4,49	0,150	2.001679	2.468043	K2	42,40	0,049
1.006846	1.023622	PHI1	7,07	0,156	2.468044	7.000000	M3M6	3,64	0,037
1.023623	1.035250	TET1	5,21	0,132					

From the comparison we can notice that there is a big discrepancy in K1 frequency. We can claim that classical manner based on the least squares method better separate P1, K1 and S1 waves. The same conclusion could be pointed out: wavelet transform of this signal did not separated correctly S2 and K2 waves (see fig. 6).
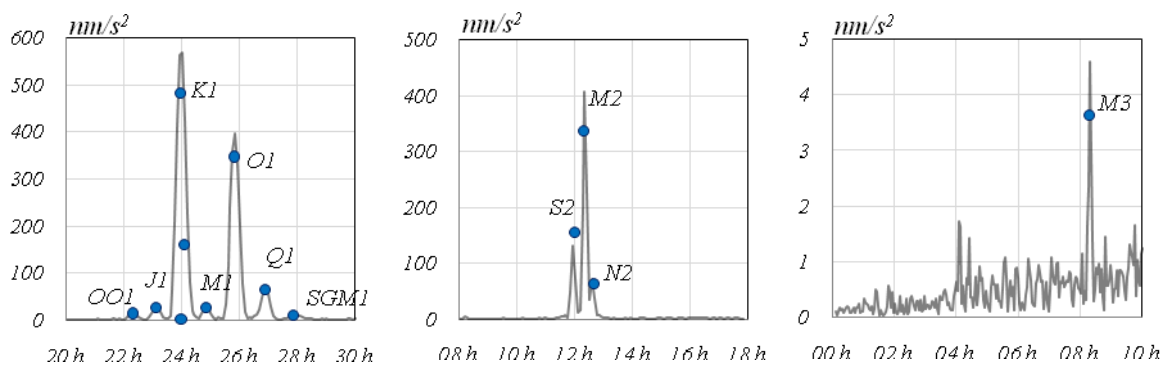
Fig. 6. Tidal waves amplitudes (solid line – CWT, 1st July 2007, ● ETERNA).

