

The analysis of Long Period tides by ETERNA and VAV programs with or without 3D pressure correction

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1. Introduction.

This paper follows the paper Ducarme & al. 2006c which presented a comparison of the programs ETERNA (Wenzel, 1994) and VAV (Venedikov et al., 2001, 2003, 2004, 2005; Ducarme et al., 2004, 2005, 2006a,b) for the diurnal and subdiurnal periods.

In this connection we have carefully studied, actually for the first time, the algorithm used by ETERNA for the computation of the MSD (mean square or standard deviations), i.e. for the estimation of the precision. Although this program is based on the LS (least squares) method, it was surprising to establish that its MSD are not LS estimates. It was found that ETERNA uses an intuitive computational scheme, without a statistical basis in order to approximate the colored noise characteristics of the residues, so that the results depend on subjective assumptions and parameters, imposed by the program (Ducarme et al., 2006c).

The situation is even more complicated for the Long Period (LP) tides. ETERNA based its evaluation of the MSD on a $1/f$ hypothesis for what concerns the average noise levels inside the LP band. This assumption is not verified at least with superconducting gravimeters.

Since its first version VAV splits the different tidal bands and evaluates separately the RMS errors in the different tidal bands, but improvements have been recently introduced (Ducarme et al., 2006c). For the evaluation of the LP waves, VAV splits the LP band in two parts: the groups from MSQM to MSM are evaluated together with the Diurnal waves, while Sa and Ssa are evaluated through a so called “zero” filter, i.e. daily mean of the data. ETERNA uses original unfiltered data and allows the representation of non-harmonic phenomena such as the drift by Tschebyscheff polynomials of order n , evaluated separately in each data block.

In the following sections we shall first compare the noise evaluation methods. As the polar motion, with its Chandler term at 430 days, is the main known source of noise close to the groups Sa and Ssa, we shall compare the reactions of the two analysis methods when the pole tide is subtracted or not.

Finally the 3D pressure correction model of Neumeyer et al. (2004 & 2006) is applied and the associated noise decrease is evaluated.

2. Estimation of the LP waves noise by ETERNA.

ETERNA uses the amplitude spectrum of the residuals, in order to determine the so called average noise levels $L(i)$, $0 \leq i \leq 4$ cycles per day (cpd), as frequency dependent estimates of the precision (Table 1).

The average white noise level $L(wn)$ is directly computed from the RMS error on the unit weight σ_0 and the number of observations n by the relation (Ducarme et al., 2006c)

$$L(wn) = \sigma_0 \sqrt{\pi/n} \quad (1)$$

The average noise levels $L(i)$ at the frequencies 1, 2, 3 & 4 cpd (cycles/day) are determined by the arithmetic means of the amplitudes in the intervals, given in table 1.

For D, SD, TD and QD waves the errors evaluated by Least Squares under the white noise assumption are directly scaled by the ratio $L(i)/L(wn)$ with $i=1,2,3,4$

For LP waves the procedure is slightly different.

The noise at the tidal wave frequency f is estimated by

$$\text{noise}(f) = L(0) * f_0 / f \quad (2)$$

$f_0 = 0.1 \text{cpd}$, $L(0)$ average noise level between $0.1^\circ/\text{h}$ and $2.9^\circ/\text{h}$

Finally a scaling factor is given as

$$\text{Scale} = \text{noise}(f) / L(wn) \quad (3)$$

The underlying assumption is that we have a colored noise in $1/f$ in the LP band.

Table 1: frequency range for the determination of the average noise levels $L(i)$ by ETERNA

Name	Tidal family (cycles/day)	Angular speed (degrees per hour)	
		from	to
L(0)	LP	0.1	2.9
L(1)	1	12.0	17.9
L(2)	2	26.0	31.9
L(3)	3	42.0	47.9
L(4)	4	57.0	62.9
$L(wn)$	White noise		

We computed detailed amplitude spectra of the residues using Tsoft (Van Camp and Vauterin, 2005) and produced average values $L(lp)$ in the frequency ranges given in Table 2 and corresponding to the different wave groups. Let us consider now the evolution of $L(lp)$ inside the different LP groups, in order to check if it follows the $1/f$ dependence. The increase of $L(lp)$ is well correlated with the ratio f_0/f in the range covered by $L(0)$ with $r=0.98$, but the regression coefficient is only 0.07. The mean value of $L(0)$ for the three stations (0.202nm/s^2) corresponds to the frequency of MSF and not to the middle of the frequency range at 01cpd (Mstm group). It is normal as the dependence is in $1/f$.

