

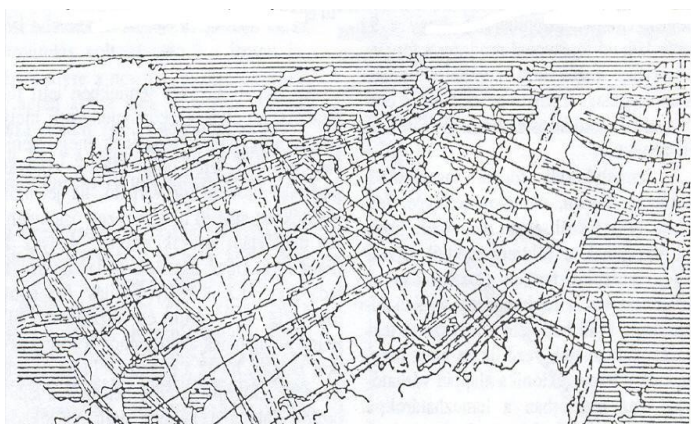
# Volcanic activity and tidal heating of Saturn's moon Enceladus

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## 1. Preliminary statements concerning dynamics of Enceladus

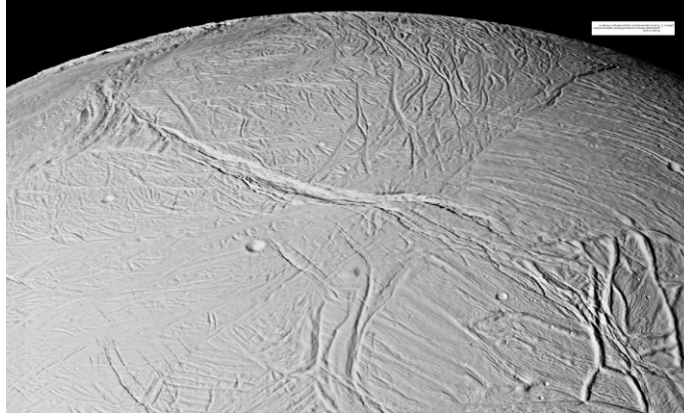
The medium size, spherical satellites of Saturn turned in synchronized rotation in the early stage of their history, what means their orbital and rotation periods are the same. Consequently their tidal bulge has no motion in a coordinate system fixed to the body of the moon, therefore there is no tidal friction influencing the axial rotation spin and dynamical processes on and within the celestial object. This passivity disappears however if the orbit is locked in the resonance between the orbital periods of members of the satellite system. This resonance generates a forced eccentricity which leads to temporal variation of the tidal bulge due to central body of the system. Enceladus is one of the innermost moons of Saturn. The most remarkable types of tectonic features found on Enceladus are faults that can run up to two hundred kilometres long, which cross older, tectonically deformed formations. The origin of these global linear structures should be related to tidal activity generated by the Enceladus-Dione 2:1 resonance. Tidal heating is resulting perhaps also from this orbital resonance with Dione, which powers the water volcanism imaged by Cassini spacecraft in 2005.



**Fig.1. Distribution of linear structures on the surface of the Earth**

Fig. 1 shows lineaments detected on the Earth over the former USSR with the use of satellite telemetric investigations during eighties of XXth century. (*Brjukanov et al.*,

1984). These structures, which were found in other parts of the Earth too, have regular distribution over the planetary surface and possibly are related to despinning of axial spin. Similar fault-lines are visible on the surface of Enceladus (Fig.2) what can be in principle also associated to resonance supported tides of Enceladus.



**Fig.2. Distribution of linear structures on the surface of the Earth**

The aim of investigation carried out by the author is to model contribution of the tidal phenomenon in dynamics of Enceladus. The mathematical modelling of the tide generated normal (radial) and tangential (tangential) distribution in the icy “mantle” of Enceladus was carried out. With the use of these results the magnitude of tidal heating was estimated.

