Determination of degree-2 Love and Shida numbers from VLBI

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Abstract

Love and Shida numbers describe the Earth's response to external forces exerted by celestial bodies due to the elasticity of the Earth. Modern space geodetic techniques, such as VLBI (Very Long Baseline Interferometry), allow the empirical validation of theoretical Love and Shida numbers. In the VLBI analysis software package OCCAM tidal displacements on the Earth's surface are modelled according to the International Earth rotation and Reference systems Service (IERS) Conventions 2003. Snapshots of corrections to the nominal displacements for the complete Earth's surface with a spatial resolution of 1°x1° are shown.
Nominal degree-2 Love and Shida numbers, h_2 and l_2, were determined from the continuous 15 days VLBI campaign CONT05. Frequency dependence was considered in the diurnal band due to the retrograde Free Core Nutation (FCN) resonance: we determined h_2 and l_2 at those diurnal tidal waves with the largest amplitudes, two of them lying very close to the resonance frequency.

Keywords: Love number, Shida number, Tidal displacement, VLBI

1 Introduction

Lunisolar gravitational attraction causes rhythmic undulations of the Earth's surface. This tidal deformation arises from the variations in the Earth's gravitational field caused by the Moon/Sun over the surface of the Earth relative to its strength at the geocentre. The Earth deforms because it has a certain degree of elasticity. The Love number h and the Shida number l are dimensionless parameters, which characterize the ability of the Earth to react to tide-generating forces. If the Earth would be a completely rigid body, h and l would be equal to zero and there would be no tidal deformation of the surface. The total range of vertical surface deformation, which is caused by the pure solid Earth tides, can reach up to tens of centimetres.
Very Long Baseline Interferometry (VLBI) measures the difference between the arrival time of a plane radio wavefront emitted by an extragalactic source at two Earth-based antennas. The tidal deformation of the Earth surface is fully projected in the variation of the antenna position. Considering the steady improvements during the last two decades in the quality of observations, VLBI determines the relative position of the stations with an accuracy of a few millimetres. Therefore it is essential to have a precise model of the solid Earth tides displacement with sub-millimetre accuracy. The idea of this work is to reestimate parameters of elasticity of the solid Earth tide model. We used the model from Mathews et al. (1995), as recommended in the IERS Conventions 2003 (McCarthy and Petit, 2004). From the 15 days VLBI campaign CONT05 we determined nominal degree-2 Love and Shida numbers, and additionally Love numbers at six different frequencies in the diurnal band in order to demonstrate the feasibility of the approach.

2 Modelling tidal displacements

2.1 Tidal displacement with nominal Love and Shida number h_2 and l_2

Considering the Earth being "Spherical, Non-Rotating, Elastic and Isotropic", what is the most basic model, the tidal displacement (u_{R}, u_{E}, u_{N}) at a given station (latitude φ, longitude λ) in the local system (Radial, East, North) is described by equations (E1) to (E3). In this model the total tidal displacement is computed as the sum of displacement components over the spherical degree n of the tide generating potential V_n. For each spherical degree exists one proportionality factor for radial displacement (Love number h) and one proportionality factor for tangential displacement (Shida number l), which determines how strong is the effect of the potential component on the displacement. The parameter g stands for the surface acceleration due to gravity force. For practical computation we used expression of the tidal potential in Cartesian coordinates. The displacement component $\boldsymbol{u}^{(2)}$ (E4) to (E6) which is generated by the second degree potential is given by, e.g., Sovers et al. (1998). The tide generating potential is expressed there by the geocentric coordinates of the attracting body (X_a, Y_a, Z_a), the geocentric coordinates of the station (x_s, y_s, z_s), the vector $\boldsymbol{R_s}$ from the geocentre to the station, the vector $\boldsymbol{R_a}$ from the geocentre to the Moon/Sun and by the ratio of the mass of the attracting body to the mass of the Earth μ_a.