7. DIURNAL AND SUB-DIURNAL WAVES

To investigate frequency of the diurnal and sub-diurnal waves Morlet wavelet cmor25-8 was used, the results are presented in fig. 7 to 9.
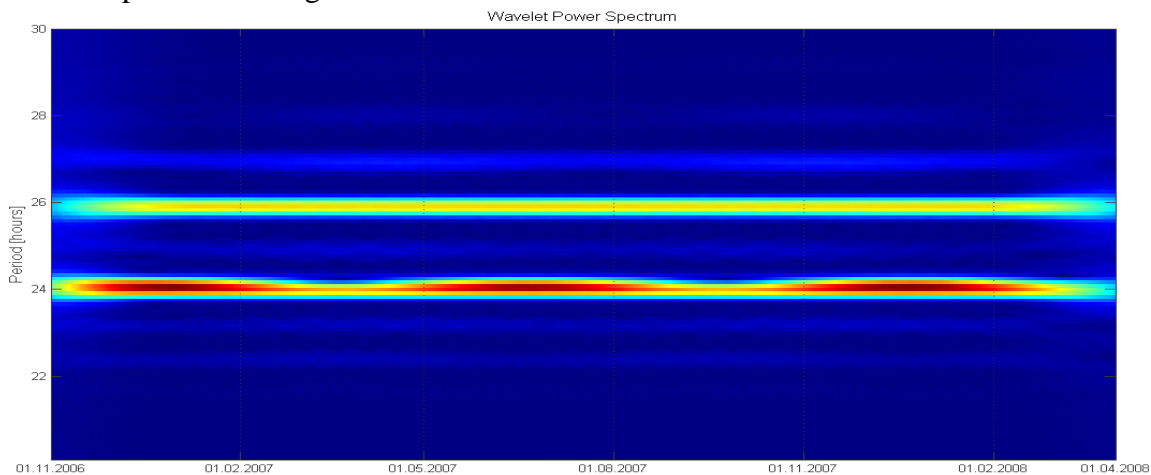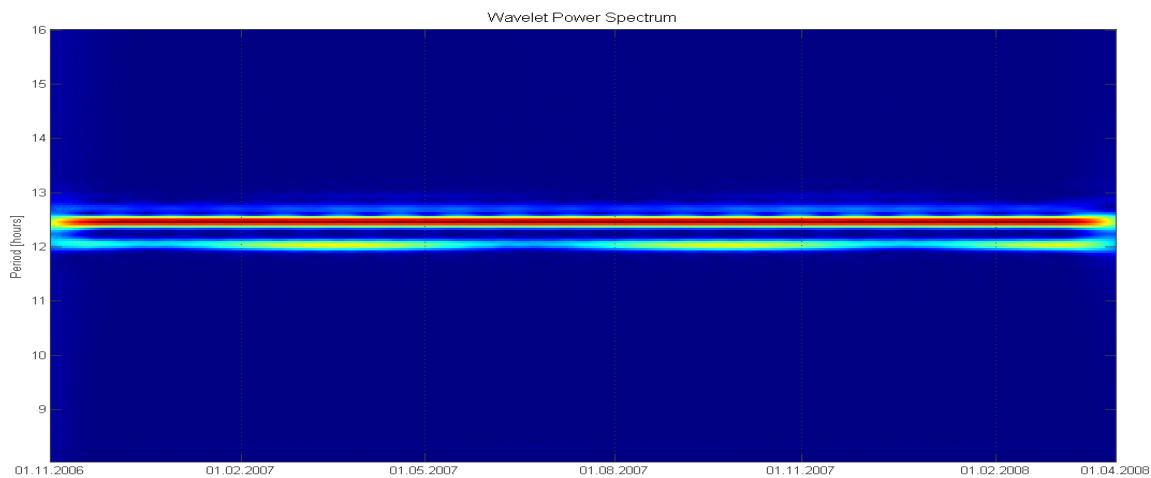

Fig. 7. Morlet Wavelet Spectrum, diurnal.

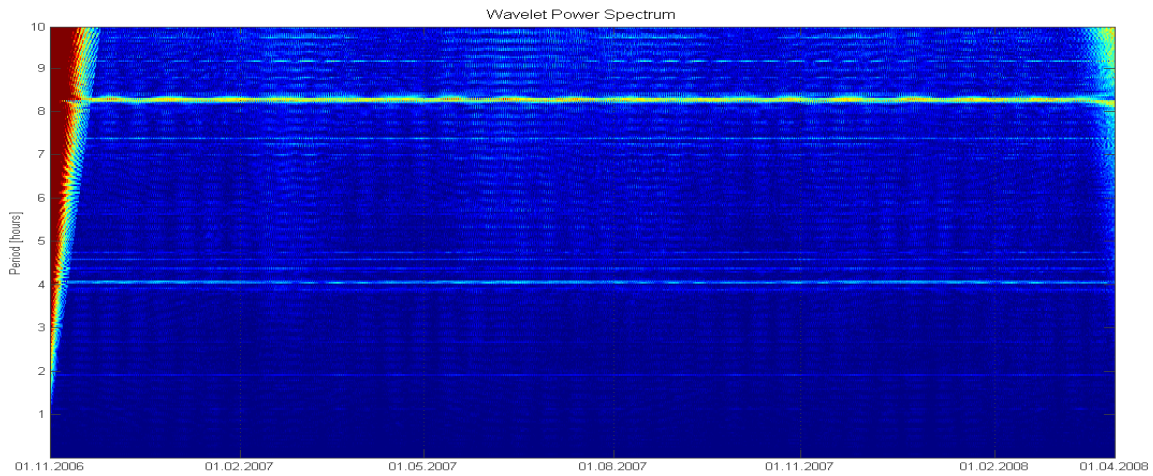Fig. 8. Morlet Wavelet Spectrum, semi-diurnal.

