A search on the gravity / height ratio induced by surface loading; theoretical investigation and numerical applications

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Introduction

Ground gravity measurements are very often collocated with measurements of surface vertical displacements. These two observables are due to deformation processes acting on different spatio-temporal scales. Combining them through a ratio may provide useful information about the physics of the acting phenomena and help to separate various contributions. A well-known theoretical value of such a ratio is the free-air gradient of -0.3086 µGal/mm used in the free-air reduction. However, because of the redistribution of the masses inside the Earth and the Newtonian attraction, this ratio for surface loading is different from the free-air gradient. For example, Wahr et al. (1995) showed that the viscous vertical displacement could be linked to the corresponding free-air gravity anomaly via a coefficient of 6.5 mm/µGal or, equivalently, that the secular variation of the total gravity could be linked to the secular vertical displacement through a ratio of -0.15 µGal/mm. This value should be constant whatever the viscosity profile inside the compressible Earth, the ice geometry and time history, as shown theoretically and numerically. These authors proposed to use this ratio to separate the effects of Present-Day Ice Melting (PDIM) from those of Post Glacial Rebound (PGR).
In this study, we aim at probing the characteristics of gravity versus vertical displacement (gravity/height) ratio in the case of an elastic loading process. We first investigate theoretically the ratio to better understand the sensitivity of its different components to the spatial wavelengths of the source (part 1). We develop a numerical approach for computing this ratio and apply it to the predictions of the hydrological loading using a global hydrological model (part 2).

1 Theoretical study of the gravity/height ratio

The theory of the loading process is considered by Farrell (1972) and Merriam (1980) for oceanic loading, and by Spratt (1982) for atmospheric loading.
The variation T of a geodetic observable at the location (θ, λ) induced by a variation of a 2D mass distribution $\sigma(\theta, \lambda)$ at the Earth's surface can be written as:

$$T(\theta,\lambda)=\sum_{n=0}^{+\infty}\tau_n\sum_{m=0}^{n}P_n^m(\cos\theta)\left(\sigma_n^{m,c}\cos(m\lambda)+\sigma_n^{m,s}\sin(m\lambda)\right)=\sum_{n=0}^{+\infty}\tau_n\,\sigma_n(\theta,\lambda)$$

where τ_n is the transfer function of degree n for the observable T. In our study T stands for gravity variation or vertical displacement.

The transfer function depends on the elastic properties of the Earth via the load Love numbers. The Earth model that is used is the Spherically Symmetric, Non-Rotating Elastic and Isotropic (SNREI) compressible Earth model PREM without oceans.

1.1 Gravity and vertical displacement induced by a 2D mass distribution at the Earth's surface

We now examine the degree n term of the transfer function in gravity (g_n) and vertical displacement ($u_{r,n}$).

1.1.1 Gravity variation

By convention, gravity is positive downwards. The gravity variation can be decomposed into a Newtonian and an elastic part.

The Newtonian part of the transfer function can be written as (Merriam, 1980) (ρ_T is the mean density of the Earth):

$$g_{N,n} = -\frac{3\,g_0}{R_T\,\rho_T}\frac{n}{2n+1} \tag{1}$$

or

$$g_{N,n} = -\frac{3\,g_0}{R_T\,\rho_T}\frac{-(n+1)}{2n+1} \tag{2}$$

depending on the location of the very local masses (resp. above (eqn. 1) and below the measurement point (eqn. 2)). Although there is an impact of the masses at global scale, gravity is dominated by the effect of these masses via the Dirac term in the Green function (Merriam, 1980). The limit when n tends to infinity is resp. $-2\pi G$ (masses above) and $+2\pi G$ (masses below) corresponding to the gravity effect of infinite plate of density σ ($\pm$ 0.042 μGal/(kg/m^2)).

The elastic part of the transfer function can be written as (Merriam, 1980):

$$g_{E,n} = -\frac{3\,g_0}{R_T\,\rho_T}\frac{2\,h'_n-(n+1)k'_n}{2n+1} \tag{3}$$

The two components in this expression are:

- the effect of free-air motion (h'_n term)
- the effect of the mass redistribution inside the Earth (k'_n term)

These two components are opposite in sign. Their sum is positive and tends to zero when n tends to infinity. It decreases (resp. increases) the sensitivity to low degrees if the local masses are below (resp. above) the surface.