## 2. Description of elastic deformations of spherical body

The stresses produced by tidal forces were studied by inhomogeneous differential equations of elasticity for a gravitating globe, which are generally used in earth tidal research. Elastic deformations in this system can be characterised by dimensionless functions of Love and Shida:  $H(r)$  describes the vertical,  $T(r)$  the horizontal displacements and  $R(r)$  gravitational potential associated with elastic deformation.  $\mu=\mu(r)$  and  $\lambda=\lambda(r)$  are the Lamé parameters,  $\rho=\rho(r)$  is the density at a  $r$  distance from the centre.  $W=W(r)$  describes the gravity potential, while  $n$  is the degree of spherical harmonics and  $G$  is the gravitational constant. If the notation system introduced by *Molodensky (1953)* is in use the corresponding differential equations are:

$$-\left[\mu\left(T'+H-\frac{2}{r}T\right)\right]' = \rho(R+W'H)' + \lambda F + 2\frac{\mu}{r}\left[2H+T'-\frac{n^2+n+1}{r}T\right]$$

$$-(\lambda F + 2\mu H')' = \rho(R+W'H)' - \rho W'F + 4\frac{\mu}{r}\left(H'-\frac{H}{r}\right) - \frac{n(n+1)}{r^2}\mu\left(T'+M-4\frac{T}{r}\right)$$

$$R'' = -\frac{2}{r}R' + \frac{n(n+1)}{r^2}R + 4\pi G(\rho F + \rho'H)$$

$$F = H' + \frac{2}{r}H - \frac{n(n+1)T}{r^2}$$

(All differentiations (') are carried out with respect to the radius  $r$ )

From among the six boundary conditions three refer to the surface ( $r=a$ ). By the definition of elastic theory of tidal deformations  $N(a)=M(a)=0$ . At the core boundary  $r=b$   $N(b)$  is equal to the hydrodynamic pressure and the value of  $M(b)$  depends on the structure of the core.

To determine the normal (radial) and tangential (horizontal) stresses the following auxiliary relations are valid

$$N = (\lambda + 2\mu)H' + \lambda \left[ \frac{2}{r}H - \frac{n(n+1)}{r^2}T \right]$$

$$M = \mu \left( T' - \frac{2}{r}T + H \right)$$

### 3. Model of Enceladus used for aims of present investigation

The data listed in Table 1 show characteristics of Saturn's moon Enceladus.

**Table 1. Characteristics of Enceladus** (*Porco et al, 2006*)

Orbital characteristics	Physical characteristics
Semimajor axis ( $R_E$ ): $2.37948 \cdot 10^8$ m	Mean radius ( $a_E$ ): $2.521 \cdot 10^5$ m
Eccentricity ( $e_E$ ): $4.5 \cdot 10^{-3}$	Mass ( $M_E$ ): $1.08 \cdot 10^{20}$ kg
Orbital period ( $T_E$ ): 1.3702 day	Volume: $6.712 \cdot 10^{16}$ m <sup>3</sup>
Mean surface temperature: 75 K	Mean density ( $\rho_E$ ): $1610$ kg·m <sup>-3</sup>
( Mass of Saturn ( $M_s$ ): $5.68 \cdot 10^{26}$ kg)	Surface gravity: $0.078$ m·s <sup>-2</sup>

If the physical parameters of Enceladus and another significant Saturn moons ( Mimas, Thetys, Dione, Rhea, Titan, Hyperion and Iapetus ) are compared it turns out that the Enceladus has a relatively high average density (see Table 1 ). It means that due to the fact that his surface is composed by water ice Enceladus should have an inhomogeneous inner structure with a dense core. For the purpose of present study a two layered somewhat arbitrary selected model was used:

- „icy” mantle  $1 \geq r/a > 0.55$
- „rocky” core  $0.55 \geq r/a \geq 0$

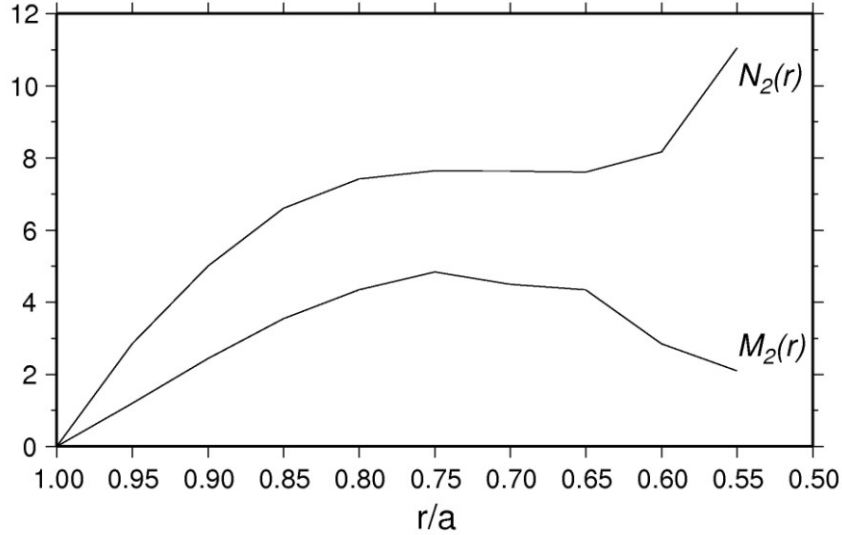
The physical parameters of such an artificial model of Enceladus are described in Table 2.

