Validating the synthetic tidal gravity parameters with superconducting gravimeter observations

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Abstract Tidal gravity parameters are modeled with a Wahr-Dehant body tide model and the ocean tide models of Schw80 for M_f and CSR3.0 for 8 main diurnal and semidiurnal waves. The tidal parameters at the GGP stations are first interpolated from a 05°x05° grid. Similarly, modeled tidal parameters are computed just at the stations. From the comparison between the parameters obtained with the two approaches, it is clear that the resolution of 0.5°x0.5° is generally sufficient for interpolation. When a station is located very close to the zero-line of a tidal family the body tide model is not well constrained. When the theoretical tidal amplitude becomes very small, the tidal loading can become proportionally very large and large fluctuations of the modeled tidal factors are produced by the interpolation procedure. On the other hand, observed tidal parameters are obtained from the harmonic analysis of the superconducting gravimeters records at the GGP stations. The comparison between modeled and observed parameters shows that synthesizing tidal gravity parameters is an effective tool for tidal gravity prediction.

Keywords tidal gravity parameters, body tide model, ocean tide model, superconducting gravimeters

1 Introduction

The superconducting gravimeter, due to its high precision, high stability and low drift, ensures the accurate determination of tidal parameters including amplitude factor and phase lead. And it has a lot of applications in geodesy, geophysics and geodynamics. Due to the identical frequency content, it is impossible to separate the earth body tide from the ocean loading tide by the analysis of tidal observations. The ocean tide loading effect is computed by a convolution of the Green's function (Farrell, 1972) and ocean tide height and then subtracted from tide observations.

Previous studies showed that the ocean tide loading is the main component in the residues of the observations. Therefore, modeling tidal parameters with ocean tide loading effect and theoretical body tide derived from an Earth's model is an efficient way to get an accurate tidal prediction. Zahran compared the synthetic tidal parameters of gravity with the ones observed in Europe, and concluded that the two values are in good agreement with each other (Zahran et al, 2005).

We use the same method as Zahran's to model the tidal gravity parameters, and compare them with the results of the superconducting gravimeters inside the GGP network (Crossley et al., 1999). The stations and the data sets are listed in table 1. Due to the high precision of superconducting gravimeters, it is expected that this comparison will provide a more realistic validation of the accuracy of the synthetic tidal parameters.

Table 1: List of stations and time span of the observations

Name		latitude (°)	longitude (°)	height (m)	time span
Ny-Alesund	NY	78.9306	11.8672	43	19990920...20041231
Brussels	BE	50.7986	4.3581	101	19820421...20000922
Membach	MB	50.6092	6.0066	250	19950804...20060403
Strasbourg	ST	48.6223	7.6840	185	19970301...20051031
Brasimone	BR	44.1235	11.1183	684	19920801...20000201
Vienna	VI	48.2493	16.3579	192	19970701...20041231
Wettzell	WE	49.1458	12.8794	612	19981104...20060630
Potsdam	PO	52.3809	13.0682	48	19920630...19981008
Moxa	MO	50.6447	11.6156	455	20000101...20060630
Metsahovi	ME	60.2172	24.3958	56	19970701...20051231
Pecny	PC	49.9170	14.7830	534	20000420...20050425
Wuhan	WU	30.5159	114.4898	89	19971220...20021127
Taiwan	TW	24.7926	120.9855	87	/
Gyungsang	GY	36.6402	128.2147	107	/
Kyoto	KY	35.0278	135.7858	60	19970701...20020731
Matsushiro	MA	36.5430	138.2070	406	19970501...20030630
Esashi	ES	39.1511	141.3318	393	19970701...20040225
Sutherland	SU	-32.3814	20.8109	1770	20001023...20050101
Bandung	BA	-6.8964	107.6317	714	19971219...20030630
Canberra	CB	-35.3206	149.0077	763	19970701...20041231
Boulder	BO	40.1308	-105.2328	1682	19950412...20031031
Cantley	CA	45.5850	-75.8071	269	19970701...20031231
TIGO	TY	-36.8470	-73.0255	100	20021205...20060630
Syowa	SY	-69.0070	39.5950	22	19970701...20030131