Fig. 9. Morlet Wavelet Spectrum, sub-diurnal.

The considered time span allowed to identify 7 diurnal waves and 4 sub-diurnal waves . They are:
- PSK1, O1, Q1, J1, M1, OO1, SIG1,
- M2, S2K2, N2, M3.

Table 2 presents differences between theoretical and obtained periods, the maximum difference did not exceed 5 minutes.

Table 2. Comparison of the waves period.

Name	Period [h]			Differences [min]
	theoretic	determined		
		range	mean value	
SGM1	27.848388	27.8333 – 28.0000	27.91667	4.1
Q1	26.868357	26.7500 – 26.9167	26.83333	2.1
O1	25.819342	25.7500 – 25.9167	25.83333	0.8
M1	24.833248	24.7500 – 24.9167	24.83333	0.0
PSK1	23.934469	23.9167 – 24.0000	23.95833	1.4
J1	23.098477	23.0833 – 23.1667	23.12500	1.6
OO1	22.306074	22.2500 – 22.3333	22.29167	0.9
N2	12.658348	12.5833 – 12.7500	12.66667	0.8
M2	12.420601	12.3333 – 12.5000	12.41667	0.2
S2K2	12.000000	11.9167 – 12.0833	12.00000	0.0
M3	8.280401	8.2500 – 8.3333	8.29167	0.7

8. MODULATION

At this stage changes of the wave's amplitudes were investigated.

Changes of the PSK1 wave's amplitude ranged from 450 to 630 nm/s^2 and are periodical. Major period is 180.5 days, minor 24-hours, 14- and 28-days, but they are of range 1 to 5 nm/s^2. O1 wave is much more stable. Changes of the amplitude are mainly half-yearly and oscillate from 398 to 408 nm/s^2. M1 wave arises from the Earth-Moon motion, so the main modulation is 27.5 days, but the amplitude is rather small: 5 to 7 nm/s^2. Conclusions from the modulation of J1, OO1 and SIG1 amplitudes are very similar. 28- and 14-day changes, but also 9- and 7-day, rather unexpected, but very small and at the level of the accuracy of the measurements. Chart of Q1's amplitude changes show strong 3-month modulation (30 nm/s^2) and 9-days, but less of importance.

M2 wave is the most stable from sub-diurnal waves. Changes of the amplitude are about 8 nm/s^2, which amount 2%. 14- and 180-days modulations could be clearly seen. The highest modulation was investigated in S2K2 wave. These oscillations are related to the thermal activity of the Sun and reach up 120 nm/s^2. Using CWT the N2 wave was also identified as the weakest possible. The amplitude varies from 55 to 90 nm/s^2 and changes with 28-days and half of the year. The last from sub-diurnal waves that were determined is M3. This is relatively weak wave, modulation of the amplitude seem to be non-regular.

The wavelet transform allows also for determination of the long-period tides and investigate its properties. As the example declinational wave Mf was taken. The amplitude is about 100 nm/s^2, but changes from 63 to 118 nm/s^2. The range of the observations was relatively short so only 60-day period of changes was found.

The wave's modulation are results from drumming near frequency's waves, that's the reason why modulation are periodic (tidal period).