$$u_{\mathrm{R}} = \sum_{n=2}^{\infty} h_n \cdot \frac{1}{\mathrm{g}} \cdot V_n \tag{E1}$$

$$u_{\mathrm{E}} = \sum_{n=2}^{\infty} l_n \cdot \frac{1}{\mathrm{g} \cdot \cos\varphi} \cdot \frac{\partial V_n}{\partial \lambda} \tag{E2}$$

$$u_{\mathrm{N}} = \sum_{n=2}^{\infty} l_n \cdot \frac{1}{\mathrm{g}} \cdot \frac{\partial V_n}{\partial \varphi} \tag{E3}$$

$$u_{\mathrm{R}}^{(2)} = h_2 \cdot \sum_a \frac{3\mu_a R_s^2}{R_a^5}\left[\frac{(\boldsymbol{R_s}\cdot\boldsymbol{R_a})^2}{2} - \frac{R_s^2 R_a^2}{6}\right] \tag{E4}$$

$$u_{\mathrm{E}}^{(2)} = l_2 \cdot \sum_a \frac{3\mu_a R_s^3}{R_a^5}(\boldsymbol{R_s}\cdot\boldsymbol{R_a})\frac{(x_s Y_a - y_s X_a)}{\sqrt{x_s^2 + y_s^2}} \tag{E5}$$

$$u_{\mathrm{N}}^{(2)} = l_2 \cdot \sum_a \frac{3\mu_a R_s^2}{R_a^5}(\boldsymbol{R_s}\cdot\boldsymbol{R_a})\left[\sqrt{x_s^2 + y_s^2}\,Z_a - \frac{z_s}{\sqrt{x_s^2 + y_s^2}}(x_s X_a + y_s Y_a)\right] \tag{E6}$$

2.2 Corrections to the tidal displacement in radial direction with frequency dependent Love numbers $h_{21(f)}$

Under consideration of a more precise Earth model with fluid core and elastic mantle, the tidal response of the solid Earth becomes frequency dependent in the diurnal band. The rotational axis of the fluid core is slightly inclined with respect to the axis of rotation of the elastic mantle. In this situation forces arise at the elliptical core-mantle boundary, which try to realign the two axes and this leads to a resonance with the tidal force.
Corrections to radial displacement (E7) coming from the harmonic terms of the second degree tidal potential in the diurnal band (i.e., from the first order of the potential) are given in, e.g., McCarthy and Petit (2004).

$$\delta u_{\mathrm{R}(f)}^{(21)} = -\frac{3}{2}\sqrt{\frac{5}{24\pi}} H_f \,\delta h_{21(f)} \sin(2\varphi)\sin(\theta_f + \lambda) \tag{E7}$$

with	H_f	Cartwright-Tayler amplitude of the tidal term,
	$\delta h_{21(f)}$	difference of $h_{21}(f)$ from the nominal value h_2,
	φ, λ	station coordinates (in latitude and longitude),
	θ_f	tidal harmonic argument.

2.3 Effect of inexact value h_2 on radial displacement

To get an idea, how large the effect of an inexact Love number is, we computed the radial displacement at two stations once with the correct Love number $h_2 = 0.6078$ (solid line) and once with the wrong Love number $h_2 = 0.5078$ (dotted line). As follows from figure (F1) the difference of 0.1 in the nominal Love number h_2 causes a displacement error up to 6 cm (station HartRAO, South Africa, at 11:00 UTC on January 1st, 2006).

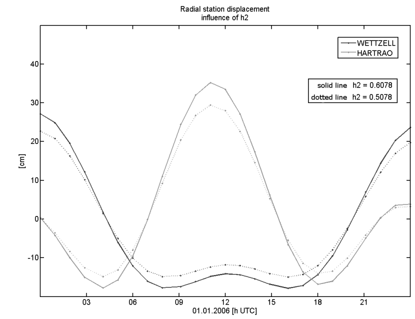

Figure (F1): Effect of wrong degree-2 Love number. By a difference of 0.1 in the nominal value reaches the displacement error in radial direction up to 6 cm.

2.4 Modelled radial components of tidal displacement following the IERS Conventions 2003

Currently, the recommended model in the IERS Conventions 2003 (McCarthy and Petit, 2004) for the computation of the variation of the station coordinates due to the solid Earth tides consists of a two-step procedure. In the first step, corrections are applied in the time domain, with one real nominal value of the Love number for all degree-2 tides ($h_2 = 0.6078$). The out-of-phase displacement due to the imaginary parts of the Love numbers is computed with one value for the diurnal tides ($h_{21}^{I} = -0.0025$) and one value for the semidiurnal tides ($h_{22}^{I} = -0.0022$). In the second step, corrections caused by the intra-band variation of the real degree-2 Love numbers in the diurnal and long-period band are taken into account. The contribution to the displacement from the variation of the imaginary parts of the Love numbers is significant only in the long-period band and from the K_1 tide in the diurnal band, as stated in Mathews et al. (1997).