2 Earth's tidal gravity

Conventionally, the tide generating potential is expanded with cosine functions. For the degree 2, the tidal generating potential is represented as (Fang, 1984)

$$T_2 = \sum_p D_{i_1} K_p \cos(i_1\tau + i_2 s + i_3 h + i_4 p + i_5 N' + i_6 p_s) \quad 0 \le i_1 \le 2 \tag{1}$$

where p represents different constituents, i_1 is the order of the constituents, i.e. 0 for long period tide (LP), 1 for diurnal tide (D) and 2 for semidiurnal tide (SD). For higher degrees of the potential, there are shorter periods third-diurnal (TD), quarter-diurnal (QD),….terms. And the bracket part is depending on time through the 6 Doodson arguments (Melchior, 1983). A main difference with the conventions of the oceanographers is that the arguments are referred to the longitude of the point of interest. K_p is the normalized amplitude coefficient of the tidal expansion. For negative amplitudes one considers usually the absolute value and adds 180° to the phase of the constituents. D_0, D_1 and D_2 are the latitude dependent geodetic coefficients of LP, D and SD tides respectively. They can be written as follows

$$\begin{Bmatrix} D_0 \\ D_1 \\ D_2 \end{Bmatrix} = D_C \begin{cases} (\frac{r}{R})^2 (\frac{1}{3} - \sin^2\phi) \\ (\frac{r}{R})^2 \sin 2\phi \\ (\frac{r}{R})^2 \cos^2\phi \end{cases} \tag{2}$$

where r is geocentric height, R the mean radius of the Earth, ϕ the latitude and D_C the Doodson constant.

In the spherical approximation, the dependence of geocentric height r on latitude vanishes. Hence Earth's tidal gravity can be computed as the derivative of potential with respect to r (positive downward), that is for LP, D and SD waves respectively:

$$G = \frac{\partial T_2}{\partial r} = D_C K_p \begin{cases} \frac{2r}{R^2}(\frac{1}{3} - \sin^2\phi)\cos(i_2 s + i_3 h + i_4 p + i_5 N' + i_6 p_s) \\ \frac{2r}{R^2}\sin 2\phi \cos(\tau + i_2 s + i_3 h + i_4 p + i_5 N' + i_6 p_s) \\ \frac{2r}{R^2}\cos^2\phi \cos(2\tau + i_2 s + i_3 h + i_4 p + i_5 N' + i_6 p_s) \end{cases} \tag{3}$$

As a consequence, the LP gravity tide changes its sign across latitude of $\pm 35.26°$ and the diurnal gravity tide changes its sign across equator.

For realistic application, an Earth tide model is usually not rigid but elastic. Therefore a factor δ_2 called amplitude factor should be applied to the gravity tide, which is (Melchior, 1983):

$$\delta_2 = 1 + h_2 - \frac{3}{2}k_2 \tag{4}$$

where h_2 and k_2 are the Love numbers for degree 2.

3 Ocean tide loading

Ocean tides due to astronomic forces changes the distribution of sea water masses, which results in variation of the Earth's gravity field by means of direct attraction. Meanwhile for the elasticity of the Earth, ocean tides also deform the Earth and then changes the gravity filed. The effect from attraction and deformation is called ocean tide loading effect. From Farrell (Farrell, 1972), this effect can be numerically calculated by convolution of ocean tides and Green's function. That is

$$\Delta g = \rho R^2 \int_0^{2\pi} \int_0^{\pi} G(\psi) H(\psi, A) \sin\psi \, d\psi \, dA \tag{5}$$

in which Δg is gravity variation due to ocean tide load, ρ density of sea water, $G(\psi)$ Green's function, which is relative to angular distance ψ between load and station of interest, and $H(\psi,A)$ ocean tidal height as a function of angular distance ψ ($0 \le \psi \le \pi$) and azimuth A ($0 \le A \le 2\pi$) of the direction from station to load.

