# Pressure Admittance Function from Least Squares Product Spectrum of Surface Gravity and Pressure

Mahmoud Abd El-Gelil[1], Spiros Pagiatakis[1], and Ahmed El-Rabbany[2]
[1]York University, Toronto, Canada, Department of Earth and Space Science and Engineering
[2]Ryerson University, Toronto, Canada, Department of Civil Engineering

**Abstract**: Atmospheric pressure is one of the most significant disturbing noises to surface gravity measurements. This effect is usually removed by two different methods that are based on the availability of pressure data and their distribution in both time and space. The first and most accurate method is the convolution of the global surface pressure field with an appropriate Green's function; its downside is that it requires short sampling intervals currently unavailable. The second method determines a transfer function between pressure and gravity known as barometric pressure admittance. In this paper, we adopt an alternative approach for the determination of the admittance that is based on the least-squares (LS) product spectrum of the atmosphere pressure and gravity time series. It represents a reasonable gravity response to air pressure fluctuations in specific frequency bands (frequency dependent admittance), that is derived from a five-year data set from the Canadian Superconducting Gravimeter Installation – CSGI (Cantley, Canada). The results show improvements in the gravity spectrum, particularly in the diurnal and semi-diurnal bands and in higher frequency bands.

## 1. Introduction

Recent rapid improvements in technology have created extremely precise and accurate measuring systems that are sensitive to minute physical phenomena that were once much too small to be detected. The superconducting gravimeter is no exception; it is sensitive to changes in gravity that reach the nanogal level (1nGal = $10^{-11}$ m/s$^2$). Gravity variations are caused by many physical phenomena e.g. lunar and solar tides, Earth's rotation changes, atmospheric and ocean loading, and others (Hinderer and Crossley, 2000; van Dam and Wahr, 1998). This amazing device, with its high sensitivity and stability covers a very wide spectrum with periods from seconds to years (Crossley and Xu, 1998). It is, with no doubt, a challenge to the geophysicists to identify and/or separate minute signal(s) of interest in a specific band of interest (Sun et al, 2002).

In the last three decades, geophysicists and other researchers have been searching for very weak signals in the superconducting gravimeter records e.g., core modes and the Slichter triplet (Hinderer and Crossley, 2004; Jensen et al, 1995; Smylie and Jiang, 1993). Unfortunately, however, these weak signals can not be detected before removing or modeling other disturbances which are considered as noise. Otherwise, misleading results or wrong interpretation may arise. The atmospheric pressure is one of the most significant environmental phenomena that affect the surface gravity records. The atmospheric pressure effect on gravity is about one order of magnitude smaller than the solid Earth tides. For certain stations, this effect reaches 30μgal (Virtanen, 2004). There are basically two approaches to estimate or model the atmospheric pressure effect on the gravity records. The first one is the physical model and the second an empirical or experimental one using an admittance (transfer) function. In the physical model, the atmospheric parameters (e.g. pressure, temperature, and other) are measured on the Earth surface or at different altitudes and with very good spatial distribution. A Green's function is then used to estimate the loading and attraction effect (Boy et al, 2001; Boy et al, 1998; Kroner and

Jentzsch, 1999; Merriam, 1992a; Mukai et al, 1995; Neumeyer et al, 2004; Sun et al, 1995). The admittance approach deals with local atmospheric pressure measurements obtained in parallel with the gravity data (same time and location).

In the admittance approach, the regression analysis is usually applied to estimate a very simple transfer function between gravity and pressure. However, it is believed that this single scalar function is not adequate to remove the pressure effect because it is local in character and usually frequency and season dependent. The frequency dependant admittance that was first introduced by Warburton and Goodkind (1977) and elaborated by Crossley et al., (1995) and Neumeyer (1995) showed that it increases smoothly and monotonically from 0.2 μgal/mbar at low frequencies to about 0.35 μgal/mbar at high frequencies. However, Sun et al., (2002) found that the frequency-dependent admittance is 0.378 μgal/mbar at low frequencies and reduces to 0.147 μgal/mbar at high frequencies. Hu et al., (2005) applied the wavelet analysis to decompose the gravity and pressure signals into 14 bands. Different admittances were estimated for all the specific bands using regression between the decomposed signals. However, it is expected that a highly accurate empirical model can remove about 90 percent of the total pressure effect (Mukai et al, 1995; Spratt, 1982).