We decided to compute and visualise the individual contribution to the total tidal displacement in radial direction for the complete Earth's surface with a spatial resolution of 1°x1° on Figures (F2) to (F9). The snap shots are taken for January 1st, 2006 at 0:00 UTC. The configuration of the perturbing bodies is shown in Figure (F10), where the Sun's ephemerides are reduced by a factor of 100. Moon and Sun just passed through their closest position, when they were aligned at new Moon. So, January 1st is the first day after which the amplitude of the tidal displacement reached the maximum and is now slowly decreasing.

In Figures (F2) and (F3) the displacement arising from the second degree tidal potential is divided into the contributions of Moon and Sun, respectively. It can be seen, that the bulges, which are caused by the Moon are slightly more than twice as large as those caused by the Sun. The same separation is shown in (F4) and (F5), where the forming potential is of third degree. In Figure (F5) it can be seen, that concerning degree 3 the contribution of the Sun is quite ignorable. The largest displacement is only 0.01 mm. The out-of-phase contributions to radial displacement are computed with nominal values for the whole band: diurnal (F6) and semidiurnal (F7). In Figure (F8) the correction from 11 constituents in the diurnal band is plotted, showing individual contributions with more than 0.05 mm in amplitude. The variation in frequency arises from the resonance behaviour of the Earth, caused by the presence of the fluid core; its total effect can amount up to ±15 mm. In the long period band (F9) the frequency dependence arises from the mantle anelasticity. Contributions of five terms having a radial correction of more than 0.05 mm in amplitude were considered causing a total displacement of up to ±1 mm. The list with the constituents (including real and imaginary parts) was taken from McCarthy and Petit (2004).

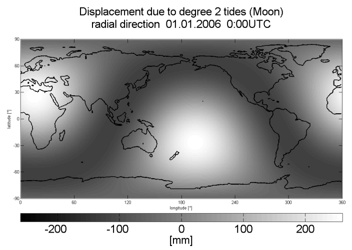

Figure (F2): Displacement in radial direction due to second degree tides on January 1st, 2006 at 0:00 UTC, only Moon's contribution.

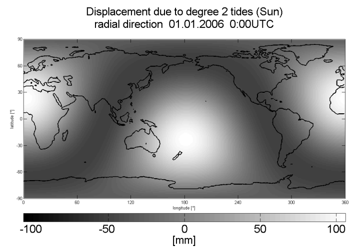

Figure (F3): Displacement in radial direction due to second degree tides on January 1st, 2006 at 0:00 UTC, only Sun's contribution.

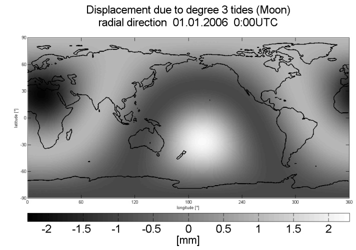

Figure (F4): Displacement in radial direction due to third degree tides on January 1st, 2006 at 0:00 UTC, only Moon's contribution.

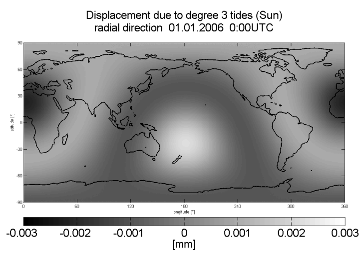

Figure (F5): Displacement in radial direction due to third degree tides on January 1st, 2006 at 0:00 UTC, only Sun's contribution.

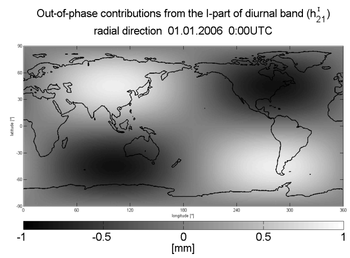

Figure (F6): *tesseral part*
Corrections to displacement in radial direction for the out-of-phase part of diurnal band on January 1st, 2006 at 0:00 UTC.