4 Synthetic tidal parameters

The synthetic tidal parameters are obtained by adding ocean tide loading effect to the theoretical body tide from an Earth tide model (Timmen, et al, 1994; Zahran, et al, 2005). For an elastic Earth model, the phase lead is zero which respect to the local tidal potential. To get the Greenwich phase, the following equation can be used.

$$P_G = P_L + i_1 \times lon \tag{6}$$

where P_G and P_L are Greenwich and local phases respectively, i_1 has the same meaning as in eq.1 and *lon* is longitude (positive to the East) of the station of interest. Traditionally, phase lag is positive in oceanography while phase lead is positive in geophysics. Therefore, for the same location, the sign of phase should be changed. Fortunately, in the NLOADF program written by Agnew (Agnew, 1997), this problem has been taken into account, i.e positive values correspond to a phase lead. Therefore the relation of equation (6) also holds for ocean tide loading effect.

Conventionally, the harmonic analysis of gravity tide provides phase differences with respect to the local tidal potential and amplitude factor always with positive sign, independently of the sign of the corresponding tidal force. As mentioned in section 2, the parallels of $\pm 35.26°$ and 0° are zero-lines of long term and diurnal tides respectively. Therefore the gravity tides at the stations over different sides of these parallels have opposite signs. Hence in synthesizing the tidal parameters, this should be taken into account, by adding adequate phase shifts (Table 2).

Table 2 phase shift for different kinds of waves of Earth's tidal gravity

Earth tides	Latitude domain	Phase shift LP	D	SD
	35°16′~90°	180°	0°	0°
gravity	0°~35°16′	0°	0°	0°
	-35°16′~0°	0°	180°	0°
	-90°~-35°16′	180°	180°	0°

For convenience, we always keep the phases of theoretical tides as 0 in the local reference frame. Therefore the phase shifts in table 2 should be subtracted from the phases of the ocean tide loading vectors.

5 Results and discussions

The theoretical values of Earth's tidal gravity factors are obtained using the program 'predict' from Eterna software (Wenzel, 1996). For gravimeter records, a priori amplitude factors for the tidal waves within one wave group are used from the WAHR-DEHANT (Dehant, 1987) elliptical, uniformly rotating oceanless Earth with inelastic mantle, liquid outer core and elastic inner core (PREM elastic Earth model with mantle dispersion from ZSCHAU and WANG 1987). Ocean tide loading is calculated with the Schw80 (Schwiderski, 1980) for M_f constituent and CSR3.0 (Eanes, 1996) for 8 main constituents (Q_1, O_1, P_1, K_1, N_2, M_2, S_2, K_2). And the integral Green's function method is adopted (Goad, 1980; Agnew, 1996; Agnew, 1997).

We calculate the synthetic tidal gravity parameters at the grid points around the GGP stations. There are four points for each station, and the spatial resolution is 0.5°. And then the tidal parameters of the stations are calculated using bilinear interpolation according to the ones of the grid points. Meanwhile, the tidal parameters are directly calculated by adding the ocean tide loading effect to theoretical body tides at those stations. The comparison between the interpolated values and calculated ones will effectively verify the adequacy of the spatial resolution.

Fig.1 gives the differences of M_f for all the stations. The differences of the "in phase" component are smaller than 0.012 except for Tigo (TI) in Chile, and the ones of the "out of phase" component are smaller than 0.02. The reason why Tigo is obviously an outlier is that this station with latitude of -36.847° locates near the zero-point.

Fig.2 gives the latitude dependence of the amplitude factor. For Tigo station, the two latitudes used for interpolation are -36.75° and -38.25°. By linear interpolation, the numerical result is between the two values. Unfortunately, the real value is quite different. The largest out of phase residue is found in Canberra (CB), where the M_f amplitude is also very small.