For what concerns the Very Long Period (VLP) tides Ssa and Sa the observed noise is well below the value extrapolated using a $1/f$ law, for which the noise level of VLP tides should be 12 or 25 times larger than the noise at Msf frequency.

It is thus not justified to apply this $1/f$ correction and we recommend the use of the same formula for LP waves as for the other tidal families. To rescale the estimated errors it is only necessary to divide them by the factor f_0/f given in the last column of table 2.

Table 2 : Average Noise Level $L(lp)$ evaluation of the residues for the LP groups. Pole tide is subtracted. Local pressure correction is applied.

a) The spectral amplitudes are computed using Tsoft (Van Camp and Vauterin, 2005) and averaged in the frequency range indicated in columns 2 and 3.

Group	Low	High	Vienna 199707-200212	Strasbourg 199703-200305	Membach 199508-200306	Mean	f_0/f
	cpd	cpd	nm/s ²	nm/s ²	nm/s ²	nm/s ²	
Sa	.0025	.0041	1.193	3.820	1.837	2.283	36.606
Ssa	.0041	.0209	0.611	0.922	0.902	0.812	18.262
Msm	.0209	.0326	0.270	0.263	0.381	0.305	3.181
Mm	.0326	.0547	0.191	0.311	0.266	0.256	2.755
Msf	.0547	.0692	0.188	0.197	0.200	0.195	1.477
Mf	.0692	.0913	0.129	0.153	0.169	0.150	1.366
Mstm	.0913	.1055	0.154	0.169	0.140	0.154	.945
Mtm	.1055	.1228	0.159	0.143	0.114	0.139	.913
Msqm	.1228	.1450	0.110	0.134	0.118	0.121	.710
Mqm	.1450	.1830	0.104	0.115	0.098	0.106	.686

b) The spectral amplitudes are computed by ANALYZE with a fixed increment of 0.0033cpd (0.05°/hr).

L(0)	.0066	.1933	0.190	0.208	0.209	0.202	
L(00)	.0040	.0400	0.597	0.771	1.025	0.798	

However this solution is not optimal for the VLP tides Sa and Ssa. For these groups the $L(lp)$ value is much higher than $L(0)$.

It is not astonishing as the annual period and its harmonics are much perturbed by meteorological phenomena. Moreover the pressure correction based on local pressure observations is no more sufficient at periods larger than 50 days (Hu et al., 2006).

It could be possible to introduce in ETERNA program a quantity $L(00)$ estimated between 0.0040cpd and 0.040cpd and thus centered at 0.022cpd. The lower frequency limit corresponds to 250 days in order to avoid the inclusion of the Chandler and annual periods. The higher frequency is inside the MM group. The frequency range has to be large due to the low resolution (0.05°/h) of the spectrum given in the output of the program ANALYZE. This $L(00)$ noise level is 3 to 5 times larger than the $L(0)$ one. It is compatible with the noise ratio between the VLP waves and Mm.

3. Determination of the RMS errors on LP waves by VAV.

The main algorithm of VAV, is based on the partition of the data into N intervals $I(T)$ of equal length ΔT and central epochs $T = T_1, T_2, \dots, T_N$.

