

# Limitations of High Precision Tidal Prediction

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**ABSTRACT :** The most demanding applications are tidal gravity predictions. We examine if the  $\text{nms}^{-2}$  precision can be reached. It correspond roughly to a  $4 \cdot 10^{-4}$  of the tidal range (TR) at mid latitude. High precision tidal prediction requires either tidal factors derived from tidal observations or modelled tidal factors based on the response of the Earth to tidal forces and on the ocean tides contribution. Both methods rely on a precise knowledge of the astronomical tides. The accuracy of the astronomical tides is very large and different tidal prediction programs agree within  $10^{-5}\text{TR}$ . A reduced tidal development (1200 terms in Tamura) still insures a precision of  $2 \cdot 10^{-4}\text{TR}$ . For tidal predictions based on observations the calibration is the main limiting factor and 0.1% remains a target still difficult to reach. The records length limits the separation of the different tidal groups. If the tidal factors of different tidal waves within the same group are not the same, systematic errors are introduced. For example neglecting the resonance around  $\psi_1$  in the  $K_1$  group, can introduce an error at the level of  $3 \cdot 10^{-4}\text{TR}$ . For tidal predictions based on modelled tidal factors the choice of the model for the response of the Earth to tidal forces is critical as differences between recent models are slightly larger than 0.1%. The best models seem to fit the observations within  $5 \cdot 10^{-4}$ . The evaluation of the indirect effect of the ocean tides is critical and general conclusions are only valid at distances larger than 100km from the coast, where improved grid is not compulsory for tidal loading computations. In the best cases we can reach a precision of  $5 \cdot 10^{-4}\text{TR}$ . Our conclusion is that the accuracy of 0.1% is generally difficult to reach and that  $5 \cdot 10^{-4}$  is nowadays the limit of accuracy using long series of observations of regularly calibrated instruments.

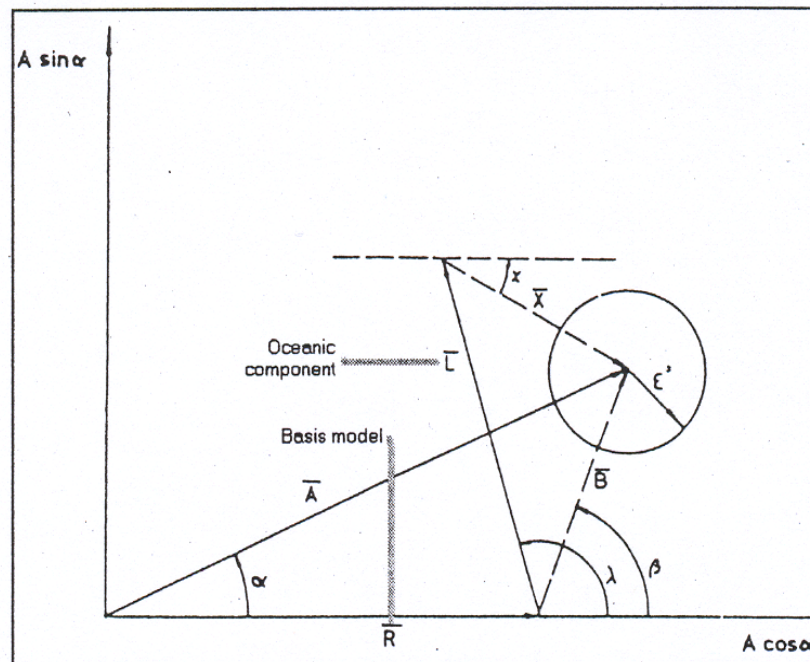
**Keywords:** tidal predictions, body tides, astronomical tides, ocean tides loading

## 1. Introduction:

Among the different applications of tidal prediction tidal gravity is the most demanding one. Absolute gravity measurements reach nowadays a precision of  $10^{-9}\text{g}$  or  $10\text{nms}^{-2}$  ( $1\mu\text{gal}$ ). A good metrological practice requires an accuracy 10 times better for all the corrections to be applied, including tidal gravity corrections. Tidal predictions should reach an accuracy of  $1\text{nms}^{-2}$ , which corresponds roughly to a  $4 \cdot 10^{-4}$  of a  $2,500\text{nms}^{-2}$  ( $250\mu\text{gal}$ ) tidal range (TR). Moreover long period (LP) tides have to be included. However as high precision absolute measurements require observations during one day or more, a large part of the tidal effect is averaged out, but not the LP tides. For gravity prospecting the measurements are always differential and the precision required is one order of magnitude lower. As a matter of fact tidal predictions with an accuracy of 0.1% will generally be sufficient. However in the following considerations we shall check if the level of the  $\text{nms}^{-2}$  ( $4 \cdot 10^{-4}\text{TR}$ ) can be reached. The considerations developed for gravity tides can be easily extrapolated for the other tidal components. Another point is that we should consider here the maximum discrepancy and not the standard deviation of the tidal prediction. The reason is that in gravity prospecting we

consider isolated values. As pointed above absolute measurements are the exception as they average the short period tides.

Astronomical tides are very accurately computed from tidal potential developments (see section 2), but for an elastic Earth it is necessary to take into account the deformation of the Earth and the additional change of potential induced by this deformation. The result is known as “body tides”. For gravity, the amplitude change is expressed by the ratio  $\delta_E$  between the tides on the elastic Earth and the amplitude of the astronomical tides  $A_{th}$ . As the tidal forces are applied also to the fluid parts of the Earth i.e. the ocean and the atmosphere, the reaction of these fluids produces additional gravity, tilt and strain changes superimposed exactly on the frequencies of the body tides. After correction of the atmospheric effects, the different constituents of the tidal effects at a given tidal frequency can be represented by rotating vectors (Fig. 1).