Fig.3 gives the differences of 4 diurnal constituents (Q_1, O_1 ,P_1, K_1) for all the stations. They are smaller than 0.01 for both in phase and out of phase parts. The differences for Esashi (ES), Taiwan (TW) and Syowa (SY) are apparently larger than the ones for the other stations. The reason is that there are some grid points locating over ocean area, for which the ocean tide loading effect may not have accurately been modeled. As a consequence, there will be much bias in interpolation comparing with the value directly calculated.

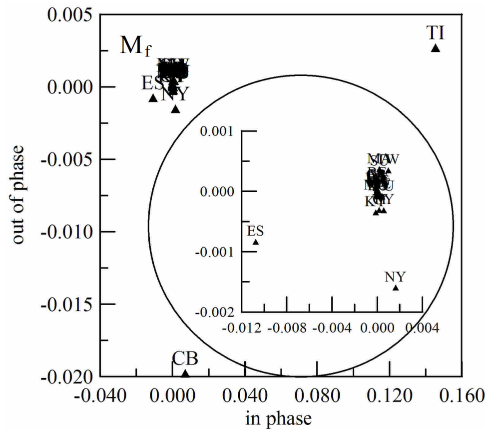

Figure 1: Differences between interpolated and calculated amplitude factors for M_f

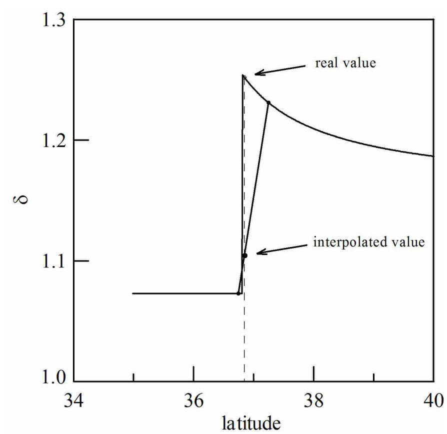

Figure 2: amplitude factor's variation as function of latitude near zero-point for long term tide

Fig.4 gives is the same as Fig3, but for the 4 main semi-diurnal constituents (N2, M2, S2, K2). As for diurnal, the differences are smaller than 0.01 for in phase and out of phase parts. Esashi (ES), Taiwan (TW) and Syowa (SY) are stations with larger differences. Additionally, the differences for Ny-Alesund (NY) station are also large due to the fact that the amplitude of the SD waves becomes very small at high latitude. For the other stations, the differences are very small.

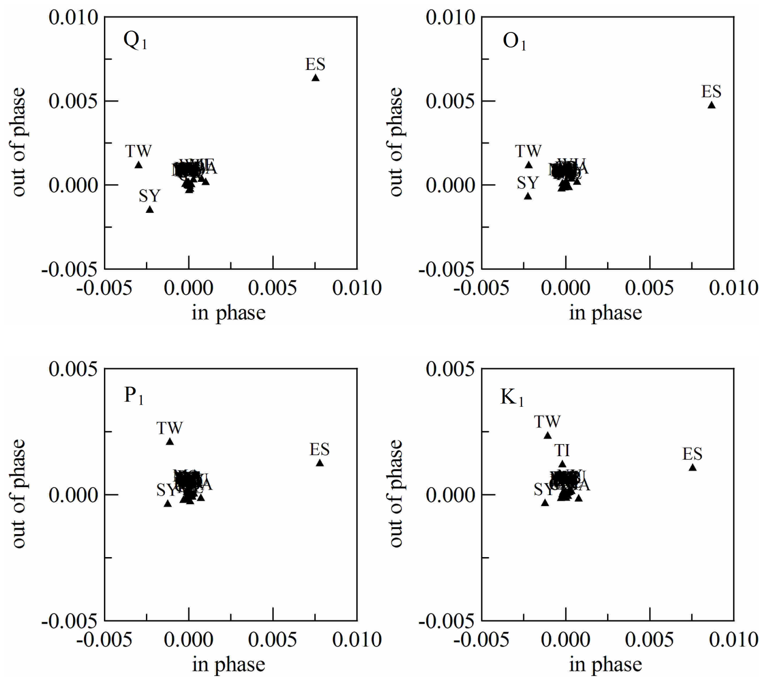

Figure 3: Differences between interpolated and calculated amplitude factors for diurnals