**Table 2. Physical properties of the model**

	Mass (kg)	Volume (m <sup>3</sup> )	Density (kg·m <sup>-3</sup> )	$\mu$ GPa	$\lambda$ GPa
„icy” mantle	$6.15 \cdot 10^{19}$	$5.59 \cdot 10^{16}$	1100	5	5
„rocky” core	$4.64 \cdot 10^{19}$	$1.12 \cdot 10^{16}$	4150	20	20

In the above model the Lamé parameters are taken equal ( $\mu = \lambda$ ) and the effect of increasing hydrostatic pressure along the radius was not taken in consideration. The numerical solution of the sixth order differential equation in case of  $n=2$  for the auxiliary functions ( expressed in relative units ) within the “icy mantle” gives auxiliary functions  $M_2(r)$  and  $N_2(r)$  in the form shown in Fig. 3.

Fig. 3. Distribution of the auxiliary functions  $M_2(r)$  and  $N_2(r)$  within the mantle



The experience of the study of very different models of the Earth shows, that the general trend remains similar for essentially different models of the mantle in case when the depth of the core-mantle boundary remains the same (Varga, 1985, 1988, 1992). If the core radius decreases ( i.e. in case of reduced inhomogeneity of the planetary body ) the relative maximums of  $N_2(r)$  and  $M_2(r)$  are displaced to the deeper parts of the mantle.

#### 4. Tidal heating of Enceladus

Due to synchronous rotation of Enceladus the tidal bulge varies only due to forced eccentricity caused by 2:1 orbital tidal resonance with Dione. The corresponding variation of second degree tidal potential can be given with equation

$$\Delta W_2 = \frac{G \cdot M_S \cdot a_E^2}{R_E} \left( \frac{3}{2} \cos^2 \Psi - \frac{1}{2} \right) \left( \frac{r}{a_E} \right)^2 \frac{2e_E}{(1-e_E)R_E}$$

The normal and tangential second degree tidal stresses can be expressed with the use of auxiliary functions  $N_2(r)$  and  $M_2(r)$  (Fig. 3)

$$\sigma_N(r) = N_2(r) \frac{\Delta W_2(r)}{g_E \cdot r^2} \cdot \left( \frac{r}{a_E} \right)^2$$

$$\sigma_M(r) = M_2(r) \frac{\partial(\Delta W_2(r))}{\partial \Psi} \cdot \frac{1}{g_E \cdot r^2} \cdot \left( \frac{r}{a_E} \right)^2$$

The stress generated energies  $E_N$  or  $E_M$  in a volume  $V(r)$  can be obtained with

$$E(r) = \frac{\sigma(r)}{2\mu} V(r)$$

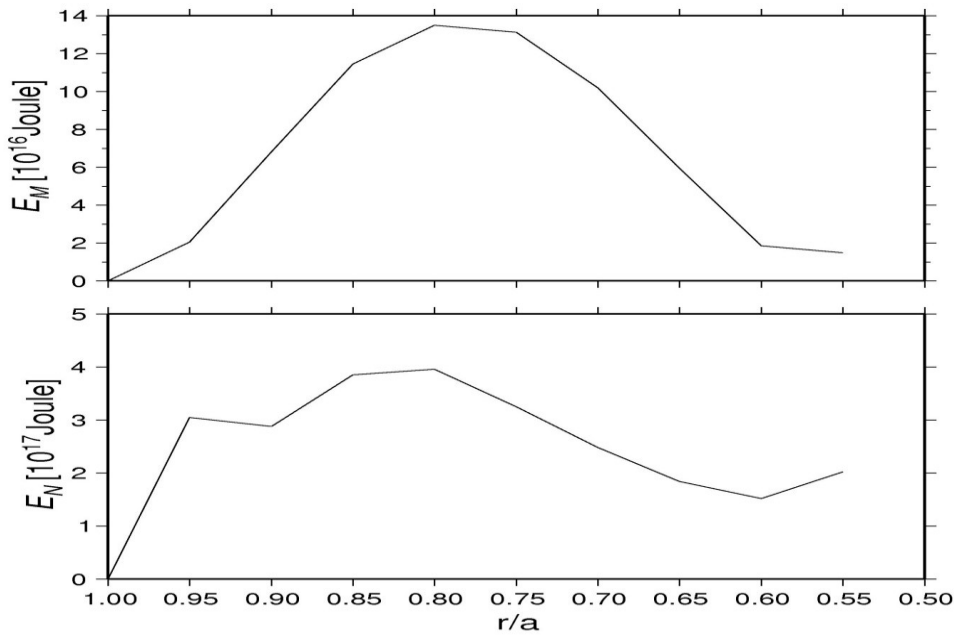
Results of calculations carried out are plotted on Fig. 4.