**Figure 1:** Phasor plot at a given tidal frequency showing the relationship between the observed tidal amplitude vector  $\mathbf{A}(A,\alpha)$ , the Earth model  $\mathbf{R}(R,0)$ , the computed ocean tides load vector  $\mathbf{L}(L,\lambda)$ , the tidal residue  $\mathbf{B}(B,\beta)=\mathbf{A}-\mathbf{R}$  and the corrected residue  $\mathbf{X}(X,\gamma)=\mathbf{B}-\mathbf{L}$ , after Melchior (1994). See the text for further explanation.

Let us consider:

- the observed amplitude vector

$\mathbf{A} = (\delta A_{th}, \alpha)$ , where  $\delta$  is the observed tidal amplitude factor and  $\alpha$  is the observed phase difference;

- the body tides tidal amplitude vector

$\mathbf{R} = (\delta_E A_{th}, 0)$ , where  $\delta_E$  is the expected tidal amplitude factor according to a given Earth model;

- the ocean load vector

$\mathbf{L}(L,\lambda)$  indirect effect computed from a given ocean tide model

For tidal prediction we can follow two approaches:

- a direct approach based on the tidal factors  $(\delta, \alpha)$ , derived from tidal records.

- an indirect approach based on predicted tidal factors  $(\delta_m, \alpha_m)$ , derived from the modelled tidal vector

$$\mathbf{A}_m(\delta_m A_{th}, \alpha_m) = \mathbf{R}(\delta_E A_{th}, 0) + \mathbf{L}(L, \lambda) \quad (1)$$

The two approaches should be equivalent if:

- the instrument is well calibrated;
- the Earth response and the tidal loading are well modelled.

We shall first consider the different factors influencing the precision of the tidal prediction.

The accuracy of the determination of observed tidal factors depends on:

- the calibration of the instrument (section 3.1)
- the astronomical tides (section 2)
- the length of the tidal record (section 3.2)

For the predicted tidal factors we should take into account:

- the response of the Earth to tidal forces (section 4.1)
- the ocean tides contribution (section 4.2)
- the astronomical tides (section 2)

For tilt and strain it should be necessary to model also the topography and cavity effects besides ocean tides contributions.

As the astronomical tides computation is a common factor we shall first consider this topic.

## 2. Astronomical tides computations

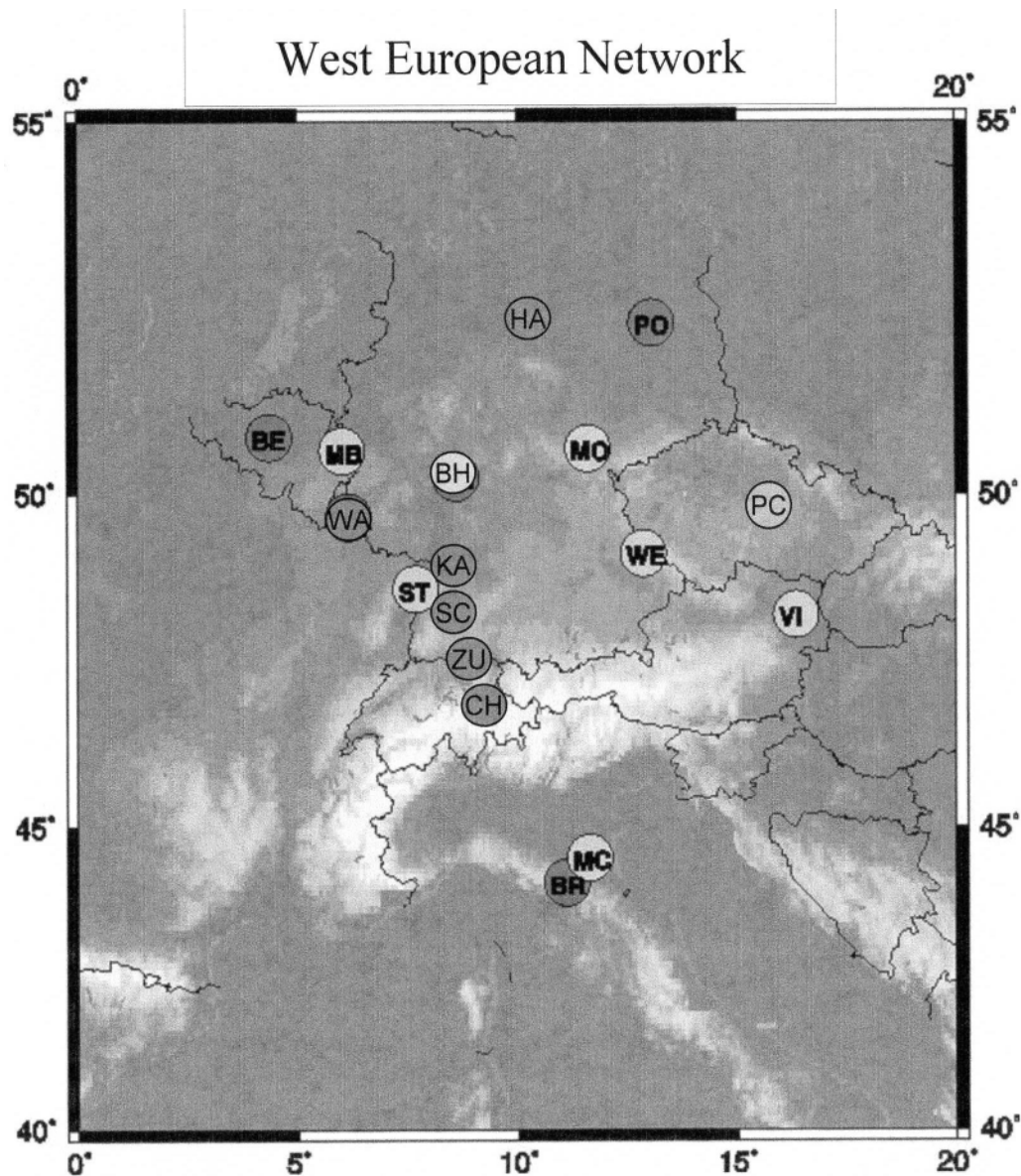
The first factor determining the precision of the tidal predictions is the number of terms or tidal waves considered in the tidal development. A recent study of the most recent tidal developments by Kudryavtsev (2004) confirmed the increase of precision with the number of terms: RATGP95 (Roosbeek, 1996, 6,499 terms, 5ngal), HW95 (Hartmann and Wenzel, 1995, 12,935 terms, 1.23ngal), KSM03 (Kudryavtsev, 2004, 28,806 terms, 0.39ngal). HW95, used as a standard by the ETERNA software (Wenzel, 1996), insures thus a precision of  $5 \cdot 10^{-6}$ TR. A previous tidal development TAM1200 (Tamura, 1987, 1,200 terms) is already correct at the level of  $2 \cdot 10^{-4}$ TR (Ducarme, 2006). It is still widely used in BYTAP-G (Tamura et al., 1991), VAV (Venedikov and Vieira, 2004), T-soft (Van Camp and Vauterin, 2005) and ICET software.