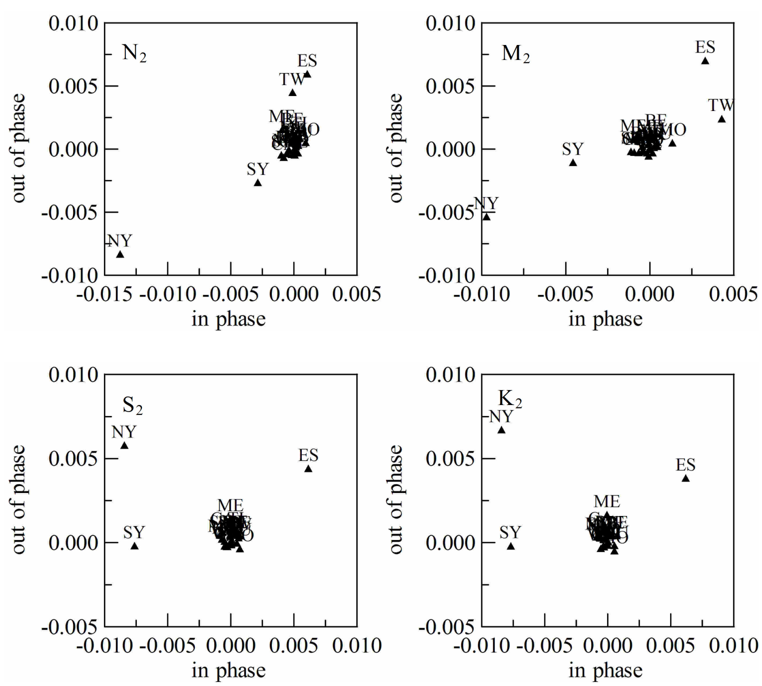

Figure 4: Differences between interpolated and calculated amplitude factors for semi-diurnals

Table 3 gives the statistical characteristics of the differences between the interpolated and directly computed values. The mean and standard deviation of the differences are computed including all the stations except for M_f for which station Tigo is excluded. The differences on the mean amplitude factors are small i.e. less than 0.0004 with standard deviation less than 0.003.

Table 3: Differences between interpolated and calculated tidal parameters for GGP stations

δ: amplitude factor, α: phase difference

	dδ		dα (°)	
wave	mean	std	mean	std
M_f	0.0003	0.0028	0.0516	0.2212
Q_1	0.0002	0.0019	0.0137	0.0625
O_1	0.0002	0.0020	0.0099	0.0463
P_1	0.0003	0.0017	0.0051	0.0245
K_1	0.0003	0.0016	0.0065	0.0258
N_2	0.0001	0.0008	0.1012	0.3991
M_2	0.0004	0.0028	0.0361	0.1098
S_2	0.0002	0.0022	0.0315	0.0964
K_2	0.0002	0.0022	0.0305	0.0968

Additionally, we compared also the tidal parameters interpolated and calculated for the 8 main waves for about 1000 stations over the world. For a better precision, only the stations of which the 4 points for interpolation around the station are all in land are adopted for the comparison. There are 638 stations satisfying this condition. Table 4 gives the statistic characteristics of the comparison. Unfortunately the result is still not yet satisfactory.