Hence ground gravity is sensitive to the entire spectrum of the source (through the Newtonian term).

1.1.2 Vertical displacement

By convention, vertical displacement is positive upwards. The reference frame is centered at the center of mass of the system {Earth + load}.

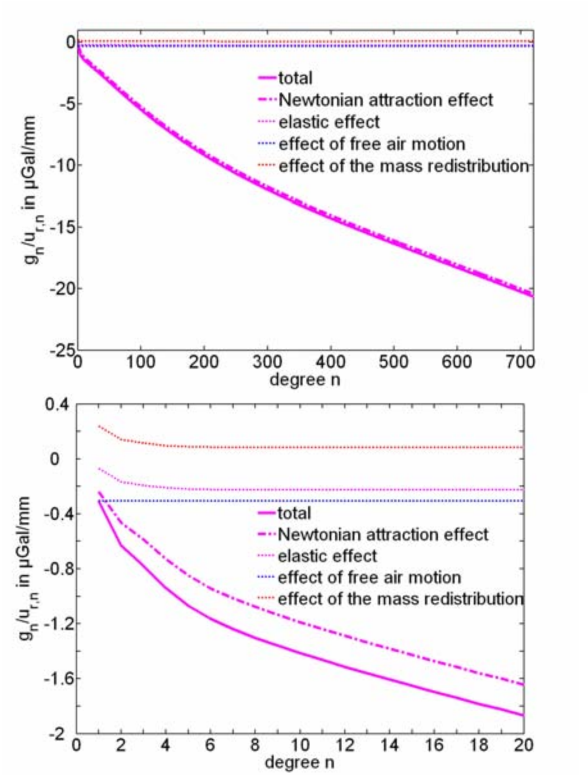
The degree n term of the transfer function can be written as (Farrell, 1972):

$$u_{r,n} = \frac{3}{\rho_T}\frac{1}{2n+1}h'_n \tag{4}$$

This transfer function is a decreasing function of the degree n and tends to zero when n tends to infinity. Hence vertical displacement is mainly sensitive to the lowest degrees (inferior to 20) of the source and behaves as a low-pass filter. Thus a load of small spatial extension will induce almost no vertical displacement.

1.2 Ratio of the degree n terms of the transfer functions or Green functions

A way to compare the different sensitivities of the transfer functions for gravity and displacement (or Green functions) is to compute the ratio of their degree n terms. Because the ratio is not a linear function of the source, it is not the transfer function for the gravity/height ratio. So the ratio of the degree n terms of the transfer functions has not to be interpreted as the degree n term of the gravity/height ratio.



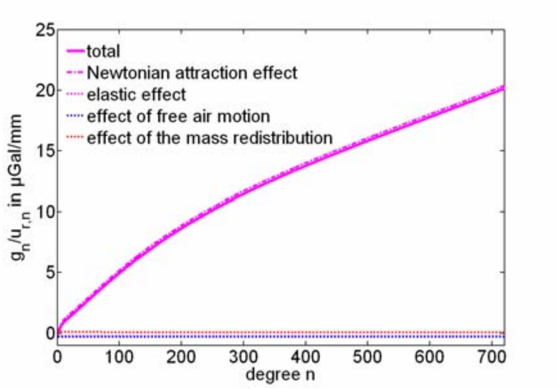


Figure 1. Ratio g_n/u_{rn} of the degree n terms of the transfer functions in gravity and vertical displacement. Left (top): local masses below the surface; zoom on the first 20 degrees (left bottom). Right (top): local masses above.

As shown on Figure 1, the ratio is dominated by the Newtonian attraction term due to the local masses (magenta full line) which determines its sign (depending on the location of the very local masses). It corresponds to the ratio of equations (1) or (2) to equation (4). Since vertical displacement tends to zero when n tends to infinity, the ratio diverges rapidly.
This constitutes a strong limitation in the interpretation of the gravity/height ratio. Because of the location of the gravimeters with respect to the ground, the sign of the gravity variations due to hydrology can be opposite as well as the sign of the ratio.
It is interesting to focus on the ratio of the elastic part of gravity to vertical displacement (magenta dotted curves) because of their similar behavior as a function of the degree of the source. It can be written as follows (equation (3) divided by equation (4)):

$$\frac{g_{E,n}}{u_{r,n}} = -\frac{g_0}{R_T}\frac{2h'_n - (n+1)k'_n}{h'_n} = -\frac{2g_0}{R_T} + \frac{g_0}{R_T}\frac{(n+1)k'_n}{h'_n} \qquad (5)$$