The first step of the tidal prediction is the precise evaluation of the direct influence of the Moon, the Sun and the planets, generally called the “astronomical tides”. It is based on the developments of the tidal potential (Melchior, 1978). To derive a tidal prediction we have to consider a scale factor often referred as “Doodson” constant, a geometrical part depending on the position at the surface of the Earth (geodetic coefficients), which is different for each tidal component, and the harmonic part, which is a sum of sinusoidal terms. The development of the tidal potential provides for each term a normalised amplitude and an argument which is a linear combination of the astronomical arguments of the celestial bodies. Only 6 arguments, chosen by Doodson, are required for the Luni-solar tides. Concerning the planetary influences, Tamura was the first to introduce tidal terms coming from Jupiter and Venus, arriving to a total of 8 arguments. Roosbeek and Hartmann-Wenzel introduced additional arguments for Mars, Mercury and Saturn to arrive to a total of 11 astronomic elements.

Comparisons between the ICET and ETERNA software can be found in Ducarme, 2006. The tidal prediction computed using the TAM1200 potential is equivalent in PREDICT (ETERNA) and MT80TW (ICET) to better than  $10^{-5}$ TR.

### 3. Precision of the observed tidal factors

The main uncertainty on the observed tidal factors comes from the calibration of the instruments. If the record length is less than one year the liquid core resonance will produce spurious effects inside the  $K_1$  group. It should be noted also that the LP tides are generally not well determined as they require very long tidal records. It is always possible to use modeled tidal factors to replace missing observed values. A discussion of the modeling of the LP tides will be given in section 4.2.



**Figure 2.** Selected stations in the West European network (Ducarme et al., 2008)

New GGP: MB (Membach), ST (Strasbourg), BH (Bad Homburg), MC (Medicina), MO (Moxa), WE (Wetzell), VI (Vienna).

Older stations: BE (Brussels), WA (Walferdange), KA (Karlsruhe), SC (Schiltach), ZU (Zürich), CH (Chur), HA (Hanover), PO (Potsdam), PC (Pecny).

BR (Brasimone) not used

### 3.1 Calibration of the gravimeters

It is necessary to model the instrument transfer function in amplitude and phase, at least at the tidal frequencies. Very precise techniques have been developed for the determination of the transfer function (Richter and Wenzel, 1991 ;Van Camp et al., 2000). Time lag corrections are precise at the level of the second i.e.  $0.01^\circ$  on  $M_2$  or  $2 \cdot 10^{-4}$  TR at the equator. An extensive study including 16 tidal gravity station in Western Europe (Fig. 2) arrived to the conclusion that the level of 0.1% is already difficult to reach for the amplitude calibration (Ducarme et al., 2008b). This network is subdivided in two parts: 7 stations of the Global Geodynamics Project (GGP) (Crossley et al., 1999) equipped with modern “compact tidal” (CT) and “double sphere” (CD) superconducting gravimeters (SG) and 9 other ones, where older model T SG’s or spring gravimeters were used.

The GGP SG’s have been calibrated using parallel tidal recording with absolute FG5 gravimeters, as described in Francis (1997). Most of the instruments used in the additional stations have been calibrated against the Hanover vertical calibration line (Kangieser and Torge, 1981; Kangieser et al., 1983), either directly or indirectly.

The precision of a single calibration can be derived from the difference between the tidal factors obtained with collocated instruments. SG’s simultaneously calibrated using the same absolute FG5 gravimeter show that the agreement is of the order of 0.05% (Ducarme et al., 2008b). Regularly repeated calibrations in Strasbourg lowered the RMS error to 0.03% (Rosat et al., 2008).

Referring to Figure 1, we can define the so called “corrected” tidal parameters: amplitude factor  $\delta_c$  and phase difference  $\alpha_c$ , by the relation.

$$\mathbf{A}_c(\delta_c A_{th}, \alpha_c) = \mathbf{A} - \mathbf{L} \quad (2)$$

As the ocean tides loading is well constrained in this part of Europe, the variations observed in the corrected amplitude factors can be considered as reflecting the calibration errors. The standard deviation of the seven GGP stations calibrated with FG5 instruments is 0.08%. The dispersion of the nine additional stations is only slightly larger (Ducarme et al., 2008b).