Table 4: Difference between interpolated and calculated tidal parameters for 638 stations

δ: amplitude factor, α: phase difference

	dδ		dα(°)	
wave	mean	std	mean	std
Q_1	-0.0021	0.0483	0.0623	1.3645
O_1	-0.0015	0.0443	-0.0478	3.0121
P_1	-0.0007	0.0586	0.1080	2.1945
K_1	-0.0008	0.0611	0.1067	2.2546
N_2	0.0049	0.0584	-0.0506	3.4700
M_2	0.0030	0.0578	-0.2546	3.5520
S_2	-0.0010	0.0613	-0.0988	2.2238
K_2	-0.0010	0.0604	-0.0590	2.3818

The discrepancy in table 4 is about one order of magnitude larger than in table 3. There are several reasons for that. One of the problems is that, even if many coastal stations have already been rejected when one of the 4 interpolation points is not on the land, there are still cases when one interpolation point is still close from one of the grid points of the cotidal map. Then the computed load vector is not reliable. There is a specific problem for gravity stations near equator, the zero line of diurnal tides, or at high latitude. When the ocean loading is still large, the interpolation error of the load vector produces large fluctuations of the tidal factors. We should directly compare the load vectors in stations where the theoretical amplitude of a wave is very small.

To verify the efficiency of the synthetic tidal parameters, the comparison between observed parameters and synthetic ones is carried out for the GGP stations. Table 5 gives the numerical results. As there is no observation available for stations of Taiwan/China and Gyungsang/South Korea, these two stations are excluded.

The means of the δ differences are smaller than 0.01 except for M_2. The mean phase differences are small for diurnals while they are much larger for semi-diurnals. Because of the high latitude for Ny-Alesund station, it is difficult to determine accurate tidal parameters for semi-diurnal waves. Therefore the result excluding the station of Ny-Alesund is also given in table 5 with asterisk marked left. The differences decrease with mean dδ values close to 0.005 and mean dα values close to 0.25°. The two results are thus in reasonable agreement, given the fact that the errors on the body tides model, the error of the ocean tides model and the calibration errors (Ducarme et al., 2007, 2008) are included in the comparison.

Table 5: Difference between interpolated and observed tidal parameters at GGP station

δ: amplitude factor, α: phase difference

	dδ		dα (°)	
wave	mean	std	mean	std
Q_1	0.0017	0.0106	0.0226	0.1926
O_1	0.0022	0.0052	0.0227	0.3276
P_1	0.0030	0.0088	-0.1049	0.1313
K_1	0.0062	0.0081	-0.0334	0.1204
N_2	0.0058	0.0123	-13.9732	64.8423
M_2	0.0206	0.0735	2.8623	13.3675
S_2	0.0082	0.0208	0.4334	3.7178
K_2	0.0076	0.0225	0.6500	3.4655
$^*N_2$	0.0065	0.0121	-0.1488	0.2021
$^*M_2$	0.0051	0.0116	0.0127	0.2300
$^*S_2$	0.0063	0.0194	-0.3588	0.1183
$^*K_2$	0.0090	0.0221	-0.0866	0.2728

* Ny Alesund excluded

6 Conclusions

In the best conditions, it is possible to interpolate the tidal parameters at one station, using synthetic tidal parameters on a 0.5°x0.5° grid with a precision better than 0.003 for the amplitude factor and 0.1° for the phase, as demonstrated by the GGP stations. The synthetic tidal parameters are comparable with the observations of the superconducting gravimeters. This demonstrates that interpolation is an effective way to get the tidal gravity parameters for tidal prediction. However a general comparison involving 638 stations shows that large errors on the amplitude factors and phase differences are possible e.g. when the theoretical amplitude of the wave becomes very small. However the effect on the tidal prediction becomes then also small. For these stations the comparison should be done directly on the load vector. When the interpolation grid points are too close from the cells of the ocean tides model the computed load vector is not reliable.

Special attention should be paid to the stations located near the zero-line of a tidal family, because the interpolation may give unrealistic results for the body tides amplitude factors.

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