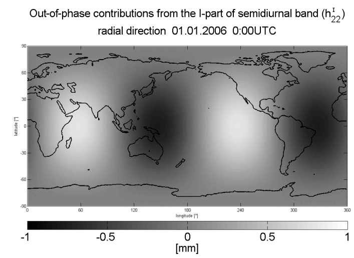

Figure (F7): *sectorial part*
Corrections to displacement in radial direction for the out-of-phase part of semidiurnal band on January 1st, 2006 at 0:00 UTC.

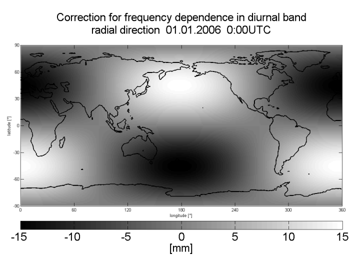

Figure (F8): *tesseral part*
Corrections to displacement in radial direction for frequency dependence in the diurnal band on January 1st, 2006 at 0:00 UTC.

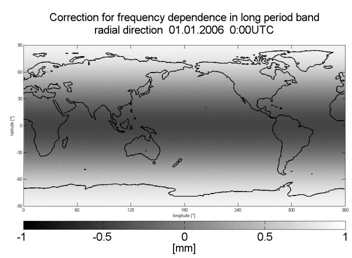

Figure (F9): *zonal part*
Corrections to displacement in radial direction for frequency dependence in the long period band (in-phase and out-of-phase) on January 1st, 2006 at 0:00 UTC.

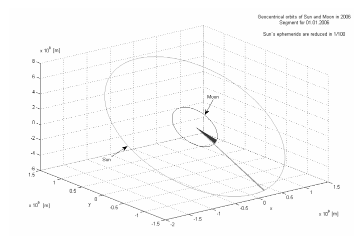

Figure (F10): Geocentric orbits of the Sun and the Moon on January 1st, 2006. Sun's ephemerides are reduced by a factor of 100.

3 Estimation of Love and Shida numbers from VLBI

3.1 VLBI data analysis

The partial derivatives of the tidal displacement w.r.t. the real part of the nominal value of the degree-2 Love and Shida numbers and w.r.t. the real parts of six frequency-dependent Love numbers in the diurnal band have been added to the VLBI software package OCCAM (Titov et al., 2004). We used data from the CONT05 campaign. CONT05 was a two-week campaign of continuous VLBI sessions, scheduled for observing in September 2005 and coordinated by the International VLBI Service for Geodesy and Astrometry (IVS). The observations started on September 12th and ended on September 27th. The station network consisted of 11 stations.

The main settings of the software for the VLBI data analysis were:
- Catalogue of the radio sources: ICRF-Ext.1,
- Catalogue of the stations: ITRF2000,
- Cut-off elevation angle: 5.0°,
- No estimation of source coordinates.

3.2 Results from the CONT05 campaign

a) Nominal degree-2 Love and Shida numbers

Several configurations of the computational approach were applied. Table (T1) shows the mean values of estimated nominal degree-2 Love and Shida numbers for individual (sequential) sessions. The values in the first column refer to the free network solution, where the station coordinates were estimated with an NNR/NNT condition (no net rotation/no net translation). The results obtained from parameter estimation with fixed station coordinates are given in the second column. In both columns, h_2 and l_2 were estimated in parallel. The following two columns refer again to a free network and fixed network, respectively. However, in this case only the Love number h_2 (or only the Shida number l_2) was determined. Figure (F11) shows that the values obtained from the fixed network and from the free solution are shifted by an offset of about 0.02 for the Love number h_2. For the Shida number l_2 (F12) the values are nearly the same. Focusing on the solid and dashed lines it follows, that a simultaneous estimation of the Love and Shida numbers provides almost the same results as a separate estimation of h_2 and l_2.

mean values of estimated nominal degree-2 Love and Shida numbers				
	parallel estimation of h_2 and l_2		separate estimation of h_2 or l_2	
	free network	fixed network	free network	fixed network
h_2	0.6184	0.5899	0.6193	0.5917
standard deviation	±0.0070	±0.0058	±0.0068	±0.0055
l_2	0.0823	0.0824	0.0817	0.0828
standard deviation	±0.0009	±0.0008	±0.0009	±0.0008

Table (T1): Mean values of h_2 and l_2 estimated from the CONT05 campaign.

