Rigorous Combination of Superconducting and Absolute Gravity Measurements with Respect to Instrumental Properties

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Abstract

Precise monitoring and a conclusive interpretation of temporal gravity variations at a given station is based upon accurate knowledge about the properties of the used instruments. The combination of concurrent sets of superconducting and absolute gravity measurements allow both. Whereas the absolute gravimeters provide the scale and reference level, the superconducting gravimeters enable to investigate gravity variations and to exploit the sources of the changes. The method proposed here permits to derive the scale function and zero drift of the superconducting gravimeter as well as a reliable survey of the instrumental stability of different absolute meters with high precision without the need of gravity reductions.

keywords: absolute gravity, calibration, superconducting gravimeter, site of regional comparison.

Motivation

For a conclusive interpretation of temporal variations in gravity obtained from superconducting gravimeters as well as to establish the gravity standard with absolute gravimeters, the instrumental properties of both meter types are of fundamental importance. While the sensor of the superconducting gravimeters (SG) shows a low but very stable drift rate, which must be precisely estimated to expose long-term and secular changes in gravity, absolute gravimeters (AG) are based on direct realization of physical standards. Thus drift can be excluded and calibration is not needed, but regular supervision of the instruments is necessary and – especially after maintenance – checks for possible offsets are essential. In contrast, the continuous time series of highest precision obtained from the SG has due to the relative nature of the measuring principle no absolute scale. This makes calibration of the sensor unavoidable.

Taking advantage of the different characters of both gravimeter types, a combination of their measurements should allow to resolve their instrumental properties and, in a later step, the determination of long-term gravity changes. The large amount of parallel gravity observations with different absolute gravimeters at SG stations available at BKG permits such an investigation. While a comparison at relatively short periods with the only objective to derive a constant scale factor for the SG is a common approach, see e.g. [Hinderer et al., 1991, Francis et al., 1998, Falk et al., 2001, Amalvict et al., 2002, Harnisch et al., 2002, Imanishi et al., 2002], a more complex combination of the data of several epochs can be carried out, in order to simultaneously estimate drift, check for offsets and determine the scale function for the SG as well.

Such an approach should be suitable to overcome the problem of different, possibly inconsistent reductions for the time varying gravity field. Instead of calculating a reduced mean from all drop values of an epoch of absolute gravity measurements, it should be alternatively possible to proceed with every single drop instead and average their values implicitly. To verify such a basic approach, first attempts based on synthetic data as well as data from the Station Bad Homburg were carried out.

Functional relationships

The instrumental properties of the SG under consideration are functions of scale[1] and drift[2]. The relation between the voltage equivalent to the current through the feedback, U_{SG} and gravity g_{SG} can be written in the general form

$$g_{SG} = g_0 + f_E(t)\left[f_D(t) + U_{SG} + \sum \Delta U\right], \tag{1}$$

where f_E and f_D are arbitrary functions of time for scale and drift. The steps ΔU occurring after disturbances must be determined separately during preprocessing and are not subject of this study. To leave open the possibility to extent the approach, simple polynomial representations were chosen for scale and drift:

$$f_E(t) = \sum_0^{n_E} e_i t^i, \quad f_D(t) = \sum_0^{n_D} d_i t^i.$$

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[1] mostly considered constant and called calibration factor

[2] change of the systems zero point with time

Most likely, the scale can be assumed to be a constant function ($n_E = 0$) and the drift should be linear[3] ($n_D = 1$). This results in a linear relationship between gravity and observations. If the observations can not be fitted adequately, a time dependence of higher order should be introduced.

For AG, the observations are the direct measurements of time t and distance s from which the gravity values $\bar{g}_{AG}$ are derived by a procedure described in [Niebauer et al., 1995]. Each single drop experiment results in a gravity value which is used further instead of the original observations. Outliers in the drop-data are assumed to be detected by suitable criteria and removed during preprocessing. Despite their absolute characteristics, offsets between different instruments are possible and stronger changes caused by maintenance of equipment, mechanical wear or misalignment may result in an offset with respect to previous observations or other instruments. The observation equations can be written as

$$g_{AG} = \bar{g}_{AG}\left(s, t\right) + O_j, \tag{2}$$

with possible instrumental offsets

$$O_j = \begin{cases} c_j \quad \bigvee \quad t \in [t_j, t_{j+1}] \\ 0 \end{cases},$$

which are expected to be constant during predefined intervals $[t_j, t_{j+1}]$ (e.g. maintenance-intervals).