In this paper, a different procedure is adopted to estimate the pressure admittance that is based on the product spectrum from the least squares spectrum analysis. The common peaks in both gravity and atmospheric pressure are used to define and estimate the admittance. The significance level of the peaks in the product spectrum is well defined based on the probability density function derived from the LS spectrum. The common spectral peaks in both gravity and pressure series are suppressed in monthly data segments to estimate their amplitudes and phases in the band 700-2h. Finally, the weighted LS regression is used to estimate a smooth admittance as a function of frequency. The new admittance is then used to correct the gravity series (residual series) in an attempt to improve the signal to noise ratio for the purpose of obtaining better estimates of other effects or searching for weak signals.

## 3. Least Squares Spectral Analysis

We use the Least Squares Spectral Analysis (LSSA) to estimate the spectra of the gravity and atmospheric pressure series and subsequently produce their product LS spectrum. The benefits of the LSSA vs. Fourier analysis have already been presented widely but most recently in Craymer, (1998), Pagiatakis, (1999), and Pagiatakis, (2000) and need not be repeated here. We only present the fundamental formulas and emphasize the statistical properties of the LS spectrum.

We consider a time series $f(t_i)$ observed at discrete times $t_i$, $i$ = 1, 2, … $n$, not necessarily evenly spaced, which is essentially equivalent to the presence of gaps. This time series has also a variance-covariance matrix $C_f$ that describes the uncertainty of the observed values. The LSSA spectrum is described by the percentage variance $s(\omega)$ of the spectral content at a specific frequency $\omega$:

$$s(\omega) = \frac{f^T C_f^{-1} \hat{p}(\omega)}{f^T C_f^{-1} f}, \tag{1}$$

where $\hat{p}(\omega)$ is the projection of $f(t)$ onto the model space spanned by different base functions that can render the series stationary while simultaneously producing the LS spectrum (trigonometric base functions). The range of $s(\omega)$ is from 0 to 1. Pagiatakis (1999) showed that the probability density function (*pdf*) of the LS spectrum $s(\omega)$ follows the *beta* distribution defined by two parameters $\alpha$ =1 and $\beta$ = ($m$-$u$-2)/2 where $m$ is the number of data points and $u$ is the number of unknown parameters estimated by the LS procedure.

**4. Least Squares Product Spectra and Response function**

Common peaks or common features in both gravity and atmospheric pressure can easily be defined through their product spectrum. Knowing that each individual factor spectrum follows the beta distribution, we derive the probability distribution of the product spectrum using standard statistical approaches (e.g. Hogg and Craig, 2005). After some development, we obtain the *pdf* for the sum of the natural logarithm of two spectra {$z = \ln(s_1)+\ln(s_2)$}, given by:

$$f(z) = \int_z^0 \beta_1 \beta_2 e^z \left(1 - e^{z-s_2}\right)^{\beta_1 - 1} \left(1 - e^{s_2}\right)^{\beta_2 - 1} ds_2 \tag{2}$$

where $\beta_i = 0.5(m_i - u_i - 2)$, $m_i$ and $u_i$ were defined earlier. The above *pdf* that underlines the product LS spectrum can be used to identify the significance of the common peaks in both series by using their product spectrum. Figure 1 shows the *pdf* for $\beta_1$ *=8500 and* $\beta_2$ = 3200. The vertical line in Figure 1 (at -15.73) shows the 95% confidence level.

