Influence of the equilibrium ocean pole tide on the gravity field

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Summary: Although there is a lot of investigations on the ocean pole tide, there are not so many publications of its contribution to the gravity field. One of the reasons may be that both the ocean pole tide and the gravity pole tide are not easily determined by observations, as they are weak and, being close to the annual period, disturbed seriously by other geophysical phenomena, such as atmospheric and oceanic global circulation. With the development of the observation techniques, these two kinds of signals can be determined now. Results from the TOPEX/Poseidon (T/P) satellite altimetry for detecting the ocean pole tide confirm that the long-wavelength component of the geocentric pole tide deformations due to the Chandler wobble is consistent with the self-consistent equilibrium response model and the classical equilibrium one can only explain 70% of the variations. In addition, the superconduting gravimeters (SG) are now able to record and determine the gravity effect caused by polar motion. It is due to their high precision (as high as $10^{-11}$ m.s$^{-2}$) and very low drift (only at the level of 10 nm.s$^{-2}$ per year). Thus it is meaningful to estimate the contribution of the ocean pole tide to the gravity field now.

On the assumption that the ocean pole tide is in equilibrium, the loading gravity effects of both the classical and the self-consistent ocean pole tide are calculated at each of the 9 SG stations where gravity pole tide has been determined by Ducarme & al. [2006]. Then, after introducing a well-chosen phase lag, the amplitude of the ocean pole tide gravimetric vector at each of these stations is estimated with a linear fit method. Numerical results demonstrate that the amplitude of the loading gravity effect of the ocean pole tide is at the level of $10^9$ ms$^{-2}$ and that its phase lag is not negligible. The contribution of the self-consistent ocean pole tide to the gravity field variations cannot be ignored as it can increases the amplitude of the pole tide gravimetric factor by more than two per cent, but its effect on the phase remains negligible. The self-consistent model predicts effects 10% larger than the classical one.

1. Introduction

Both the ocean pole tide and the gravity pole tide are weak signals disturbed by many geophysical factors. Therefore, they are difficult to determine. The ocean pole tide is the ocean response to the variation of the centrifugal force. The equilibrium ocean pole tide assumes that the displaced ocean surface is in equilibrium with the equipotential surface [Lambeck 1988; Munk and Macdonald, 1960]. The classical equilibrium ocean pole tide is an equilibrium ocean pole tide conserving mass in non-global oceans overlying an elastic earth [Haubrich and Munk 1959; Dickman 1985; Desai 2002]. The self-consistent ocean pole tide further includes the tidal loading and the self-gravitation, in addition to the classical equilibrium ocean pole tide [Dahlen 1976; Agnew 1978; Desai 2002]. The assumption of an equilibrium ocean pole tide is controversial in the investigation of the ocean pole tide [Proudman 1960; Munk and Macdonald, 1960; Wunsch 1974; Naito, 1979, 1983; O'Connor 1983; Dickman 1985]. Recent results from TOPEX/Poseidon (T/P) satellite altimetry demonstrate that the long-wavelength
component of the geocentric pole tide deformations at the Chandler period is consistent with the theoretical self-consistent equilibrium response and can explain 70% of variations of the theoretical classical equilibrium response [Desai, 2002]. In addition, the superconducting gravimeters (SG) are able to record and precisely determine the gravity effect caused by polar motion, thanks to their high precision (as high as $10^{-11}$ ms$^{-2}$) and very low drift (only at the level of 10 nms$^{-2}$ per year). Thus the deployment of SG’s in the framework of the Global Geodynamics Project (GGP) provides an opportunity to determine accurately the pole tide gravity effect [Crossely 1999; Loyer 1999; Xu J. Q. 2004; Harnisch 2006; Ducarme et al., 2006]. Consequently, we have the reason and the possibility to estimate the influence of ocean pole tide on gravity field now.