One important problem of the combination of AG and SG measurements are non-uniform gravity reductions used in practice to remove the time dependent parts of the gravity signal. Besides different reduction models and parameters for Earth tides, atmospheric effects, loading effects etc., even different software and processing schemes may lead to different residual gravity values. Especially with respect to the determination of drift and offset parameters with highest precision, primary observations instead of incomparable residuals will be used. This is possible because both sensors are collocated and are sensitive to the same (time dependent) gravity signal. The resulting gravity value of each single drop experiment from the AG is associated with a value from SG registration by means of a simple linear interpolation[4]. This results in a huge amount of observations with a large scatter, but as it will be shown, scale and drift as well as offset parameters are separable.

The observation equations for the combination are achieved by treating only the absolute gravity values as the measured quantity, whereas the SG data are considered as error-free parameters. This can be motivated by the distinct precision levels: A single drop has a standard deviation of about $100 nms^{-2}$, while the SG measurements have a sensitivity of $0.01 nms^{-2}$. From equations (1) and (2) follows

$$g_{AG} + v = g_0 + \sum_0^{n_E} e_i t^i \left[\sum_0^{n_D} d_i t^i + \tilde{U}_{SG} \right] + O_j, \tag{3}$$

where $\tilde{U}_{SG}$ are the step-free SG data. Based on this functional model, a least squares adjustment ($\sum v^2 \rightarrow Min$) of the parameters d_i, e_i and O_j can be carried out. Because the AG measurements do not have a uniform accuracy level, a suitable weighting scheme depending on standard deviation and scatter between drops respectively groups of drops (so called sets) should be used to attenuate less reliable observations.

The vast number of observations should not suggest a highly over-determined equation system. On the contrary, the resolution of drift and offset parameters does not depend on the quantity of single drops but on the number of independent measurement epochs. While one AG measurement epoch is typically consistent of 10 to 25 sets with 150 drops, only the scale function is retrievable from these several hundred observations[5]. For each drift parameter d_i and instrumental offset O_j one additional AG measurement epoch is necessary. Therefore, the minimum number of independent epochs is $n = 3$.

Tests with synthetic data

Because of the large scatter of the AG drop-data, the question arises, whether all unknown instrumental parameters are resolvable within a least squares procedure. To check this, synthetic data on the basis of predicted tidal signals for four distinct epochs (2001-2005), each lasting about 48h hours with a sampling-rate of $60\,s$, were generated.

The simulated observations were obtained by adding Gaussian noise with an amplitude of $2 nms^{-2}$ for SG and $100 nms^{-2}$ for AG observations. The input data are shown in figure 1 and the chosen parameters with their corresponding adjustment results are documented in table 1. It is to be seen, that within the limits of the single standard deviation, all parameters can be retrieved. An exact reproduction of the initial values cannot

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[3] initial nonlinear run-in effects are not considered here

[4] interpolation is only necessary because of different sampling rates and/or a time shift between sampling instances

[5] furthermore dependent on tidal amplitude and temporal extent of the AG measurement

Table 1: Chosen parameters and the corresponding adjustment results for synthetic data

	predefined	resultant
constant scale $e_0[nms^{-2}V^{-1}]$	750.00	750.55 ± 1.31
linear drift $d_1[nms^{-2}a^{-1}]$	50.00	50.37 ± 1.05
offset $O_1[nms^{-2}]$	-20.00	-22.70 ± 4.76

be expected since every time limited random sequence has a systematic component, which effects a change in the originally chosen parameters. Thus it is demonstrated, that the algorithm should be capable to determine the instrumental parameters from the scattered AG observations.

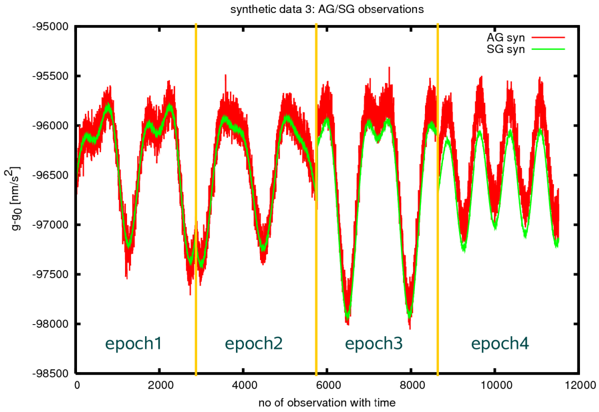

Figure 1: Synthetic data derived from tidal prediction over 4 epochs, each lasting 48 hours.