The first term (blue dotted line) is the famous free-air gradient whose value for a spherical Earth is -0.3086 µGal/mm. Since it is not dependent on the degree n, this value can be used in the spatial domain to correct gravity variations from the free-air motion effect.
The second term (red dotted line) is due to the mass redistribution inside the Earth which partly compensates the deformation of the surface. It tends rapidly to a limit when n tends to infinity.
The limit of both terms (computed at degree n=9000) is equal to -0.2320 µGal/mm. This value may explain smaller absolute values of the gravity/height ratio far away from the loads, as the local Newtonian effect is then equal to zero.

1.3 Generalization to any load

Real loads are not constituted by one single degree but by a long series of degrees in order to reproduce their complex spatial geometry such as the hydrological basin limits or the ocean-continent limit.
At the location (θ, λ), the gravity/height ratio is then the ratio of two combinations of degrees, characterized by two different transfer functions:

$$\frac{g(\theta,\lambda)}{u_r(\theta,\lambda)} = \frac{\sum_{n=0}^{+\infty} g_n \, \sigma_n(\theta,\lambda)}{\sum_{n=0}^{+\infty} u_{r,n} \, \sigma_n(\theta,\lambda)}$$

We consider now a load whose spatial distribution changes with time:

$$\sigma(\theta,\lambda,t) = \sum_{n=0}^{+\infty} \sigma_n(\theta,\lambda,t)$$

Since generally the geometry of the load is not the same according to time, we cannot separate time and space dependencies and the gravity/height is time-dependent (however in the case of tidal waves of fixed frequency, it is possible to make such a separation by considering the ratio as a complex number with an amplitude in µGal/mm and a phase).

2 Numerical study of the gravity/height ratio

As shown above, the ratio gravity/height is a complex function of space and time. It may diverge at some time t or location (θ, λ) because of a vanishing vertical displacement.

2.1 Methodology

At a point (θ, λ), we first compute a gravity/height ratio for a given period of time D by a linear regression using the least squares method (Figure 2). Time is thus removed. This method avoids taking into account abnormally large values of the ratio as it would be the case by computing the arithmetical mean. However, a shortcoming of this method is that it fails to give the right ratio if gravity and vertical displacement are not in phase (case of tidal waves).

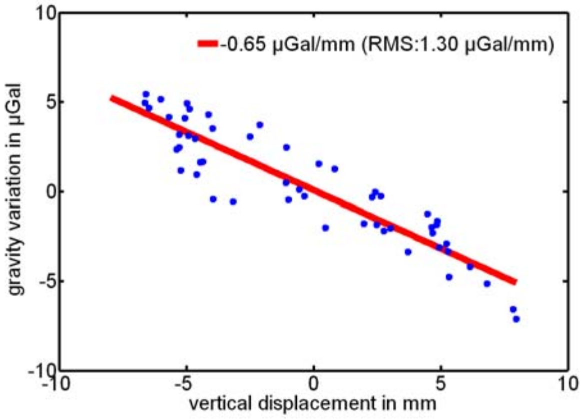


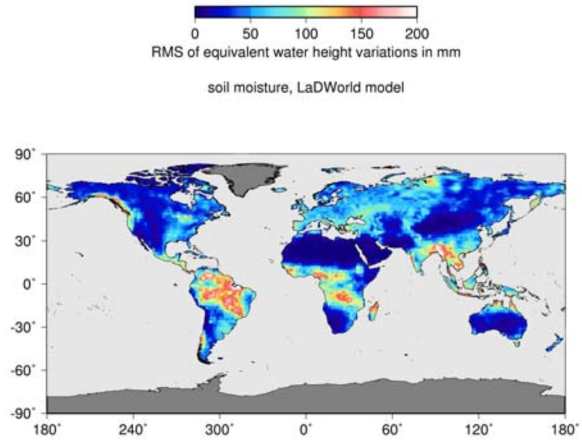
Figure 2. Linear regression between vertical displacement and gravity variation in Strasbourg predicted by the global hydrological LaD model (Milly & Shmakin, 2002). Each blue dot represents a monthly time sample.