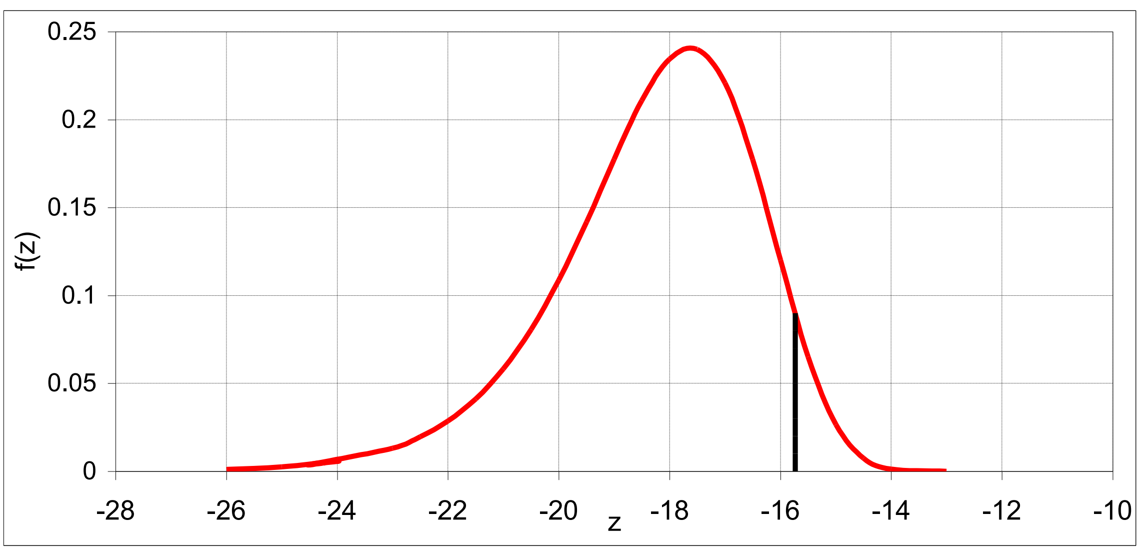


Fig. 1. The probability density function of the product spectrum

After producing the product spectrum between gravity and atmospheric pressure and using its *pdf* as a guide, we can identify statistically significant peaks above the 95% confidence level or higher. These peaks (actually their periods) are suppressed separately in the gravity and atmospheric pressure series to estimate their amplitude and phase:

$$g_i = a_{iG} \cos\left(\omega_i t - \varphi_{iG}\right), \tag{3}$$

$$p_i = a_{ip} \cos\left(\omega_i t - \varphi_{ip}\right), \tag{4}$$

where $a_{iG}$, $a_{ip}$ are the amplitudes of gravity and atmosphere pressure constituents respectively, and $\varphi_{iG}$, $\varphi_{ip}$ are their phases. All the above parameters have an associated covariance matrix estimated from the LS procedure. Only the statistically significant amplitudes and phases are used to estimate the pressure admittance. The magnitude and phase of the pressure admittance is then estimated from

$$\alpha(\omega_i) = \frac{a_{iG}}{a_{ip}}, \tag{5}$$

$$\Delta\varphi_i = \varphi_{iG} - \varphi_{ip}. \tag{6}$$

Applying the covariance law to Eq. (5) and (6) we calculate the standard deviation $\sigma_\alpha$ of admittance and $\sigma_{\Delta\varphi}$ of its phase:

$$\sigma_\alpha = \left[\frac{1}{a_p^2}\left(\sigma_{a_G}^2 + \sigma_{a_p}^2\left(\frac{a_G^2}{a_p^2}\right)\right)\right]^{1/2} \tag{7}$$

$$\sigma_{\Delta\varphi} = \left[\sigma_{\varphi_G}^2 + \sigma_{\varphi_p}^2\right]^{1/2} \tag{8}$$

We note here that the admittance is calculated only at specific frequencies at which both gravity and atmospheric pressure data have significant amplitudes. We avoid the calculation of the admittance at frequencies where other phenomena may be present (i.e. in the semidiurnal or diurnal band) since they may contaminate it.