Preliminary results with observed data at Station Bad Homburg in 2005

The station Bad Homburg (BH) was developed during the last years to a station of regional comparison [Wilmes and Falk, 2006]. Beside the continuous gravity time series obtained from the dual sphere superconducting gravimeter SG30, regular absolute gravity measurements with various meters and high repetition rates were carried out. The objective is to ensure the gravity standard, monitor long-term gravity changes and to supervise diverse AG. With 18 measurement epochs during the year 2005 performed with five different instruments partially operating side by side, this station seemed ideally suited for the described comparison.

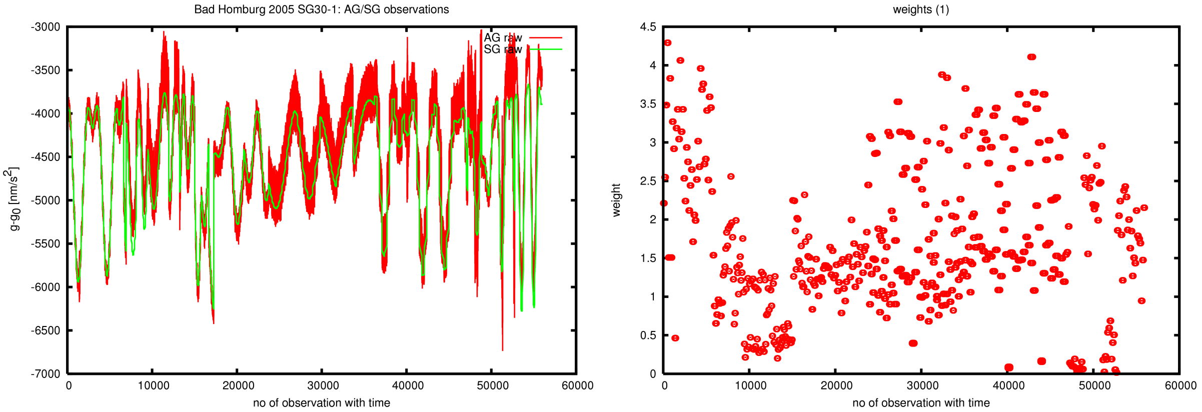

Figure 2: a) Dedicated AG and SG observations for station BH. b) weights for AG observations derived from drop scatter.