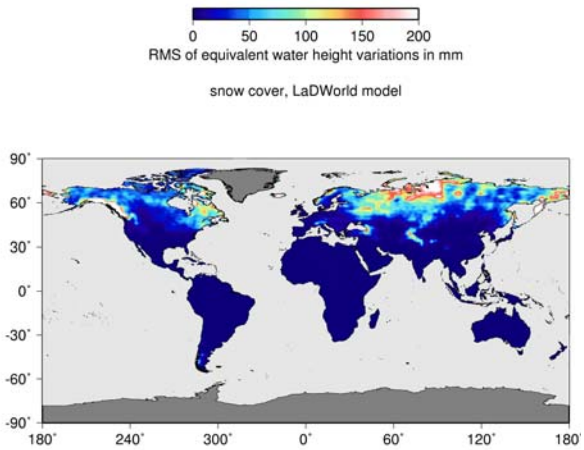
A second step is the computation of the spatial mean of the ratios calculated as explained above on a given domain (continents, hydrological basins).
We could also make a global linear regression removing both time and space. So we compared the resulting ratio to the one obtained by the former method and found very close values. However, the spatial integration of such a ratio is not meaningful on a global scale but rather on smaller domains we will identify below.

2.2 Application to a global hydrological model

We use the global LaD model from Milly & Shmakin (2002) given with a temporal sampling of one month on a 1° square grid. The investigated time period ranges from January 2000 to April 2004. We use the predictions of snow cover and soil moisture (as sum of the water in the root zone and ground water). The Root Mean Square (RMS) of the temporal variations for each quantity and their sum are mapped in Figure 3.
In glaciated areas such as Antarctica and Greenland, the model fails to give realistic values of snow cover (too much accumulation of snow) so we removed their contribution to global hydrological loading.

Areas with a strong signal are located in the equatorial zone where strong precipitations occur especially during the monsoon. The corresponding hydrological basins are Orinoco, Amazon and Tocantins basins in South America, Niger, Chad and Congo basins in Africa and Brahmaputra and Mekong basins in South East Asia.
It should be reminded that there is a phase-lag of 6 months because of the meteorological equator. Thus the Orinoco and Amazon basins are not in phase and so are the Chad and Congo basins. As a consequence, the effective loads are smaller than those showed on Figure 3.
In Europe, the signal is not so strong than in the equatorial area but is coherent on a large scale. It is due only to the variations of water contained in the soil.
In Russia and particularly in Siberia, the predicted variations of snow cover dominate the total signal which is stronger than the one predicted in Amazon.
In the northern part of the country, the model predicts a strong signal related to high precipitations whereas almost no signal is predicted for the rest of the country.





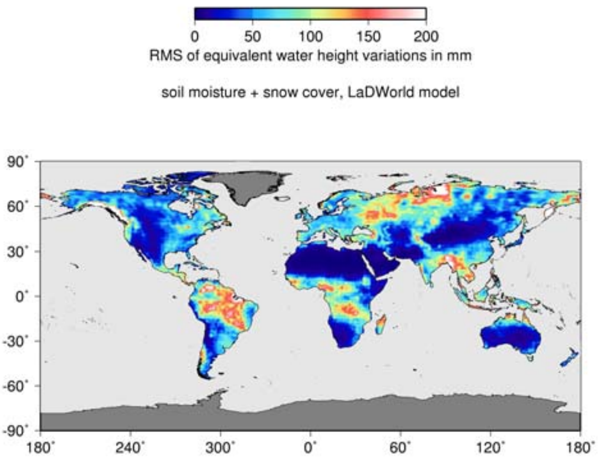


Figure 3. RMS of the monthly variations of a hydrological load predicted by the LaD model (Milly & Shmakin, 2002) from January 2000 to April 2004 for different contributions (in mm of equivalent water height). Above, left: soil moisture. Above, right: snow cover. Below: sum of soil moisture and snow cover.

The spectral energy of the variations of the total hydrological load (Figure 4) is maximum around degree 5 corresponding to a spatial half wavelength of 4000 km. However the ocean-continent limit introduces other degrees and the load cannot be assumed as a pure degree 5. Moreover the distribution of energy as a function of degree is changing with time.
The two peaks per year (around March and September) correspond to the annual cycle which is out of phase of 6 months between the Northern and Southern hemispheres.