It can be concluded that most of tidal energies are concentrated in the middle of the somewhat arbitrary chosen mantle. It is so - independently from the selected model -

because of features of  $N_2(r)$  and  $M_2(r)$  (Fig. 3) and due to  $\left(\frac{r}{a}\right)^2$  in expression

for the tidal potential. This energy concentration in the middle of the mantle can be illustrated by the distribution of focal depths of quakes of the Moon (Nakamura, 1983).

Fig. 4. Energy due to tangential ( $E_M$ ) and normal ( $E_N$ ) stress within the mantle



For energy dissipation within the Enceladus ( $M_E = 1.08 \cdot 10^{20}$  kg) one gets

$$-\left[\frac{dE}{dt}\right]_E = J \cdot \sigma \cdot K \cdot M_E$$

Here  $J$  is the mechanical equivalent of heat ( $=4.182$  Joule),  $\sigma$  expresses the specific heat ( $\approx 0.2$ ). To calculate the warm up  $K$  (in degree/s) we got for dissipation from  $\Delta E \approx 10^{17}$  Joule and  $\Delta T = 1/2 T_E = 0.6851$  day

$$-\left[\frac{dE}{dt}\right]_E = 8.4 \cdot 10^{11} \text{ Joule/s.}$$

This way for the warm up we got  $K = 1.4 \cdot 10^{-8}$  degree/s. This means that during 300-350 year the temperature enlargement in the "middle mantle" can be  $\approx 200$  Kelvin. (For a comparison for the Earth  $K = 1.4 \cdot 10^{-16}$  degree/s i.e. 3 Kelvin during  $10^9$  year)

##### 5. Tidal friction due to irregular axial rotation along eccentric orbit.

Applying Kepler's third law to the Saturn\_Enceladus system orbital velocity of Enceladus in case of circular motion is

$$n_E = \sqrt{G(M_S + M_E)a_E^3}$$

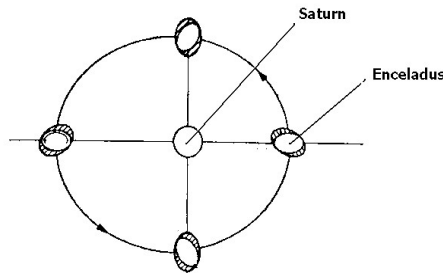
In case when the axial rotation is synchronized  $\omega_E = n_E$  and due to the forced eccentricity caused by Dion the magnitude of orbital speed variation is

$$\Delta n_E = 2G^{1/2}(M_S + M_E)^{1/2}a_E^{3/2}[1 - (1 - e_E^2)]^{3/4}$$

$$\Delta n_E \sim 10^{-3} \text{ } ^\circ/\text{s}$$

This means that the librational motion of Enceladus on its orbit around Saturn (Fig. 5) is  $\pm 8.6^\circ$ .

**Fig.5. Librational motion of Enceladus**



This motion produces an additional heating independently from the inner structure of Enceladus. The amount of heating generated in this way will be discussed in a future contribution.

## 5. Conclusions.

The tidal stresses and energies as it is shown by earlier model calculations for a spherical non-rotating, elastic and isotropic Earth are generated chiefly in the middle part of the mantle. This conclusion holds in case of another bodies of the solar system too. This circumstance allows extend the traditional field of earth tide studies to the problems of planetology. By definition tidal stresses are equal to zero both at the surface and the centre of celestial bodies: at the planetary surface  $N(r)=M(r)=0$  while at the centre the tidal potential  $\Delta W=0$ .

The tidal heating can warm up and melt the inner part of the Enceladus at the depth interval 15-160 km. This way, the fountain-like plumes detected by the Cassini mission can be explained. In case of reduced of reduced inhomogeneity (expressed in lower mean density values), the tide generated heat moves to the deeper parts of

the moon. Possibly that is the reason why in case of other moons of Saturn there is no similar volcanic activity observed.

Tidal friction due to irregular axial rotation along eccentric orbit can be another source of tidal heating, which needs further investigations.

### Acknowledgements

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