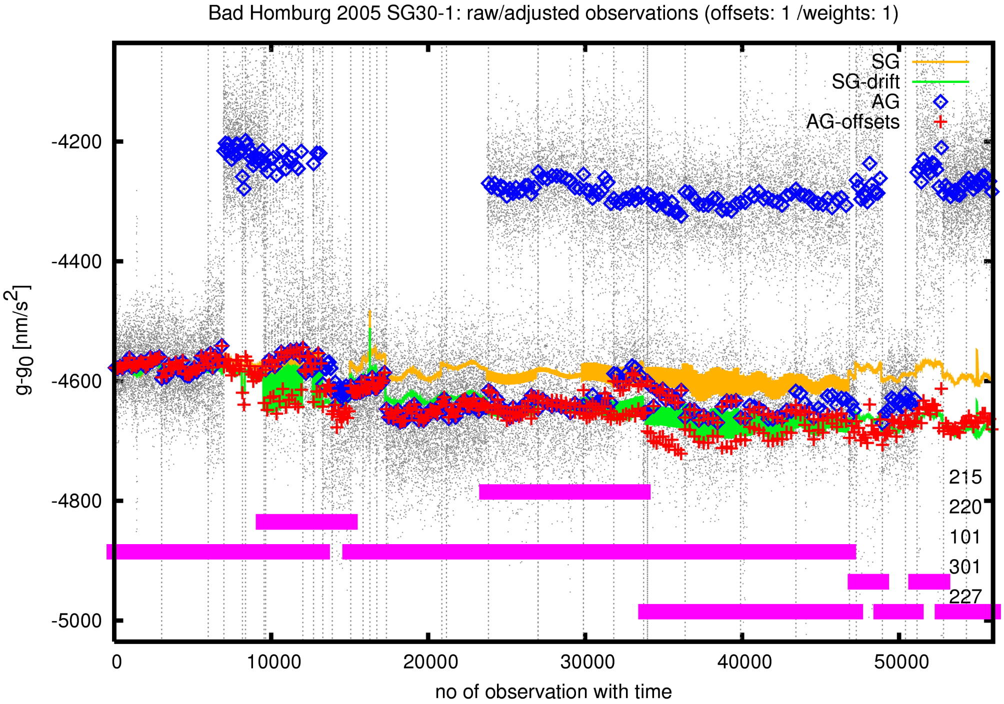

Figure 3: Observations before and after adjustment: SG: orange (grey) resp. green (light grey); AG: blue (dark grey) diamonds resp. red (grey) pluses. Single AG observations are denoted with gray dots, while markers indicate mean values of a set. During parallel operation of AG, different monuments had to be used, resulting in a constant offset in gravity. Dashed vertical lines indicate the beginning of a measurement epoch. The (pink) bars at the lower part of the plot symbolize the presence of different FG5 gravimeters annotated on the right. Values at the abscissa are the observation numbers, ordered with increasing time, not the time itself.

Preprocessing of SG data was done with the help of the well-known remove-restore approach. Tidal and atmospheric constituents are removed from the signal only for the purpose of data correction (spikes, gaps, disturbances) which were performed with the program TSoft[Van Camp and Vauterin, 2005] and restored afterwards. The manual identification and removal of steps is probably the most important part of the preprocessing since the SG values are treated error free and unrecognized steps influence drift as well as AG offsets. Because of the different accuracy levels of AG and SG measurements, step amounts can not be obtained from the combined approach. However, a raw identification of overlooked steps is still possible.

AG data from four different FG5 instruments (FG5-101, FG5-227, FG5-301 from BKG, FG5-215 from Geodetic Observatory Pecny, Czech Republic and FG5-220 from University Hannover, Germany) were reprocessed with same reductions, preferences and software versions to ensure homogeneous datasets and consistent outlier-criteria. The elimination of outliers was done on the basis of the $3-\sigma$ criterion with respect to the mean over a set[6]. Beside statistical also different deterministic outlier criteria based on instrumental properties were investigated. The dedicated observations and the weights derived from drop scatter are shown in figure 2.

In case of parallel operation of AG, different monuments had to be used, causing a constant offset in gravity. In this case, the functional model from equation (3) has to be extended by an additional offset parameter M_k, describing the gravity difference between distinct points k.

Table 2: *Preliminary* instrumental parameters for double sphere gravimeter SG30 from combination with different AG for year 2005.

	SG30-1 (lower sphere)	SG30-2 (upper sphere)
constant Scale $e_0[nms^{-2}V^{-1}]$	-741.23 ± 0.31	-682.06 ± 0.29
linear drift $d_1[nms^{-2}a^{-1}]$	-92.33 ± 1.25	-83.05 ± 1.27

The advantage of the dual sphere system SG30 lies in the possibility to check the results independently. The preprocessing of the time series of both sensors (including determination of steps) is autonomous and will affect

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[6] as a rule: 1 set = 150 drops

the adjustment results in a different manner. Further, the instrumental properties of both sensors are different, but the resulting offsets for the AG should be the same.
Although procedure and data are still under investigation, preliminary even though promising results can be given here (see figure 3). The adjustment of more than 55000 drops leads to (constant) values for the SG scale very near to those actually used. However, both values are slightly larger about 1.7 resp. 2.8 $nms^{-2}V^{-1}$. The SG drift rates for both sensors look widely similar and reasonable, the values are shown in table 2.
The residuals after adjustment show no significant deviations from a normal distribution. Merely some measurements have a distinctly larger scatter, but are symmetric as can be verified by the histogram presented in figure 4. Hence it can be infered, that no significant systematic errors nor model deficits distort the parameter estimation.

Table 3: *Preliminary* results for AG offsets from combination with SG30 for year 2005. Instrument FG5-101 held fixed.

j	serial	SG30-1		SG30-2	
		O_j $[nms^{-2}]$	rms $[nms^{-2}]$	O_j $[nms^{-2}]$	rms $[nms^{-2}]$
1	FG5-101	0.00	–	0.00	–
2	FG5-227	36.31	0.81	36.36	0.82
3	FG5-301	42.15	3.18	43.40	3.19
4	FG5-220	37.62	2.21	34.20	2.22
5	FG5-215	14.46	0.90	22.41	0.90

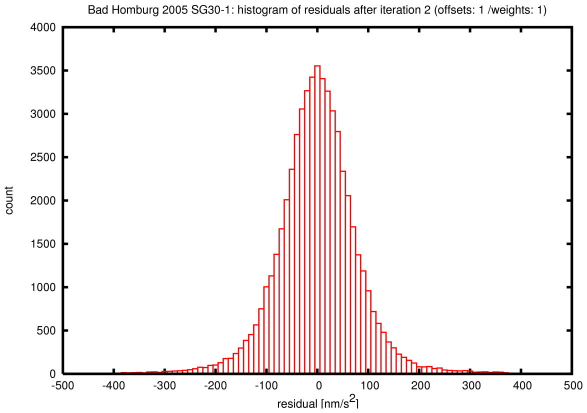

Figure 4: Histogram of the residuals after adjustment for SG30-1.