But from previous results (Chojnicki, 1996; Bogusz and Klek, 2008) we can claim that some part of this modulation is not artificial and represents real, geophysical effect.
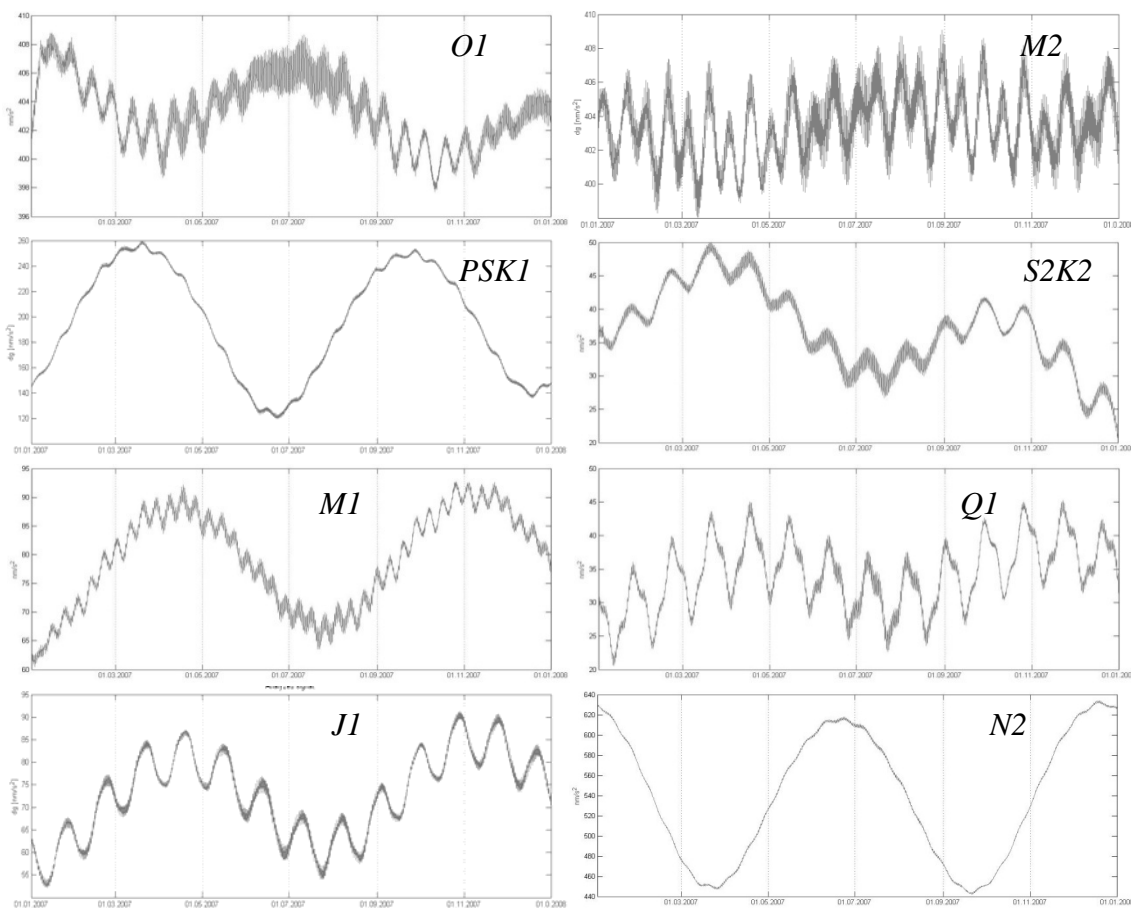

Fig. 10. Amplitude's seasonal modulation.

9. CONCLUSIONS

This investigation was aimed at application of the wavelet transform to the Earth tides observations analyses. It was done upon the data collected in Astro-Geodetic Observatory at Jozefoslaw by LC&R Et-26 gravimeter. Wavelet transform was made using Morlet functions with different parameters to recognise its usefulness to this type of data. Calculations were made in the Matlab environment. The results were compared to the previously obtained by different method. Good consistence was found in frequencies (with theoretical) and amplitudes (compared to Eterna) as well. A big advantage of WT is the ability of amplitude's seasonal modulation investigation. Seasonal changes of the main diurnal and sub-diurnal tidal waves were presented. Disadvantage is lack of phase determination, obtainable in least square method. WT could be also implemented to investigation of the long-period tides. Wavelet analysis is now a very popular tool for the analysis of non-stationary signals and after careful setup can be implemented to the selected analyses of the Earth tides observations.

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