**5. Data processing and Analysis**

Two five-year-long time series of gravity and atmospheric pressure respectively starting 1st January 1998 are used to estimate the atmospheric pressure admittance from the Canadian Superconducting Gravimeter Installation – CSGI (Cantley, Canada). These five year records are segmented into monthly time series. First of all, the solid Earth tide effect is removed from the 1s gravity records using GWAVE (Merriam, 1992b). Secondly, the 1s gravity residual series (tide free) is then filtered using a Parzen weighting scheme that produces unequally spaced series along with their standard deviation at a sampling interval ranging from 2 to 5 minutes. The atmospheric pressure and its associated standard deviation are calculated by using the same scheme but at the sampling interval of 15 minutes. Thirdly, the ocean loading effect is also removed by least-squares fitting of eight most significant periods; this is done simultaneously with the estimation of the gravity spectrum using the LSSA software.

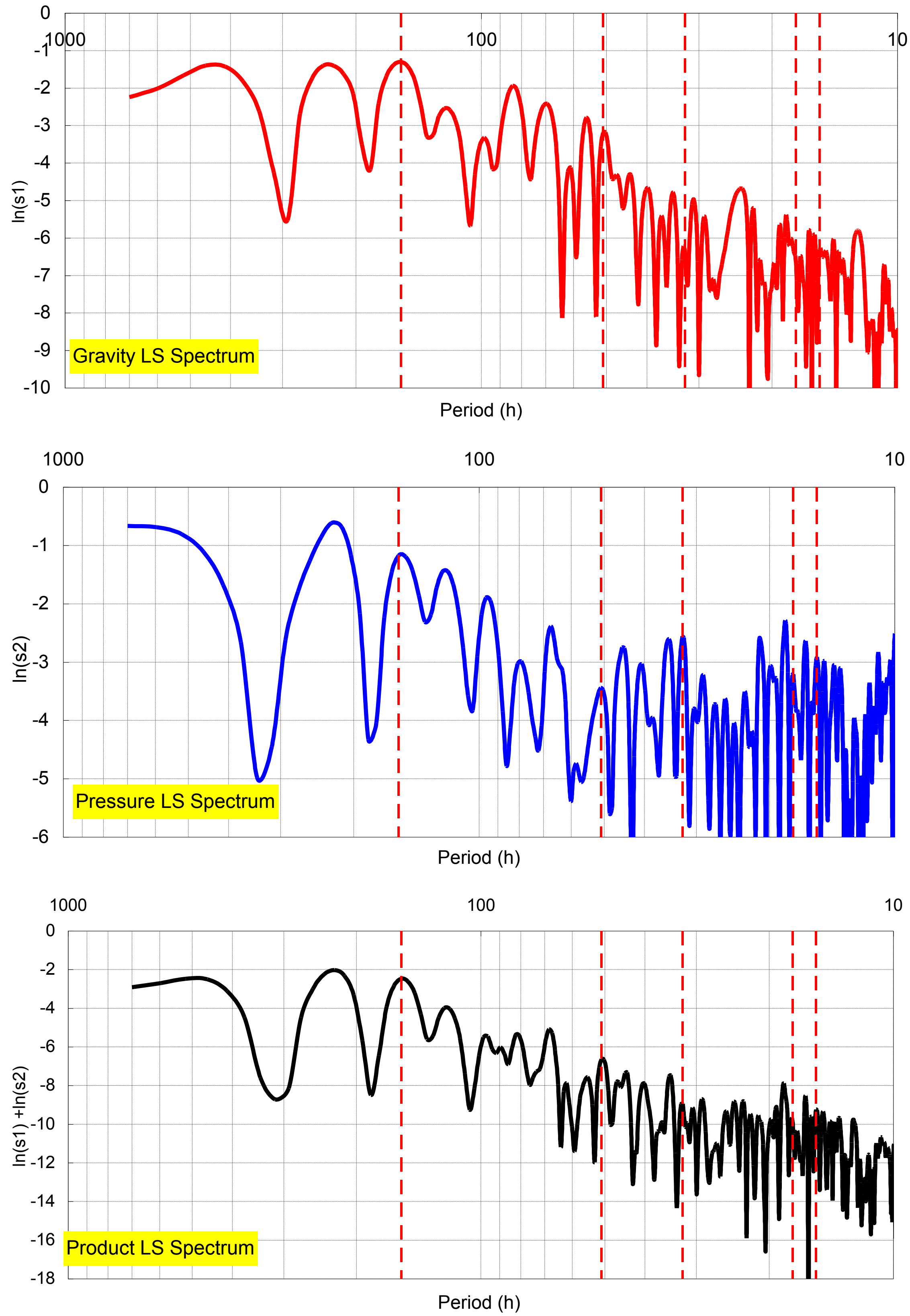


Fig. 2. LS spectra of gravity, pressure and of the product spectrum

The following steps are followed to estimate the admittance:

1. We produce monthly LS spectra for both gravity and pressure in the band 700-2h.
2. The natural logarithms of both spectra are taken and summed. The identification of the significant peaks in the product LS spectrum at 95% confidence levels is done rigorously using the probability density function of the product LS spectrum (Fig. 2).
3. The peaks in the product must be statistically significant at 95% confidence level in both gravity and pressure (Figure 2) to be considered in the next step.
4. We suppress these peaks in the monthly data to estimate their amplitudes and phases.
5. We use Eqs. (5)-(8) to estimate the admittance (amplitude and phase) along with their associated standard deviations.
6. The weighted LS regression is used to define the best fit to get a smoothed gravity response for pressure changes (Fig. 3)

$$\alpha(f) = 0.312501 - 0.291637\, e^{-\left(\frac{1}{32f}\right)}, \tag{9}$$