A promising approach for the calibration of gravimeters is the use of inertial accelerations. For that purpose the instrument is placed on a platform and submitted to vertical accelerations at different frequencies. Such a platform was developed for spring gravimeters (van Ruymbeke, 1989; van Ruymbeke et al., 2008). A precision of 0.1% has been achieved. For SG’s the three usual supports are replaced by step motors which are gently lifting up and down the instrument in a sinusoidal way (Richter et al., 1995; Wilmes et al., 2008). All elements of the system are designed to insure a precision of  $10^{-4}$ . However the precision is still limited to 0.05%.

As a conclusion we can state that the best precision achieved nowadays is:

- Superconducting gravimeters
  - parallel registration with absolute gravimeters  
 $3 \cdot 10^{-4}$  to  $10^{-3}$
  - inertial accelerations  
 scheduled  $10^{-4}$       effective  $0.5 \cdot 10^{-3}$
- Spring gravimeters
  - vertical baseline  $10^{-3}$
  - inertial accelerations  $10^{-3}$

It should be noted that the normalisation of spring gravimeters at a fundamental station, as it was realised for example during the Trans World Tidal Gravity Profiles (Melchior, 1994), did not generally insure a precision better than  $3 \cdot 10^{-3}$  (Ducarme et al., 2008a).

### 3.2 Effect of the record length

As the ocean tides loading is strongly frequency dependant, we cannot extrapolate the tidal factors obtained for one wave to a neighboring one. It is thus important to resolve a maximum of tidal groups to avoid systematic errors. Records shorter than 6 months should be avoided as the main waves  $P_1$  and  $K_2$  cannot be separated from their neighbors  $K_1$  and  $S_2$ .

Moreover, due to the liquid core resonance, the Nearly Diurnal Free Wobble (NDFW) modifies the body tides amplitude factors inside the diurnal band (Melchior, 1978; Dehant et al., 1999). The resonance effect is concentrated inside the  $K_1$  group. A minimum time span of one year is required to resolve the complex tidal structure of this group, which includes the two annual ( $\psi_1$  and  $S_1$ ) and semi-annual ( $\phi_1$  and  $P_1$ ) modulations of  $K_1$ .  $P_1$  amplitude factor is reduced of 0.45%, and  $K_1$  of 1.7%, while  $\psi_1$  is amplified of 10% and  $\phi_1$  of 1.4%. If the record length is shorter than 6 months, the error will reach  $7 \cdot 10^{-4}TR$  at a latitude of  $50^\circ$ , due to the differential resonance between  $K_1$  and  $P_1$ . Tidal records shorter than 1 year will not allow the separation of the annual modulations inside the  $K_1$  group and produce residues at the level of  $3 \cdot 10^{-4}TR$ . However this effect can be strongly reduced by the introduction of a resonance model inside the group. It has been implemented in MT80TW.

## 4. Precision of predicted tidal factors

Besides the astronomical tides evaluation, the precision of the predicted tidal factors depends on the precision of the  $\mathbf{R}$  and  $\mathbf{L}$  vectors i.e. the precision of the body tides model and of the tidal loading computation.

### 4.1 The body tides models

Different body tides models are used by the different tidal prediction software.

- PREDICT is using latitude dependent tidal parameters for an elliptical, rotating, inelastic and oceanless Earth computed from the Wahr-Dehant-Zschau model (Dehant, 1987).

- MT80TW computes tidal predictions with  $\delta_E$  values extracted from

- either (Dehant et al., 1999) :

- the DDW99 elastic (H)

- the DDW99 non-hydrostatic/inelastic (NH) models

- or the MAT01/NH inelastic (Mathews, 2001) model.

These models differ at the level of  $10^{-3}$  (Table 1).

To discriminate the different theoretical models we compare the values of  $\delta_E$  with the experimentally determined corrected amplitude factor  $\delta_c$  computed by the relation (2).

A study based on the global GGP network (1997-2003) (Ducarme et al., 2007) provided a mean value  $\delta_c(O_1) = 1.1546 \pm 0.0006$

It agrees within 0.1% with:

- the value  $\delta_E = 1.1543$  computed from the DDW99/NH model

- and the value  $\delta_E = 1.1540$  given by MAT01

A more recent study of the West European network (Ducarme et al., 2008b, Fig. 2) gave:

- For O1 the value  $\delta_c = 1.15340 \pm 0.00023$  falling between the DDW99/H (1.1528) and the MAT01/NH (1.1540) inelastic models.
- For M2 the value  $\delta_c = 1.16211 \pm 0.00020$  fitting very well the DDW99/NH (1.1620) and MAT01/NH (1.1616) inelastic models.
- For K1 the mean result  $\delta_c = 1.13525 \pm 0.00032$  fitting the MAT01/NH (1.1349) inelastic model to better than 0.05%.

The conclusion is that MAT01/NH inelastic model seems to be the best choice, with an error close to  $5 \cdot 10^{-4}$ .

**Table 1:** Theoretical amplitude factors at 45° latitude

	O1 $\delta_{th}$	K1 $\delta_{th}$	M2 $\delta_{th}$	O1/K1	M2/O1
DDW/H	1.1528	1.1324	1.1605	1.0180	1.0067
MAT01/NH	1.1540	1.1349	1.1616	1.0168	1.0066
DDW/NH	1.1543	1.1345	1.1620	1.0174	1.0066

## 4.2 The tidal loading computation

In continental stations the loading effect is generally at the level of a few microgal for the main waves, but one can observe huge effects in coastal areas. Moreover the variation of the tidal factors for a given value of the load vector depends of the amplitude of the astronomical tides at this latitude. As the diurnal gravity tides vanish at the equator, the corresponding tidal factors are not reliable at low latitude. It is the same at very high latitudes for both diurnal and semi-diurnal tides. It is thus difficult to issue general statements concerning the precision of modeled tidal factors and our examples are taken from middle latitude stations.