In a first stage of VAV the hourly data $y(t)$ in every $I(T)$ are transformed through filtration into even and odd filtered numbers (u, v) , as shown by (4):

$$(u_f(T), v_f(T)) = \sum_{\tau=-\theta}^{\theta} F_f(\tau) y(T + \tau) \quad (4)$$

For hourly data we can define 12 frequency bands $0 \leq f \leq 11$ cpd
VAV applies LS on (u, v) as if (u, v) are the observations. As a result it provides the estimates of the unknowns, in which we are interested, the adjusted (\tilde{u}, \tilde{v}) of the observed (u, v) and the residuals

$$\Delta u_f(T) = u_f(T) - \tilde{u}_f(T) \ \& \ \Delta v_f(T) = v_f(T) - \tilde{v}_f(T) \text{ for all values of } T \text{ and } f. \quad (5)$$

If we had data with White Noise, all u_f & $v_f, f = 1, \dots, \mu$ would have one and the same standard deviation. Then it would be estimated by the RMS for unit weight

$$\sigma_0 = \sqrt{\sum_{f=1}^{\mu} \left(\sum_T \Delta u_f^2(T) + \sum_T \Delta v_f^2(T) \right) / (2N\mu - m)} \quad (6)$$

where m is the number of unknowns.

VAV solves the problem of the Colored Noise by using separately the residuals $(\Delta u_f, \Delta v_f)$ for getting the RMS $\sigma_f(u, v)$, as an estimate of the standard deviation of (u_f, v_f) . Namely, we use the RMS of the data at frequency f computed through

$$\sigma_f(u, v) = \sqrt{\left(\sum_T \Delta u_f^2(T) + \sum_T \Delta v_f^2(T) \right) / (2N - m_f)} \quad (7)$$

where m_f is the number of unknowns in each frequency band.

For Sa and Ssa the RMS σ_{LP} at $f=0$ is evaluated from the residuals after application of the “zero filter”, while for the other LP groups the value σ_D of the diurnal filters ($f=1$) is used.

It was found recently that the distribution of the residues $\Delta u_f(T), \Delta v_f(T), T = T_1, T_2, \dots, T_N$ was not a normal one. This problem was overcome by the introduction of a weight on the filtered numbers $u_f(T)$ & $v_f(T)$ (Ducarme et al., 2006c).

In section 5 we shall compare the results obtained with and without introduction of the weights.

4. Results obtained with ETERNA3.4 (program ANALYZE)

Table 3 shows the results obtained with the time series of the Superconducting Gravimeter (SG) C025 at Vienna between 1997/07/01 and 2002/12/31, with or without pole tide correction. The data set is subdivided in 3 blocks (1997/07/01-2002/08/28, 2002/08/30-2002/10/17, 2002/10/18-2002/12/31). A second degree Tschebyscheff

polynomial is adjusted on the first block. Local pressure correction is applied. The errors are estimated according to eq. 3 or directly under white noise assumption.

Table 3: estimation of the LP tidal factors at Vienna by ETERNA3.4 analysis program

Wave (ampl.)	no pole tide correction $L(0)=0.1928, L(wn)=0.1553$				pole tide corrected $L(0)=0.1898, L(wn)=0.0700$			
	$1/f$ noise (eq. 3)		White noise		$1/f$ noise (eq. 3)		White noise	
(nm/s^2)	δ	α	δ	α	δ	α	δ	α
Sa	3.0131	80.50	3.0131	80.50	2.3130	14.68	2.3130	14.68
(3.164)	± 1.7888	± 33.53	± 0.0395	± 0.74	± 1.7797	± 45.64	± 0.0180	± 0.46
Ssa	1.1636	-8.58	1.1636	-8.58	1.1647	-7.81	1.1647	-7.81
(19.928)	± 1.436	± 7.02	± 0.0063	± 0.31	± 1.414	± 6.90	± 0.0029	± 0.14
Msm	1.1000	4.43	1.1000	4.43	1.1346	4.61	1.1346	4.61
(4.327)	± 0.0218	± 1.07	± 0.0271	± 1.41	± 1.052	± 5.30	± 0.0122	± 0.61
Mm	1.1499	0.51	1.1499	0.51	1.1486	0.77	1.1486	0.77
(22.624)	± 0.0182	± 0.91	± 0.0053	± 0.27	± 0.0179	± 0.90	± 0.0024	± 0.12
Mf	1.1484	0.22	1.1484	0.22	1.1460	0.19	1.1460	0.19
(42.826)	± 0.0055	± 0.27	± 0.0032	± 0.16	± 0.0054	± 0.27	± 0.0015	± 0.07
Mtm	1.1193	0.47	1.1193	0.47	1.1287	0.40	1.1287	0.40
(8.200)	± 0.0194	± 0.994	± 0.0171	± 0.88	± 0.0191	± 0.97	± 0.0077	± 0.39
Msqm	1.1914	-0.09	1.1914	-0.09	1.1807	-0.25	1.1807	-0.25
(1.309)	± 0.0931	± 4.24	± 1.057	± 4.81	± 0.0916	± 4.02	± 0.0476	± 2.09
$\sigma_0(\text{nm/s}^2)$	19.21				8.66			