Further, the estimated gravity difference $361 \pm 1[nm/s^2]$ between the monuments AA and BA matches very well the value $356 \pm 15[nm/s^2]$ obtained from relative measurements with spring gravimeters. Therefore it can be assumed, that the obtained offsets for the AG (table 3) are expedient as well. The instrument with the longest tradition, FG5-101 held fixed[7]. All offsets are significant with respect to their standard deviations and they agree between both calculations within their bounds of error, except for instrument FG5-215, for which the difference seems to be significant with respect to the (too optimistic) error estimation. The results will soon be examined and validated with an extended dataset, since some instruments are included only with a small number of independent measurement epochs.

Conclusions

The proposed method of a strict combination based on most primary observations without any gravity reduction applied, works successfully. The simultaneous estimation gives reasonable results. For SG, a constant scale and a linear drift can be resolved well, while for AG, plausible offsets for fixed periods are obtained. Additionally, gravity differences between different points could be reliably specified. It can be concluded, that this method is an eligible basis for the analysis of temporal gravity variations and the supervision of AG.
In future investigations, correlation between drift and offsets should be examined more in detail and the combination should be extended to common subsets of AG at different SG locations. For the SG, different periods of constant parameters should be defined in order to take into account changes in the instrumental properties.

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[7] this choice is arbitrary

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References

M. Amalvict, J. Hinderer, P. Gegout, S. Rosat, and D. Crossley. On the use of AG data to calibrate SG instruments in the GGP network. *Bulletin d'Information des Marées terrestres*, 135:10621–10626, 2002. URL `http://www.astro.oma.be/ICET/bim/text/amalvict.pdf`.

R. Falk, M. Harnisch, G. Harnisch, I. Nowak, B. Richter, and P. Wolf. Calibration of the Superconducting Gravimeters SG103, C023, CD029 and CD030. *Journal of the Geodetic Society of Japan*, 47(1):22–27, 2001.

O. Francis, T. Niebauer, G. Sasagawa, F. Klopping, and J. Gschwind. Calibration of a superconducting gavimeter by comparison with an absolute gravimeter FG5 in Boulder. *Geophysical Research Letters*, 25(7):1075–1078, 1998.

M. Harnisch, G. Harnisch, and R. Falk. Improved Scale Factors of the BKG Superconducting Gravimeters, derived from Comparisons with Absolute Gravity Measurements. *Bulletin d'Information des Marées terrestres*, 135:10627–10638, 2002. URL `http://www.astro.oma.be/ICET/bim/text/harnisch.htm`.

J. Hinderer, N. Florsch, J. Makinen, H. Legros, and J. Faller. On the calibration of a superconducting gravimeter using absolute gravity measurements. *Geophysical Journal International*, 106:491–7, Aug. 1991.

Y. Imanishi, T. Higashi, and Y. Fukuda. Calibration of the superconducting gravimeter T011 by parallel observation with the absolute gravimterer FG5 #210 - a Bayesian approach. *Geophysical Journal International*, 151:867–878, 2002. URL `http://www.blackwell-synergy.com/doi/abs/10.1046/j.1365-246X.2002.01806.x`.

T. Niebauer, G. Sasagawa, J. Faller, R. Hilt, and F. Klopping. A new generation of absolute gravimeters. *Metrologia*, 32:159–180, 1995.

M. Van Camp and P. Vauterin. Tsoft: graphical and interactive software for the analysis of time series and Earth tides. *Computers & Geosciences*, 31:631–640, 2005.

H. Wilmes and R. Falk. Bad Homburg - a regional comparison site for absolute gravity meters. In O. Francis and T. van Dam, editors, *International Comparison of Absolute Gravimeters in Walferdange (Luxembourg) of November 2003*, volume 26 of *Cahiers du Centre Européen de Géodynamique et de Seismologie (EGCS)*, pages 29–30, Luxembourg, 2006.