where $f$ is the frequency cycle/hour (cph) and the phase difference $\Delta\varphi = 85.86 \pm$ [illegible]$.53$ degrees.

The results obtained from Eq. (9) are plotted in Fig. (3), which shows the admittance from 5-year data (monthly segments) with their error bars ($1\sigma$). Different fitting functions are tested ($2^{nd}$, $3^{rd}$ order polynomials and sine wave); however, Eq. (9) is the best fit with minimum quadratic norm. The red line shows a weighted LS regression that is expected to describe a smooth response over all frequencies without any resonance. The estimated frequency-dependent admittance tends to be constant for periods longer than about 100h and decreases exponentially in the band 100-3h. This means that the new admittance in the diurnal and semi-diurnal or higher frequency bands is smaller than the one estimated from previous studies (0.3μgal/mbar). The phase difference (phase response) is estimated from the weighted average.

Different tests are carried out to assess and validate this new admittance. LS gravity spectra of June, 1998 before and after the removal of the pressure effect using the constant (0.3 μGal/mbar) and the newly estimated frequency-dependent admittance are used in this comparison. Fig. 4 shows the spectrum in the band 100-10h, whereas Fig. 5 shows the 10-3h band. It is obvious that the frequency-dependent admittance improved the peaks but with more significant improvements in the 10-3h band. From Fig. 4, the gravity peak at period 56.689h, which is originally from the pressure (pressure spectrum not shown here) is reduced by 35% and 54% when applying the constant admittance and the new one, respectively. Craymer (1998) showed that the reduction of the LS spectrum peaks is equivalent to the inverse of the square of the signal-to-noise ratio (SNR). This means that the SNR is improved. Also, Fig. 4 shows the peak at 26.252h is improved when applying the new admittance (red color); however, this peak does not exist in the pressure spectrum. Fig. 5 shows significant improvements in the peaks in that band (3-10h) when applying the new admittance compared with the constant one.