There are only a few publications on the tidal loading effect of the ocean pole tide. Assuming that the pole tide is in equilibrium and has no phase lag, Hinderer and Legros point out that the pole tide gravimetric factor increases by 0.04, in the case of global oceans without continent [Hinderer and Legros 1989]. If the non-global oceans are used, a rough computation taking into account the degree 0 order 0 of the ocean function is done by Dehant, the increase is 0.025. And the phase shifts rang from 9º to 22º with an uncertainty of about 5º [Dehant and Zschau 1989]. Loyer introduces the work of Hinderer and Dehant when he tries to explain the influence of ocean pole tide on gravity field [Loyer 1999]. And Boy shows that the amplitude of the gravimeter factor increases from the theoretical value for an elastic Earth 1.16 to 1.185 and the phase lag is less than 0.5º after taking into account a static model of oceanic pole tide [Boy, 2000]. Xu J.Q. substitutes loading vector of Chandler period with that of the long period tidal wave $S_{0\alpha}$ to estimate the influence of the ocean pole tide on the pole tide gravimetric factor, and the increase is about 0.024 [Xu J. Q. 2004]. The emphasis of all these previous works is not to estimate the influence of the ocean pole tide so that only brief introductions are given in these works.

In this paper, we give a detailed description of the influence of the equilibrium ocean pole tide on the gravity field. The theoretical gravity pole tide is computed in advance. Then the gravity loading effect of the classical and the self-consistent ocean pole tide are computed, respectively. After manually introducing a proper phase lag in the calculation of the gravity loading effect, the relative amplitude of the loading pole tide with respect to the theoretical gravity pole tide is obtained by fitting the gravity loading effect and the theoretical gravity pole tide with a linear fit method. We present here the relative gravity loading vectors (amplitude factor and phase lag) of all the GGP stations. But only station Brussels is discussed in the paper because the processing is similar to all the other stations.

2. The pole tide potential

In an Earth’s fixed rotating coordinate system with its mean rotation axis is initially aligned with the $z$ axis of the coordinate system, $x$ is oriented along the Greenwich meridian, and $y$ along the 90ºE meridian, the instantaneous rotation vector $\omega(t)$ can be expressed as

$$\omega(t) = \Omega \begin{bmatrix} m_1(t) & m_2(t) & (1 + m_3(t)) \end{bmatrix}$$

where $\Omega$ is the mean rotation rate of the Earth, $\Omega = 7.292115 \times 10^{-5}$ rad·s$^{-1}$; $m_1(t)$ and $m_2(t)$ describe the polar motion, $m_3(t)$ describes the Earth’s rotation rate. The small time depending variables $m_i(t)$ are usually defined in terms of small quantities ($m_i(t) << 1$) [Lambeck, 1988; Munk and Macdonald, 1960]. To the first order $m_1(t)$ and $m_2(t)$ are the angular offsets of the instantaneous rotation vector from the $z$ axis of the defined coordinate system, in the direction of the $x$ axis and $y$ axis of the coordinate system, respectively.
The instantaneous centrifugal potential \( V_c(x, y, z) \) at an arbitrary point on the Earth \( \vec{r} = (x, y, z) \), can be written to the first order in \( m_i(t) \), as \cite{Wahr1985}.

\[
V_c(x, y, z) = \frac{1}{2} \left[ |\vec{p}(t)|^2 - (\vec{\omega}(t) \cdot \vec{p}(t))^2 \right] = \frac{\Omega^2}{2} \left[ (1 + 2m_3(t))(x^2 + y^2) - 2z(xm_1(t) + ym_2(t)) \right]
\]  

(2)

The time dependent differential centrifugal potential \( V(x, y, z, t) \) is then obtained by subtracting the centrifugal potential \( V_c(x, y, z) \) induced by the initial mean rotation vector \( \vec{\omega}_0 \) from the instantaneous centrifugal potential \( V_c(x, y, z) \).