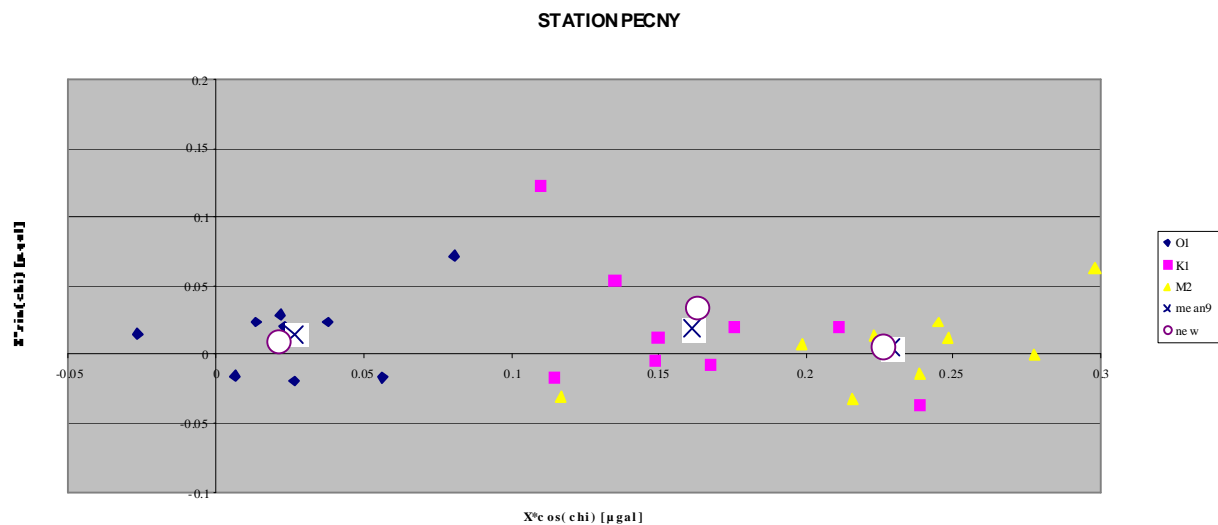
The ocean tides models provide at least the 8 main diurnal ( $Q_1$ ,  $O_1$ ,  $P_1$ ,  $K_1$ ) and semi-diurnal ( $N_2$ ,  $M_2$ ,  $S_2$ ,  $K_2$ ) and the fortnightly tide  $M_f$ . These waves cover most of the tidal spectrum. However in the diurnal band the frequencies higher than 1.024cycle/day (periods lower than 23h45m), corresponding to the small constituents  $J_1$  and  $OO_1$ , are not always available. As the contribution of these groups represents only 6.5% of the diurnal tides, we can use the body tides values as a first approximation.

The LP tides deserve a careful treatment. Two recent studies (Ducarme et al, 2004; Boy et al, 2006) showed that, for the fortnightly lunar wave  $M_f$ , the tidal loading computations based on recent ocean tides models were in agreement with tidal gravity observations of superconducting gravimeters performed in the frame of GGP. The observations cannot determine precisely enough the monthly lunar wave  $M_m$  so that it is not yet possible to confirm its modelling. However Boy et al. showed that the ratio of the tidal loading vectors  $\mathbf{L}$  for  $M_m$  and  $M_f$  is roughly equal to their amplitude ratio in the gravity tides and that the phases are similar. We can thus include  $M_m$  and  $M_f$  in one and a same group. In Siberia (Ducarme et al., 2008a) the 3 recent models (NAO99, TPX06, FES04) agree closely for  $M_f$  with a standard deviation better than 0.1% in amplitude and 0.05° in phase.

For the annual and semi-annual solar waves  $S_a$  and  $S_{sa}$  the tidal loading is not the main perturbation. The contributions from meteorological and hydrogeological sources are preponderant. Tidal gravity analyses on GGP data sets determined observed tidal factors larger than 2 for  $S_a$  (see for example Ducarme et al, 2006), while global models are required for effective pressure corrections (Neumeyer et al., 2004) and continental water storage fluctuations induce strong seasonal effects (Peter et al, 1995; Neumeyer et al, 2006). As these very long period tidal waves deserve a special treatment we suggest to use the body tides model values for tidal predictions.

The constant tidal effect called  $M_0S_0$  should be treated with a special care in order to follow the resolutions of the International Association of Geodesy (IAG). For Gravity one should follow the “zero tide” correction principle i.e. one should remove only the astronomical part of the  $M_0S_0$  tide and not the constant deformation. Clearly speaking the amplitude factor of  $M_0S_0$  should be put equal to 1.

If tidal gravity observations have been performed in the area, it is often possible to select a best fitting ocean tides model, but generally the use of the mean of several models is largely improving the precision (Zahran, 2000; Zahran et al., 2005). In Figure 3 we consider 9 different ocean tides models (ORI96, CSR3, CSR4, FES95, FES02, FES04, NAO99, GOT00 and TPX06) and a sub-group of 6 more recent models (CSR4, FES02, FES04, GOT00, NAO99, ORI96, TPX06).



**Figure 3:** dispersion of the final residue  $X$  computed from 9 ocean tides models for a continental station (Pecny, CZ)

X: mean of 9 models, O: mean of 6 recent models

Let us consider the 16 European stations of Figure 2. The standard deviation of the 9 different ocean tides models is close to  $0.3\mu\text{gal}$  for  $O_1$  and  $M_2$ , i.e. 0.1% of the amplitude of these tidal waves. The use of the mean of 9 different models could reduce the accidental error contribution down to  $3 \cdot 10^{-4}$  of the tidal range.