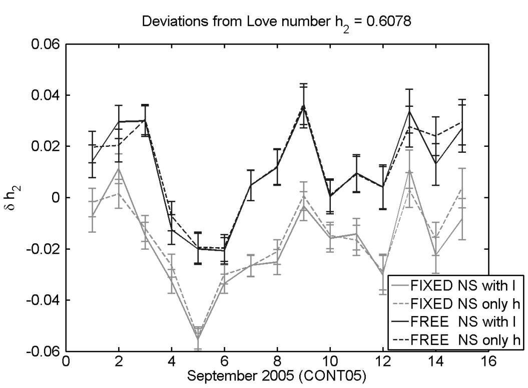

Figure (F11): Deviations from the nominal Love number h_2 estimated for daily intervals during the CONT05 campaign. Grey and black lines show the values obtained from a fixed and free network, respectively. Results from the approach, where the Shida number l_2 was simultaneously estimated are plotted in solid lines, whereas the dotted lines show the separate h_2 estimation.

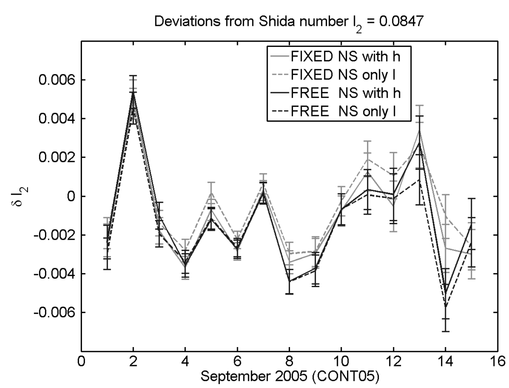

Figure (F12): Deviations from the nominal Shida number l_2 estimated for daily intervals during the CONT05 campaign. Grey and black lines show the values obtained from a fixed and free network, respectively. Results from the approach, where the Love number h_2 was simultaneously estimated are plotted in solid lines, whereas the dotted lines show the separate l_2 estimation.

b) Frequency dependent Love numbers in the diurnal band

We also tried to estimate the Love numbers corresponding to six selected tidal waves in the diurnal band (O_1, P_1, K_1, ψ_1, Φ_1, J_1). The first two digits in the argument number of the tidal wave (Table (T2), second column) represent the group number, which characterizes the block of waves separable from one month of observations. From our period of data (15 days) it would be only possible to separate the wave O_1 ($n_1n_2 = 14$) from the group $n_1n_2 = 16$, because the cipher n_2 differs by two times of the Moon's mean longitude; the first digit n_1 stays for the species number in the sense of Laplace, where $n_1 = 1$ represents the tesseral spherical harmonic function. To evade this limitation we had to use finesse in the estimation approach. Our strategy was to solve always for one wave only, while the others were kept fixed. This procedure was applied at each tidal wave, so we went six times through the whole data analysis process. Our goal was to find out, if also from a short time interval any reasonable results could be obtained knowing that our procedure is mathematically not fully correct as it neglects all correlations between the parameters of the various terms.

Figure (F13), together with Table (T2), shows the mean estimated value of frequency dependent Love numbers in the diurnal band from the CONT05 campaign (black dots). These are compared to the results achieved by Haas and Schuh (1996) who had used more than 10 years of VLBI data which are plotted in grey lines. The dotted line interpolates between the currently adopted Love number values (McCarthy and Petit, 2004) for the diurnal band. The frequency of the Nearly Diurnal Free Wobble (NDFW) in the terrestrial frame (corresponding to Free Core Nutation (FCN) in the celestial frame) was fixed. It is evident, that the formal errors of the very weak tides, close to the resonance, are larger than those of the strong tides and also larger than of the results obtained by Haas and Schuh (1996) due to the much longer time span of VLBI data used in the latter solution. Nevertheless, it is interesting that a similar shift of $h_{21}(\psi_1)$ is found which was already reported by Haas and Schuh (1996). The point is that the estimated value is opposite to the resonance curve. The "official value" is 1.0569, whereas the results of Haas and Schuh (1996) and this work provide negative values of -0.136 ± 0.228 and -1.484 ± 1.459, respectively. However, it should be paid attention to the large formal errors. Also the weak J_1 tide does not correspond to its expected value. The reason is in the low amplitude and – as already mentioned – the short time span of VLBI data used here. On the other hand, the results for the three strong tides K_1, O_1, P_1 do agree to theory within their formal errors. The differences to their theoretical values are less than twice their standard deviation.