\[
V(x, y, z, t) = V_c(x, y, z) - V_c(x, y, z) = \frac{\Omega^2}{2} \left[ 2m_3(t)(x^2 + y^2) - 2z(xm_1(t) + ym_2(t)) \right]
\]

\[
\vec{\omega}_0 = [0 \ 0 \ \Omega], \quad V_c(x, y, z) = \frac{\Omega^2}{2} \left[ x^2 + y^2 \right]
\]  

(3)

The magnitude of the variations in the rotation rate \( m_3(t) \) is at least two orders smaller than that of variations in the pole coordinates \( m_1(t) \) and \( m_2(t) \). Therefore, the contribution of \( m_3(t) \) to the differential centrifugal potential can be omitted. The remainder of the differential centrifugal potential is called the pole tide potential, which is only a function of the location of the instantaneous rotation pole. If the arbitrary point \( \vec{r} = (x, y, z) \) is expressed in geographical coordinates (colatitude \( \theta \) and longitude \( \lambda \)), the pole tide potential is then expressed as

\[
V(r, \theta, \lambda, t) = -\frac{r^2\Omega^2}{2}(m_1(t) \cos \lambda + m_2(t) \sin \lambda) \sin 2\theta
\]

\[
\vec{r} = (r, \theta, \lambda)
\]  

(4)

3. **Theoretical gravity pole tide**

The gravity change caused by the pole tide potential is given by the following formula \cite{Wahr1985; HuangC.L.1998; XuJ.Q.2004}.

\[
\Delta g(\theta, \lambda, t) = \delta \cdot r \cdot \Omega^2 \left( m_1(t) \cos \lambda + m_2(t) \sin \lambda \right) \sin 2\theta
\]  

(5)

where \( r \) is usually taken as the mean radius of the Earth, \( r = a = 6378136.6 \) m. The value of \( \delta \) is 1.0 for a rigid Earth, and about 1.16 for a spherical elastic Earth. The theoretical gravity pole tide is calculated with formula (5) with the value \( \delta = 1 \), that is for a rigid Earth. The instantaneous pole tide coordinates \( m_1(t) \) and \( m_2(t) \) are computed from the observed values \( x(t) \) and \( y(t) \) which are defined in a celestial ephemeris coordinate system. They are expressed in the form of complex notation \cite{Gross1992}.

\[
\begin{align*}
\mathbf{p}(t) &= x(t) - i \cdot y(t) \\
\mathbf{m}(t) &= m_1(t) + i \cdot m_2(t) = \mathbf{p}(t) - \frac{i}{\Omega} \dot{\mathbf{p}}(t)
\end{align*}
\]  

(6)

The transformation from \( \mathbf{p}(t) \) to \( \mathbf{m}(t) \) is used to eliminate the drift of the mean pole.
The observed instantaneous pole tide coordinates $x(t)$ and $y(t)$ are downloaded from the International Earth Rotation Service (IERS). We choose a time period January 1, 1995 to December 31, 1999, when the amplitude of the theoretical gravity pole tide covers almost its total range, in order to see the corresponding characteristic of the gravity loading effect of the ocean tide. The bold lines in Figure 1 and Figure 2 show the theoretical gravity pole tide at station Brussels.

4. **The classical and the self-consistent equilibrium ocean pole tide**

The classical equilibrium ocean pole tide $\zeta^C_0(\theta, \lambda, t)$ of an arbitrary point $r(a, \theta, \lambda)$ can be computed by the following equation [Desai, 2002].

$$
\begin{align*}
\zeta^C_0(\theta, \lambda, t) &= \gamma_2 \left( \frac{8\pi}{15} \right)^{\frac{3}{2}} \frac{\Omega^2 a^2}{g} \cdot \text{Re} \left[ m^*(t) \cdot \vec{E}^C(\theta, \lambda) \right] \\
\vec{E}^C(\theta, \lambda) &= O(\theta, \lambda) \left[ \vec{P}_{21}(\cos \theta) + K^C \right] 
\end{align*}
$$