Let us consider the trans-Siberian tidal gravity profile (TSP, Ducarme et al., 2008a). On the Siberian territory the tidal factors modelled using 9 different ocean tides models have a standard deviation close to 0.1% ( $0.05^\circ$ ) for the diurnal waves and 0.2% ( $0.1^\circ$ ) for  $M_2$ . Using the mean of 9 ocean tides models we can thus insure a precision of 0.03% ( $0.015^\circ$ ) in the diurnal band and 0.06% ( $0.03^\circ$ ) in the semi-diurnal one. The RMS error of 3 recent models of the LP tide  $M_f$  is lower than 0.025%. The global RMS error due to load computations can be kept below  $5 \cdot 10^{-4}TR$ .

The case of 4 gravity stations installed along the Atlantic coast of France (Timofeev et al., 2006), at 100km from the sea shore, is less favourable, due to the very large semi-diurnal ocean tides loading of the Gulf of Biscay. The standard deviation of the 9 different ocean tides models is close to  $0.3\mu\text{gal}$  for  $O_1$ , but it reaches  $1\mu\text{gal}$  for  $M_2$ ! It follows that the use of the mean of 9 ocean tides models will still have a global uncertainty at the level of  $0.2\mu\text{gal}$ , close to  $10^{-3}TR$ . Among the different ocean tide models the best fit with the observed tidal factors



was obtained using CSR3, CSR4 or FES02 with less than 0.05% in amplitude and 0.1° in phase.

We can conclude that the use of the mean of several tidal models can reduce the uncertainty to  $5.10^{-4}TR$  for inland stations in well constrained areas, but that at 100km from the coast the uncertainty can easily reach  $10^{-3}TR$ .

### 5. Statements concerning the final precision

For each of the two approaches we can draw a table giving the best precision that can be reached as well as the normal one. The three main contributions are listed together with the expected global precision. For the astronomical tides usual means that the TAM1200 tidal potential is used.

Table 2 presents the case of the observed tidal factors. As expected the main error source is the calibration. A precision better than  $5.10^{-4}$  was only achieved by the SG of Strasbourg. It is probably more realistic to consider a precision of 0.1% for the time being. At this level of precision a reduced tidal development is sufficient for the astronomical tides computation.

For spring gravimeters only the best instruments can insure a precision of 0.1%. A more conservative figure is 0.3%.

For the modeled tidal factors (Table 3) the two main error sources are the tidal loading evaluation and the uncertainties on the response of the Earth to the tidal forces. Even in the best case the error budget is close to 0.1%. For coastal stations the error on tidal loading evaluation is very difficult to estimate. Tidal gravity observations can help to determine the best models for the considered region.

**Table 2:** Precision on the observed tidal factors for superconducting gravimeters (SG) and spring gravimeters

Calibration		Astr. tides		Rec.length		Total	
best	usual	best	usual	>1y.	<1y.	best	usual
SG							
$\leq 510^{-4}$	$10^{-3}$	$5.10^{-6}$	$2.10^{-4}$	-	$3.10^{-4}$	$\leq 510^{-4}$	$10^{-3}$
spring							
$10^{-3}$	$3.10^{-3}$	$5.10^{-6}$	$2.10^{-4}$	-	$3.10^{-4}$	$10^{-3}$	$3.10^{-3}$

**Table 3:** Precision of the modelled tidal factors

Loading		Earth mod.		Astr. tides		Total	
inland	<100km	best	any	best	usual	best	usual
$5 \cdot 10^{-4}$	$>10^{-3}$	$5 \cdot 10^{-4}$	$10^{-3}$	$5 \cdot 10^{-6}$	$2 \cdot 10^{-4}$	$7 \cdot 10^{-4}$	$>10^{-3}$

## 6. ICET contribution

Which approach is the most efficient?

- The determination of precise observed tidal parameters is time consuming and requires expensive instruments.

- Modelled tidal parameters are inexpensive to compute but unreliable for coastal stations.

The International Centre for Earth Tides (ICET) prepared two kinds of modelled tidal factors, available from its web site <http://www.upf.pf/ICET/>.

For 1,000 stations around the world very precise tidal parameters based on different means of ocean tides models are proposed. We computed modeled tidal factors using 9 different ocean tides models (ORI96, CSR3, CSR4, FES95, FES02, FES04, NAO99, GOT00 and TPX06). The tidal loading vector  $\mathbf{L}$  was evaluated by performing a convolution integral between the ocean tide models and the load Green's function computed by Farrell (1972). The Green's functions are tabulated according to the angular distance between the station and the load. The water mass is condensed at the center of each cell and the Green's function is interpolated according to the angular distance. This computation is rather delicate for coastal stations if the models are computed on a coarse grid, as the stations can be located very close to the center of the cell. The numerical effect can be largely overestimated. To avoid this problem our tidal loading computation checks the position of the station with respect to the center of the grid. If the station is located inside the cell, this cell is eliminated from the integration and the result is considered as not reliable (Melchior et al., 1980). We can consider two groups of models, the older models up to 1996 (ORI96, CSR3, FES95) on one hand, and the new generation of models (CSR4, FES02, FES04, GOTOO, NA099 and TPX06) on the other. For the first generation of models, the effect of the imperfect mass conservation is corrected on the basis of the code developed by Moens (Melchior et al., 1980). Following Zahran's (2000, 2005) suggestion, we computed mean tidal loadings for different combinations of models: all the 9 models or only the 6 recent ones.