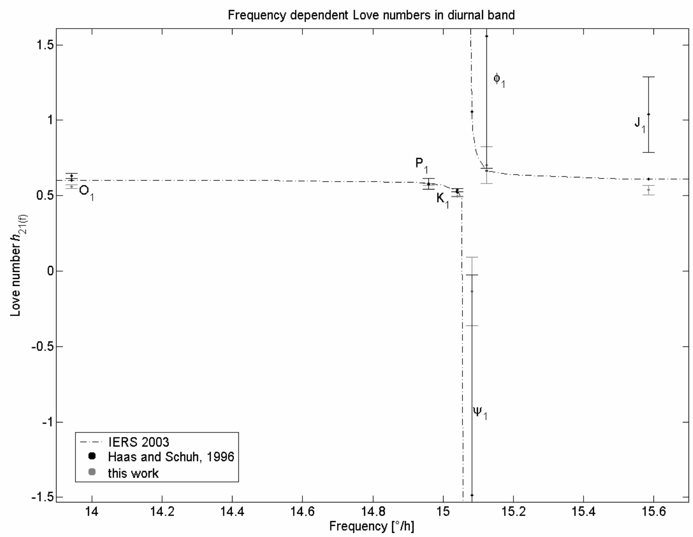

Figure (F13): Estimates of frequency dependent Love numbers in the diurnal band.

Tide	Argument number	Amplitude [mm]	Love number $h_{21(f)}$		
			IERS 2003	Haas and Schuh (1996)	This work
O_1	145.555	–262	0.6028	0.560±0.012	0.631±0.016
P_1	163.555	–122	0.5817	0.574±0.005	0.578±0.036
K_1	165.555	369	0.5236	0.496±0.002	0.537±0.012
ψ_1	166.554	3	1.0569	–0.136±0.228	–1.484±1.459
Φ_1	167.555	5	0.6645	0.702±0.121	1.559±0.879
J_1	175.455	21	0.6108	0.538±0.031	1.039±0.250

Table (T2): Estimates of frequency dependent Love numbers for six tidal waves in the diurnal band.

4 Conclusions

- Nominal second degree Love and Shida numbers of the solid Earth tide model could be already determined from fifteen 24-hour VLBI sessions of the CONT05 campaign.
- The estimated value of the nominal value h_2 is 0.618±0.007, which differs by 0.011 from its theoretical value. The estimate of the Shida number l_2 is 0.082±0.001 and has a difference of about –0.002 from its theoretical value. Better results with respect to the predicted values were found with a free network solution instead of fixed station coordinates.
- One possible reason for the larger uncertainty in the estimates of h_2 compared to l_2 is that it depends on the vertical displacements of the VLBI stations. The vertical component (i.e. height) is usually less precise in space geodetic observations than the horizontal component. It can be subject to errors caused by the time delay of the signals through the atmosphere (errors in the models for the troposphere, e.g. mapping functions) or by atmosphere loading and ocean loading corrections, which are mainly in radial direction.
- Frequency-dependent Love numbers were achieved in an iterative approach for the three largest tidal waves K_1, O_1, P_1, for which the estimated values differ from the theoretical values by less than twice their standard deviation. For the three weak tidal waves J_1, ψ_1, Φ_1, being close to the NDFW resonance, we obtained values with large standard deviations.
- It was demonstrated, that Love numbers for strong tides can be determined from VLBI data covering only a short time interval. The formal errors of the results would decrease with the number of observables used and thus the optimal accuracy could be achieved by doing a so-called global solution using all existing VLBI data since 1984.

5 References

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