(7)

where $\gamma_2 = \mathcal{P}_2 \exp(i \cdot \Delta \phi^C)$, is the Love number related to the ocean tide. $\Delta \phi^C$ denotes the phase lag between the classical equilibrium ocean pole tide and the static ocean tide, $\mathcal{P}_2 = 0.7053$. The mean gravitational acceleration at the Earth’s surface $g$ has been substituted with the ratio $GM/\alpha^2$ in the calculation, where $GM$ is the Earth’s gravitational constant, $GM = 3.986004418 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$. The complex constant $K^C$ is introduced to impose the conservation of mass on the classical equilibrium ocean pole tide. $\vec{E}^C(\theta, \lambda)$ is defined as a normalized equilibrium admittance function for the classical equilibrium ocean pole tide, so that the mass conservation can be done easily and the computation is then independent of the time. The ocean function $O(\theta, \lambda)$ has value of 1 on the ocean and 0 on the land [Muck and Macdonald, 1960]; the boundary of the ocean and the land used in the calculation is the boundary defined in ocean tide model NAO99 [Matsumoto 2000].

The self-consistent equilibrium ocean pole tide $\zeta^S_0(\theta, \lambda, t)$ of an arbitrary point $r(a, \theta, \lambda)$ can be computed by the following equation [Desai, 2002].

$$
\begin{align*}
\zeta^S_0(\theta, \lambda, t) &= \gamma_2 \left( \frac{8\pi}{15} \right)^{\frac{3}{2}} \frac{\Omega^2 a^2}{g} \cdot \text{Re} \left[ m^*(t) \cdot \vec{E}^S(\theta, \lambda) \right] \\
\vec{E}^S(\theta, \lambda) &= O(\theta, \lambda) \left[ \sum_{n=0}^\infty \gamma'_n \alpha_n \vec{E}^S_n + K^S \right] 
\end{align*}
$$

(8)

where the Love number $\gamma_2$ is also assumed with the complex form, $\gamma_2 = \mathcal{P}_2 \exp(i \cdot \Delta \phi^S)$, but $\Delta \phi^S$ denotes the phase lag between the self-consistent equilibrium ocean pole tide and the static ocean tide. The complex constant $K^S$ is also introduced here to impose the conservation of mass on the self-consistent equilibrium ocean pole tide. $\vec{E}^S(\theta, \lambda)$ is defined as a normalized equilibrium admittance function for the self-consistent equilibrium ocean pole tide. The second term in the square brackets of $\vec{E}^S(\theta, \lambda)$ describes the effects of tidal loading and self-gravitation of the ocean pole tide, where $\gamma'_n$ is load Love number of degree $n$ and $\alpha_n$ is the factor $(3/(2n+1)) \left( \rho_w/\rho_e \right)$, $\rho_w$ and $\rho_e$ define the mean densities of the ocean water and the Earth, respectively, with $\left( \rho_w/\rho_e \right) \approx 0.19$ [Munk and Macdonald 1960]. The function $\vec{E}^S(\theta, \lambda)$ can be expanded into spherical harmonic components defined by coefficients $(a_{nm} + i \cdot b_{nm})$. We obtained the spherical harmonic coefficients from Desai by personal communication. And the coefficients given by Desai are expanded to degree.
\( n = 360 \), which implies a spatial resolution of 0.5 degrees in latitudes and longitude.

\[
\hat{E}^S(\theta, \lambda) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \hat{P}_{nm}(\cos \theta)(a_{nm} + ib_{nm})e^{im\lambda}
\]  

\( 9 \)

5. **Gravity loading effect of the equilibrium ocean pole tide**

The gravity loading effect of the equilibrium ocean tide \( L(\theta, \lambda, t) \) at a given time \( t \) can be calculated by a convolution [Agnew 1997; Zhou J. C. 2005]

\[
L(\theta, \lambda, t) = \rho_n a^2 \int \int_{S_0} H(\theta', \lambda', t) G(\psi) \sin \psi d\psi dA
\]  