As many of the ocean tide models do not provide the smaller tidal constituents  $J_1$ ,  $OO_1$ ,  $M_3$ ,  $M_4$ , we provide only the theoretical amplitude factors of the corresponding groups. For the long period constituents we use always the mean of the 3 recent models NAO99, TPX06 and FES04 to compute the loading for the fortnightly tide  $M_f$  and we include the monthly tide  $M_m$  as well as the shorter period tides in one and the same group  $M_f$ . As explained in section 4.1, we use the body tides model values for the annual and semi-annual solar waves  $S_a$  and  $S_{sa}$ .

To evaluate the real precision of the prediction based on modelled tidal factors we compared it with a prediction based on observed tidal factors in one of the best calibrated stations: Moxa

(Fig. 2). The tidal coefficients are given in Table 4. Due to the large difference in the tidal parameters for the Ssa group, a strong semi-annual wave shows up with an amplitude of  $2\text{nm s}^{-2}$  in the difference between the two tidal predictions (Figure 4). For the shorter periods the differences does not exceed  $\pm 3\text{nm s}^{-2}$ , i.e.  $1.2 \cdot 10^{-3}\text{TR}$ . There is a scale difference of  $4 \cdot 10^{-4}$  producing a systematic effect of  $\pm 0.5\text{nm s}^{-2}$ . It is a mixture of the errors due to the inaccuracy of the calibration and of the body tides model. The residual error is close to  $\pm 2.5\text{nm s}^{-2}$  i.e. 0.1% and corresponds principally to the inaccuracy of the ocean tides computation. The associated standard deviation is only  $1\text{nm s}^{-2}$  ( $4 \cdot 10^{-4}\text{TR}$ ). If we do not consider the LP tides, the error on the tidal correction of absolute gravity determinations obtained by observations averaged on several days will be of the same order of magnitude. It justifies the statement made in the introduction .

For less accurate tidal predictions we propose global tidal gravity parameters on a  $0.5^\circ \times 0.5^\circ$  grid using the CSR3 or NAO99 ocean tide model for 9 waves ( $M_f$ ,  $Q_1$ ,  $O_1$ ,  $P_1$ ,  $K_1$ ,  $N_2$ ,  $M_2$ ,  $S_2$ ,  $K_2$ ). Zhou J.C. et al. (2007) used the CSR3 ocean tides model together with a purely elastic Earth model. Ocean load vectors have been computed using the Agnew (1996, 1997) software. The NAO99 model was used at ICET with the Melchior et al. (1980) software. The computed load vectors were associated to a non hydrostatic/inelastic Earth model (Dehant et al., 1999) to compute modeled tidal parameters.

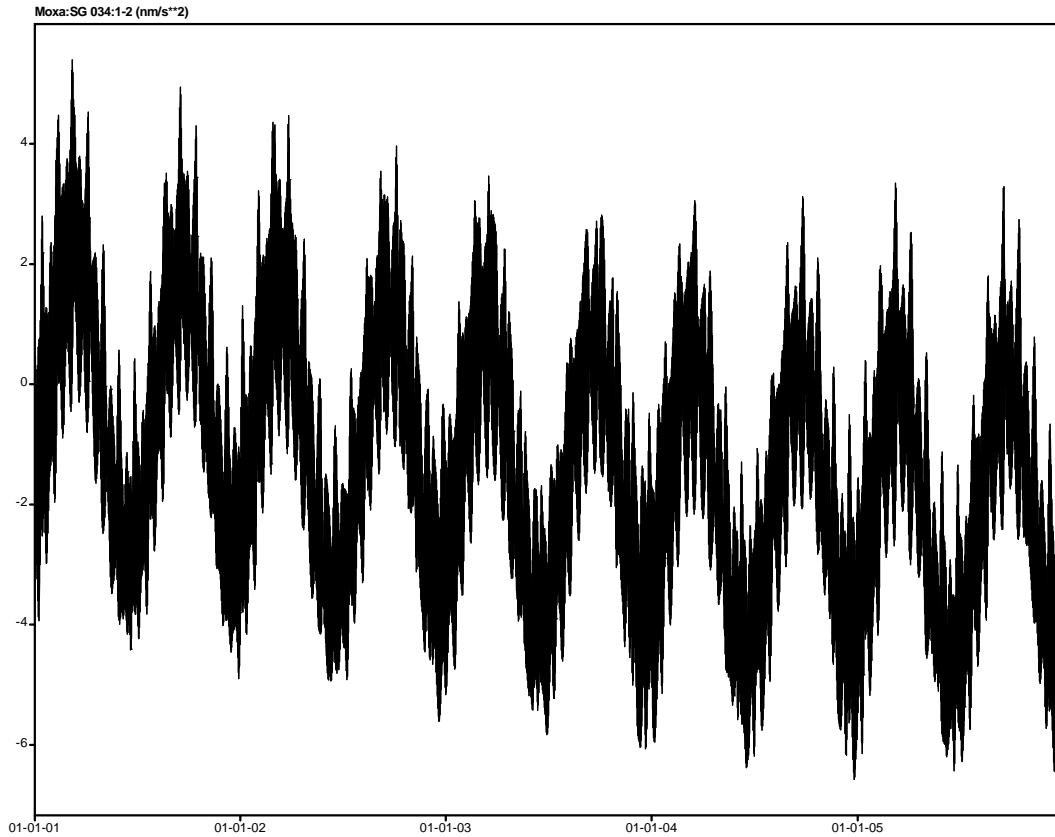
Interpolation software is proposed on the ICET WEB site to provide an output compatible with the most common tidal prediction software. The proposed software is an update of the WPAREX program developed by H. G. Wenzel for a bilinear interpolation inside the grid. If the input coordinates are not surrounded by 4 grid points error message is issued and the values at the closest point are selected..