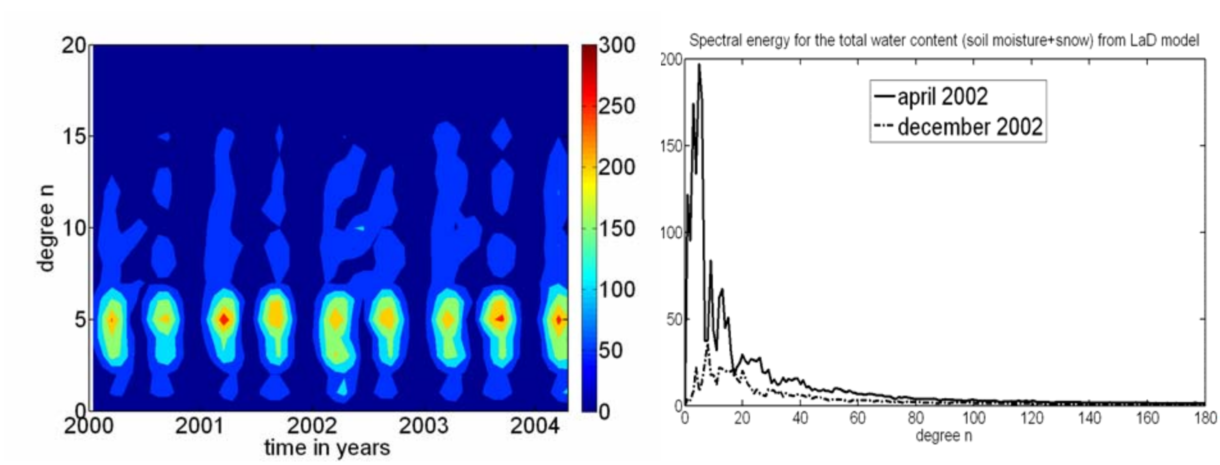


Figure 4. Spectral energy (in mm²) of the predicted total water content (soil humidity and snow cover) by LaD model versus degree and time (left) and versus degree at two different months (right).

2.3 Results

For the computation of gravity variations induced by these water content variations, we assume that the local masses are below the measurement point by using equation (2) for the Newtonian attraction effect. The 1° square grid has been interpolated to 0.5°. We truncated the development of equations (2) to (4) at degree 360.
The gravity/height values computed for each (θ, λ) point as explained in part 2.1 are mapped on Figure 5.

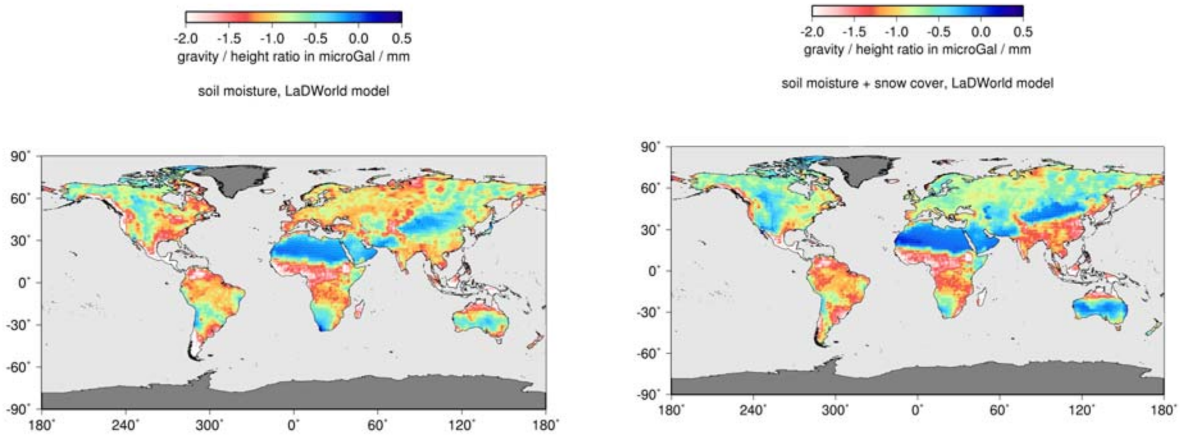


Figure 5. Gravity/height ratio in µGal/mm for the global hydrology model LaD. Left: effect of soil moisture. Right: effect of total water content (soil moisture and snow).