\( 10 \)

where \( H(\theta', \lambda', t) \) is the height of the ocean tide at the point of the colatitude \( \theta' \) and the longitude \( \lambda' \) at the corresponding time. And \( G(\psi) \) is the Green’s function; \( A \) is the azimuth from the selected point \( (\theta, \lambda) \) and the loading point \( (\theta', \lambda') \); \( \psi \in [0, \pi] \) is the angular distance between the selected point \( (\theta, \lambda) \) and the loading point \( (\theta', \lambda') \), given by the formula.

\[
\cos \psi = \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos (\lambda - \lambda')
\]  

\( 11 \)

As an example, we calculate the gravity loading effects of the classical and the self-consistent ocean pole tide at station Brussels. Figure 1 shows the comparison with the theoretical gravity pole tide before (a) and after (b) introducing a well-chosen phase lag. From Figure 1 (a) we can see that the amplitude of the gravity loading effect of the self-consistent ocean pole tide is larger than that of the classical ocean pole tide, the amplitude of both of them being larger than \( 10^{-9}\) m.s\(^{-2}\). There is a slight but noticeable phase shift between ocean pole tide loading and the theoretical gravity pole tide in Brussels. From Figure 1 (b) we can see that the difference of the amplitude between the loading effect of the self-consistent ocean pole tide and that of the classical ocean pole tide decreases slightly; when the phase shift is corrected.

![Figure 1](image-url)

**Figure 1** Comparison of the theoretical gravity pole tide and the gravity loading effects of the ocean pole tide before (a) and after (b) introducing a well-chosen phase lag. The bold line is the theoretical gravity pole tide; the fine line is the gravity loading effect of the classical equilibrium ocean pole tide; the dashed line is the same effect of the self-consistent equilibrium ocean pole tide

The gravity loading vector takes the complex forms \( \delta_L = \delta_L e^{i\Delta \varphi_L} \), where \( \delta_L \) is the amplitude and \( \Delta \varphi_L \) refers to the phase difference. Lags correspond to negative values. The estimation of the gravity loading vector is...
based on the assumption that the theoretical gravity pole tide \( \Delta g(\theta, \lambda, t) \) and the gravity loading effect of the ocean tide \( L(\theta, \lambda, t) \) at a given point \((\theta_0, \lambda_0)\) has the relationship shown in the following equation.

\[
L(\theta_0, \lambda_0, t) = \delta L \cdot \Delta g(\theta_0, \lambda_0, t) = \delta L \cdot e^{j\lambda_0} \cdot \Delta g(\theta_0, \lambda_0, t)
\]

(12)

The assumption is reasonable, as it can be seen from Figure 2 and Figure 3. Figure 2 shows the phasor plot of the gravity loading effect of the classical equilibrium ocean pole tide and the theoretical gravity tide of station Brussels, before and after introducing the phase difference. Figure 3 corresponds to Figure 2 for the self-consistent equilibrium ocean pole tide. From Figure 2 and Figure 3, we can see that phasor plot of the gravity loading effects with respect to the theoretical gravity looks like an ellipse for both the classical and the self-consistent ocean pole tide. After introducing a well-chosen phase difference, it can be completely fitted by a straight line. Therefore, we can conclude that the assumption in equation (8) is correct. If the linear fit takes the form \( Y = AX + B \), the coefficient \( A \) of the linear fit is the amplitude factor of the loading gravimetric vector.

The well-chosen phase difference is the phase of the loading gravimetric vector.

Figure 2: Linear fit of the gravity loading effect of the classical equilibrium ocean pole tide and the theoretical gravity tide of station Brussels, before (a) and after (b) introducing a well-chosen phase lag. The white straight line in (b) is the fit line.

Figure 3: Linear fit of the gravity loading effect of the self-consistent equilibrium ocean pole tide and the theoretical gravity tide of station Brussels, before (a) and after (b) introducing a well-chosen phase lag. The white straight line in (b) is the fit line.