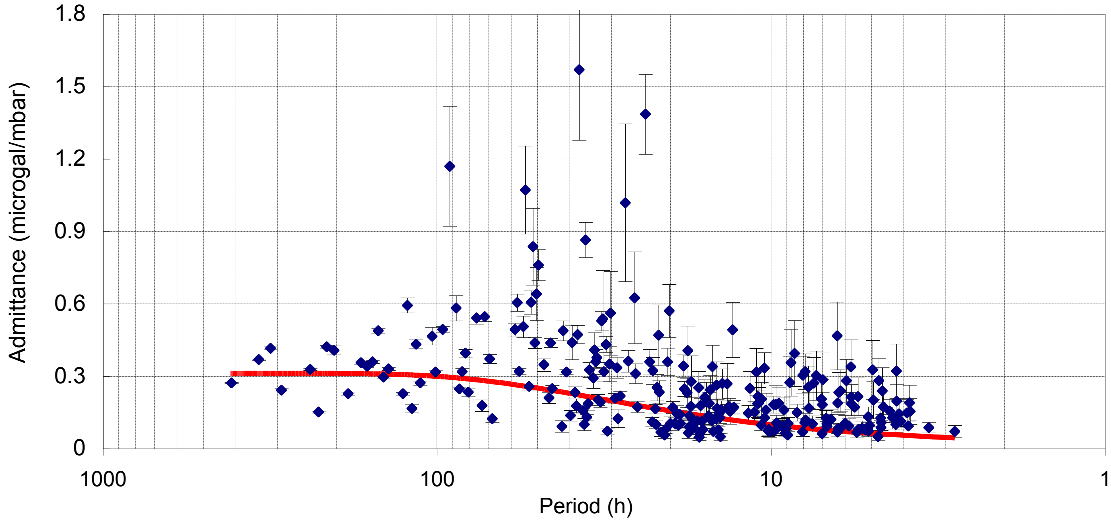


Fig. 3. Pressure admittance from the Least Squares Response Method

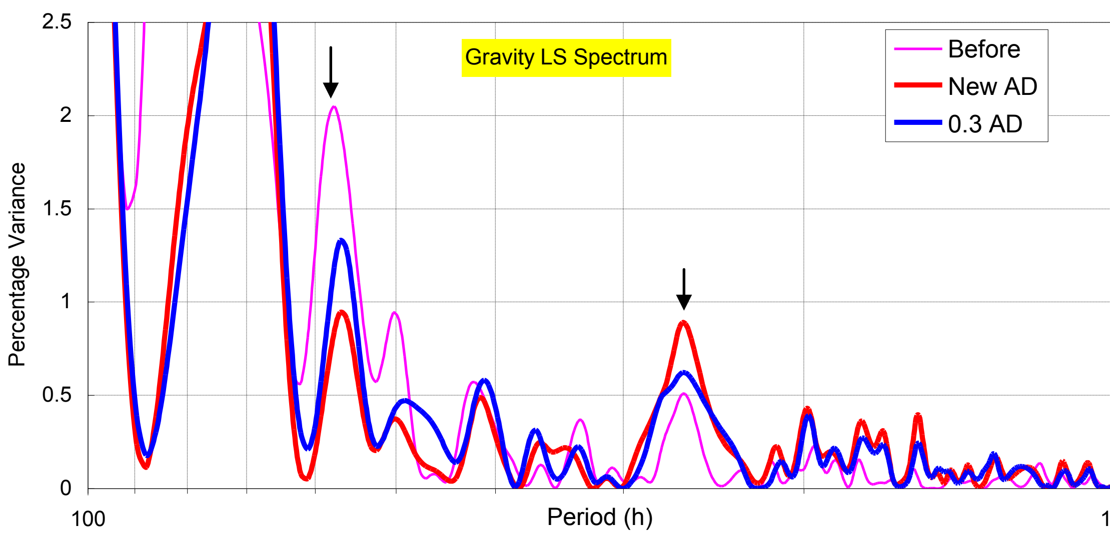


Fig. 4. LS spectrum before and after the application of the new admittance (10-100h)