If one of the grid points is too close from one cell of the ocean tides model a warning is issued, as the load vector computation is probably not accurate at this point.

**Table 4:** Observed ( $\delta$ ,  $\alpha$ ) and modelled ( $\delta_m$ ,  $\alpha_m$ ) tidal gravity factors for station Moxa.  
N: number of waves in Tamura, 1987

Tidal Group	N	Frequency range (cycle per day)		$\delta$	$\alpha$ ( $^\circ$ )	$\delta_m$	$\alpha_m$ ( $^\circ$ )
						DDW99/NH 6 recent models	
M0S0	2	.000000	.000001	1.0000	0.000	1.0000	0.000
Ssa	32	.000002	.020884	1.2358	0.760	1.1570	0.000
Mf	247	.020884	.501369	1.1454	0.450	1.1411	0.410
Q1	143	.501370	.911390	1.1461	-0.186	1.1468	-0.132
O1	106	.911391	.981854	1.1488	0.124	1.1501	0.097
P1	17	.981855	.998631	1.1493	0.179	1.1503	0.161
K1	40	.998632	1.023622	1.1363	0.224	1.1358	0.156
J1	43	1.023623	1.044800	1.1566	0.166	1.1560	0.000
OO1	102	1.064841	1.470243	1.1535	0.118	1.1560	0.000
N2	149	1.470244	1.914128	1.1762	2.167	1.1784	2.062
M2	95	1.914129	1.984282	1.1850	1.581	1.1859	1.510
S2	17	1.984283	2.002736	1.1835	0.344	1.1866	0.607
K2	116	2.002737	2.451943	1.1859	0.581	1.1838	0.546
M3	81	2.451944	3.381378	1.0695	0.454	1.0700	0.000

Systematic comparisons between the precisely computed and the interpolated tidal gravity parameters for 24 GGP stations around the world showed that the differences on the mean amplitude factors are small i.e. less than  $4 \cdot 10^{-4}$  (Zhou J.C. et al., 2007). However the interpolated tidal parameters may become questionable at very high latitude or for the diurnal waves at the equator (section 4.2).



**Figure 4.** Difference between tidal prediction using the observed tidal parameters at Moxa and a modelling based on the DDW99/NH body tides model and the mean of 6 recent ocean tides models (Table 4). Units are  $\text{nm s}^{-2}$

## 7. Conclusions

The final accuracy of tidal prediction based on previous tidal observations depends on the correct evaluation of the astronomical tides, the length of the tidal records and the accuracy of the calibration of the instrument.

Tidal predictions can also be performed on the grounds of “predicted tidal factors”. The different elements contributing to the precision of such tidal predictions are:

- the astronomical tides;
- the response of the Earth to the tidal force;
- the ocean tides contribution.

The accuracy of the astronomical tides is very large and different tidal prediction programs agree within  $10^{-5}$  of the tidal range (TR). A reduced tidal development (1200 terms in Tamura) still insures a precision of  $2 \cdot 10^{-4}$  TR.

For tidal prediction based on observation the records length limits the separation of the different tidal groups. If the tidal factors of different tidal waves within the same group are not

the same, systematic errors are introduced. The two main sources of difference between waves with close frequencies are the FCN in the diurnal band and differential ocean tides effects. Neglecting the resonance around  $\psi_1$ , can introduce an error at the level of  $3 \cdot 10^{-4}$ . Differential ocean tides effect depends on the magnitude of the local effects.

The calibration remains thus the main limiting factor and 0.1% remains a target still difficult to reach. The standard deviation on the corrected tidal factors of 16 selected stations in Europe reaches 0.1% in amplitude and  $0.02^\circ$  in phase.

For tidal predictions based on modelled tidal factors the choice of the model for the response of the Earth to tidal forces is critical as difference between recent models are slightly larger than 0.1%. Investigations based on 16 stations in western Europe showed that the MAT01 model fits the observations within 0.05% for  $O_1$  as well as for  $M_2$ .

The tidal loading evaluation is critical and general conclusions are only valid at distances larger than 100km from the coast, where improved grid is not compulsory for tidal loading evaluation. We present case studies for Europe and Siberia. In the best cases we can reach a precision of 0.05%. In these areas the global error due to Earth model and tidal loading is thus below 0.1%. This level of precision is confirmed by tests performed on one of the best GGP station.

Up to now it is thus quite impossible to reach an accuracy of  $4 \cdot 10^{-4}$ TR for tidal prediction on a real Earth. It is even difficult to reach  $10^{-3}$ TR, which is suitable for tidal correction of absolute gravity observations. At this level of precision a reduced tidal development is sufficient for the computation of astronomical tides. The modelling of tides with periods larger than 6 months is still unreliable.

As it is much less expensive to compute modelled tidal factors than to perform tidal gravity observations, the International Centre for Earth Tides (ICET) prepared two kinds of modelled tidal factors, available from its web site <http://www.upf.pf/ICET/>. For 1,000 stations around the world very precise tidal parameters based on different means of ocean tides models were computed. For less accurate tidal predictions we propose global tidal gravity parameters on a  $0.5^\circ \times 0.5^\circ$  grid using the CSR3 or the NAO99 ocean tide model for 9 waves ( $M_f$ ,  $Q_1$ ,  $O_1$ ,  $P_1$ ,  $K_1$ ,  $N_2$ ,  $M_2$ ,  $S_2$ ,  $K_2$ ). An interpolation software is also available.

## Acknowledgements

The author had fruitful discussions with O. Francis concerning tidal predictions suitable for absolute gravity measurements and with H. Wilmes on the precision reached by the inertial calibrating method for superconducting gravimeters.

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