The loading gravimetric vectors of all the other GGP stations are obtained with the same method. The numerical result for 9 stations is given in the Table 1. From table 1, we can see that the largest amplitude of
loading gravimetric vectors is the amplitude of Canberra. The amplitude factor value is 0.0363 for the classical model and 0.0429 value for the self-consistent one. The reason we considered is that the station Australia is surrounded by the oceans. The smallest amplitude of loading gravimetric vectors is found at Boulder, The amplitude factor value is 0.0137 for the classical model and 0.0152 value for the self-consistent one. The reason is that the station Boulder lies in the inner part of the North America mainland and is far away from the ocean. The mean value for all the stations is 0.0224 for the classical model and 0.0247 for the self-consistent one. Compared with the theoretical value of the gravimetric factor $\delta = 1.158$ for a non-hydrostatic and anelastic Earth model given by Dehant in 1999, these two mean values correspond to 1.9% and 2.1%, respectively [Dehant et al., 1999]. Therefore, the influence of the ocean pole tide cannot be omitted when we try to determine the gravimetric factor $\delta$ with gravity observations. The largest phase difference is found at station Vienna, an advance of 12.10 days for the classical ocean pole tide and of 15.55 days for the self-consistent one. The smallest phase difference is the phase advance of Boulder too, 4.32 days for the classical ocean pole tide and 4.75 days for the self-consistent one. Phase lags are observed in Canberra and Cantley. However it should be noted that, as the load vectors represents 2% of the direct gravity pole tide, a $10^\circ$ phase shift will only produce a $0.2^\circ$ phase difference on the observed gravity pole tide. Considering the results of Ducarme et al. [2006] it is, for the while at least, below the limit of detection. Compared to the preliminary evaluations given in Ducarme et al. [2006] there is an increase of the tidal loading of the order of 30%. The mean corrected tidal factor should become $\delta = 1.156$ for the classical ocean pole tide model and $\delta = 1.154$ with the self-consistent one. However the RMS error on the mean tidal gravity amplitude factor is still at the 0.5% level so that both values agree with the value $\delta = 1.158$ of the DDW99 model [Dehant et al., 1999].

Table 1: Loading gravimetric vectors of 9 GGP stations

<table>
<thead>
<tr>
<th>Station</th>
<th>Loading gravimetric vector (classical)</th>
<th>Loading gravimetric vector (self-consistent)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amplitude factor $\delta_L$</td>
<td>Phase diff. at 435day ($^\circ$)</td>
</tr>
<tr>
<td>Boulder</td>
<td>0.0137</td>
<td>3.60</td>
</tr>
<tr>
<td>Brussels</td>
<td>0.0264</td>
<td>7.20</td>
</tr>
<tr>
<td>Canberra</td>
<td>0.0363</td>
<td>-3.24</td>
</tr>
<tr>
<td>Cantley</td>
<td>0.0210</td>
<td>-8.46</td>
</tr>
<tr>
<td>Membach</td>
<td>0.0242</td>
<td>8.10</td>
</tr>
<tr>
<td>Moxa</td>
<td>0.0201</td>
<td>9.90</td>
</tr>
<tr>
<td>Potsdam</td>
<td>0.0210</td>
<td>9.54</td>
</tr>
<tr>
<td>Strasbourg</td>
<td>0.0214</td>
<td>8.64</td>
</tr>
<tr>
<td>Vienna</td>
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<td>10.08</td>
</tr>
<tr>
<td>Mean value</td>
<td>0.0224</td>
<td>---</td>
</tr>
</tbody>
</table>

6. Conclusions

The amplitude of the loading gravity effect of the ocean pole tide is of the level of $10^{-9}$ m s$^{-2}$, and its phase difference with respect of the gravity pole tide ranges between -11 and +16 days. This loading effect can increase the amplitude factor of gravity the pole tide by more than two percent, but its effect on the phase remains negligible. The self-consistent model predicts effects 10% larger than the classical one.

Reference