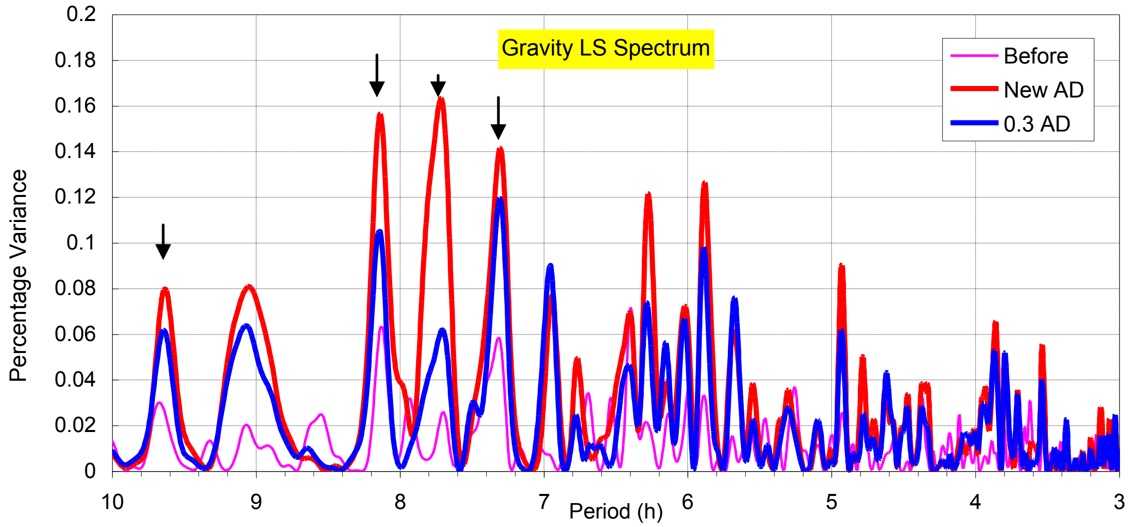


Fig. 5. LS spectrum before and after the application of the new admittance (3-10h)

## 6. Conclusion

The constant admittance is not adequate to correct the pressure effect. The new admittance is frequency dependent and it is relatively constant in the low frequency band (higher than few days), and it decreases exponentially starting from 100h. The spectrum in the high frequency band (10-3h) is improved significantly with peaks sharpened. Also, the signal-to-noise ratio of the gravity spectrum is improved with pressure peaks in the gravity either removed or reduced. Research is continuing to determine the seasonal variations of the pressure admittance.

## References

Boy J.P., Gegout, P. and Hinderer Jacques, 2001. Gravity variations and global pressure loading, *Journal of the Geodetic Society of Japan,* 47, 267-272.

Boy, J.P., Hinderer, J. and Gegout, P., 1998. Global atmospheric loading and gravity, *Phys. Earth Planet. Inter.,* 109, 161-177.

Craymer, M.R., 1998. The least squares spectrum, its inverse transform and autocorrelation function: theory and some applications in geodesy, PhD thesis, University of Toronto, Canada.

Crossley, D., Jensen, O.G. and Hinderer, J., 1995. Effective Barometric Admittance and Gravity Residuals, *Phys. Earth Planet. Inter.,* 90, 221-241.

Crossley, D.J. and Xu, S., 1998. Analysis of superconducting gravimeter data from Table Mountain, Colorado, *G. J. International,* 135, 835-844.

Hinderer, J. and Crossley, D., 2004. Scientific achievements from the first phase (1997-2003) of the Global Geodynamics Project using a worldwide network of superconducting gravimeters, *Journal of Geodynamics,* 38, 237-262.