2.3.1 Effect of snow

We computed separately the gravity/height ratio for snow cover and soil moisture effects. Since the predicted snow cover variations present a strong degree 1, it will have an impact on the ratio even far away from the sources as shown on Figure 5.
For example, the mean ratio computed in Europe (area between -10°E and 35°E in longitude and between 35°N and 75°N in latitude) is -1.03 µGal/mm if we consider only the soil moisture variations and increases to -0.71 µGal/mm if the snow cover variations are considered besides those of soil moisture.
This can be explained in two steps. On one hand, gravity is almost not affected because there is no (or negligible) snow in Europe so that the local Newtonian attraction effect is almost zero. On the other hand, snow cover in Siberia causes a flexure of the crust and therefore a vertical displacement which adds to the vertical displacement caused by soil moisture in Europe.

2.3.2 Spatial variations of the gravity/height ratio

The mean value of the ratio induced by soil moisture and snow cover variations is -0.86 µGal/mm but the RMS on the continents is 0.57 µGal/mm. The ratio ranges from -6.4 µGal/mm to 1.8 µGal/mm and only 0.6% of the computed values are inferior to -2.0 µGal/mm indicating that the mean ratio is not affected by too high values.

2.3.2.1 Comparison of the means on different hydrological basins

If we compare Figure 5 with Figure 3, we cannot find everywhere a correlation between a high signal and a high (in absolute value) gravity/height ratio. It is due to temporal phase lags between basins that are not shown in Figure 3. For example, as already mentioned in part 2.2, there is a phase lag of 6 months between the Orinoco and the other basins of South America. The latter add their effects to the loading of the area inducing a significant vertical displacement. But the Orinoco basin being smaller, the induced vertical displacement is not so important. Then, gravity variations being comparable in both areas, the gravity/height ratio is higher in Orinoco basin (-1.38 μGal/mm) than in the other basins of South America (-1.11μGal/mm).

In Northern Australia (latitudes below 20°S), the value is -1.84 μGal/mm which is twice the mean value. This is due to the small spatial extension of the source which results in a small vertical displacement and a big gravity change. It is also the case in every island where there is a significant hydrological signal and on the Western coast of Canada where the signal is concentrated on a narrow area.

In Siberia, however, the mean value is -0.82 μGal/mm although the predicted signal is one of the highest in the world.

So the gravity/height ratio does not automatically depend on the amplitude of the load, but it is closely influenced by its spatial extension. Thus, the more concentrated the source is, the smaller the vertical displacement and the higher the ratio.

2.3.2.2 Case of desert areas (outside the loads)

We are now considering the impact of soil moisture only (Figure 5, left).

We find a mean value of -0.28 μGal/mm in the area formed by Sahara and Arabia (latitudes between 15°N and 30°N).

Close values can also be found in Central Asia (Mongolia), and Central Australia.

In these areas, as shown in Figure 3, there is no hydrological signal. However, gravity variations and vertical displacements are not zero. In gravity, the Newtonian attraction part is indeed zero and only the elastic part and the global Newtonian attraction effect are non zero. The vertical displacement is not zero because of the bending of the crust induced by the strong hydrological signal in the African equatorial zone.

We do not find -0.23 (limit value of equation (5), part 1.2) because of the small global Newtonian attraction effect which enhances the effect in gravity.

Conclusion

The gravity/height ratio is strongly dominated by the effect in gravity of the Newtonian attraction of the local masses (Dirac effect) as its sign depends on the position of the local masses. The elastic effect due to mass redistribution inside the Earth and free-air motion (via the -0.3086 μGal/mm value) is much smaller and is not so sensitive to the degree n of the source. It leads to a ratio more or less close to -0.23 μGal/mm.

Numerical application for a predicted global hydrological load gives a mean ratio of -0.86 μGal/mm. This value was found by first using a linear regression through time at any continental point and then computing a spatial mean over the continents. However, the different mean values found on several hydrological basins are mainly controlled by the basin size; the more concentrated the source is, the smaller the vertical displacement and the higher the gravity/height ratio.

On the contrary, in desert areas, the ratio is dominated by the elastic processes and also by the Newtonian attraction of the surrounding masses which leads to mean values close to the elastic limit of -0.23 μGal/mm.
This study focused on the hydrological loading. However, many other elastic loading processes (due to oceans, atmosphere, ice) are present in gravity and vertical displacement measurements. Because of their different spectral content, the gravity/height ratio will be different from those presented here. Moreover values found in this study may be different from those deduced from field measurements because of the bad modeling of the very local effects not taken into account in the global loading models.

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