Hinderer, J. and Crossley, D., 2000. Time variations in gravity and inferences on the Earth's structure and dynamics, *Surv.Geophys.,* 21, 1-45.

Hogg, R.V. and Craig, A.T., 2005. *Introduction to mathematical statistics*. Prentice Hall New Jersey, (6th edition).

Hu, X.-G; Liu, L.-T; Hinderer, J.; Sun, H.P , 2005. Wavelet filter analysis of local atmospheric pressure effects on gravity variations. Journal of Geodesy, 79 (8): 447-459.

Jensen, O., Hinderer, J. and Crossley, D.J., 1995. Noise Limitations in the Core-Mode Band of Superconducting Gravimeter Data, *Phys. Earth Planet. Inter.,* 90, 169-181.

Kroner, C. and Jentzsch, G., 1999. Comparison of different barometric pressure reductions for gravity data and resulting consequences, *Phys.Earth Planet.Inter.,* 115, 205-218.

Merriam, J.B., 1992a. Atmospheric-Pressure and Gravity, *Geophysical Journal International,* 109, 488-500.

Merriam, J.B., 1992b. An ephemeris for gravity tide predictions at the nanogal level, *Geophysical Journal International,* 108, 415-422.

Mukai, A.; Higashi, T.; Takemoto, S.; Nakagawa, I.; Naito, I.., 1995. Accurate Estimation of Atmospheric Effects on Gravity Observations made with a Superconducting Gravity Meter at Kyoto, *Phys. Earth Planet. Inter.,* 91, 149-159.

Neumeyer, J., 1995. Frequency dependent atmospheric pressure correction on gravity variations by means of cross spectral analysis; Theoretical tidal model, calibration, and high precision tidal data processing [modified], *Marees Terrestres. Bulletin d'Informations,* 122, 9212-9220.

Neumeyer, J.; Hagedoorn, J.; Leitloff, J.; Schmidt, T., 2004 Gravity reduction with three-dimensional atmospheric pressure data for precise ground gravity measurements, *Journal of Geodynamics,* 38, 437-450.

Pagiatakis, S.D., 1999. Stochastic significance of peaks in the least-squares spectrum, *Journal of Geodesy,* 73, 67-78.

Pagiatakis, S.D., 2000. Application of the least-squares spectral analysis to superconducting gravimeter data treatment and analysis; Proceedings of the workshop on High precision gravity measurements with application to geodynamics and Second GGP workshop, *Cahiers Du Centre Europeen De Geodynamique et de Seismologie (ECGS),* 17, 103-113.

Smylie, D.E. and Jiang, X.H., 1993. Core Oscillations and their Detection in Superconducting Gravimeter Records, *Journal of Geomagnetism and Geoelectricity,* 45, 1347-1369.

Spratt, R.S., 1982. Modeling the Effect of Atmospheric-Pressure Variations on Gravity, *Geophysical Journal of the Royal Astronomical Society,* 71, 173-186.

Sun H-P, Ducarme, B. and Dehant V., 1995. Theoretical calculation of atmospheric gravity Green's functions, *Cahiers Du Centre Europeen De Geodynamique et de Seismologie (ECGS),* 11, 223-237.

Sun, H-P *et al.,* 2002. Tidal gravity observations obtained with a superconducting gravimeter at Wuhan/China and its application to geodynamics, *Journal of Geodynamics,* 33, 187-198.

van Dam T.M. and Wahr J., 1998. Modeling Environment Loading Effects: a Review, *Physics and Chemistry of the Earth,* 23, 1077-1087.

Virtanen, H., 2004. Loading effects in Metsahovi from the atmosphere and the Baltic Sea, *Journal of Geodynamics,* 38, 407-422.

Warburton, R.J. and Goodkind, J.M., 1977. Influence of Barometric-Pressure Variations on Gravity, *Geophysical Journal of the Royal Astronomical Society,* 48, 281-292.