It is obvious that the error evaluation under white noise assumption is very sensitive to the presence of the pole tide. The error is double, following the increase of $L(wn)$, which is proportional to the RMS error on the unit weight σ_0 . Surprisingly it is not the case for the error expressed according to eq. 3. There is no change at all. It can be understood if one considers the fact that $L(0)$ is not affected by the presence of the polar motion which is outside of the defined frequency range .0066-.0193. As explained in Ducarme et al., 2006c, eq. 5 and 6, the RMS error is directly proportional to $L(f)$ and σ_0 is eliminated from the formula. It is thus necessary, when applying the ANALYZE program, to study the spectrum of the residues to check if there is no large peak outside the frequency bands defined in Table 1.

5. Estimation of the LP waves by VAV.

The computations have been made with 4 different options: with or without pole tide correction, with or without weight on the filtered numbers (Table 4). As explained above the VLP components Sa and Ssa are computed with the “zero filter”, separately from the other LP groups, which are evaluated together with the diurnal waves. Local pressure correction is used.

We see indeed that the RMS error is always larger on Ssa than on Mm, although both waves have nearly the same amplitude. It corresponds to the results of table 2 showing the noise increase at very low frequency. Considering the “no weight” option, it is

interesting also to note that the pole tide is mainly affecting the error determination in the VLP groups. The RMS error on the unit weight in the diurnal band σ_D is not affected. The introduction of the weight rescales the RMS error on the unit weight. It slightly diminishes the associated RMS errors for the VLP groups and at periods below Mf for the other LP waves. The increase of the RMS errors on Mm and Msm is more pronounced if the pole tide is not corrected

Table 4: estimation of the tidal factors by VAV05 tidal analysis program

Wave (ampl.) (nm/s ²)	no pole tide corr. no weight		no pole tide corr. weight		pole tide corr. no weight		pole tide corr. weight	
	δ	α	δ	α	δ	α	δ	α
Sa (3.164)	2.9976 ± 1.1923	88.85 ± 3.58	3.1615 ± 1.1869	87.35 ± 3.31	1.8899 ± 0.0886	12.42 ± 2.82	1.9016 ± 0.0880	14.73 ± 2.78
Ssa (19.928)	1.2270 ± 0.0310	-6.12 ± 1.45	1.2243 ± 0.0281	-5.80 ± 1.32	1.1838 ± 0.0145	-5.62 ± 0.70	1.1854 ± 0.0142	-5.55 ± 0.69
σ_{LP} (nm/s ²)	197.0		59.1		92.0		31.0	
Msm (4.327)	1.1628 ± 0.0218	2.79 ± 1.07	1.1632 ± 0.0449	2.71 ± 2.21	1.1621 ± 0.0218	4.26 ± 1.07	1.1628 ± 0.0380	3.30 ± 1.87
Mm (22.624)	1.1495 ± 0.0042	0.31 ± 0.21	1.14981 ± 0.0077	-0.10 ± 0.38	1.1449 ± 0.0042	0.56 ± 0.21	1.1484 ± 0.0068	0.09 ± 0.34
Mf (42.826)	1.1444 ± 0.0023	0.46 ± 0.12	1.1442 ± 0.0024	0.41 ± 0.12	1.1447 ± 0.0023	0.36 ± 0.12	1.1443 ± 0.0023	0.40 ± 0.12
Mtm (8.200)	1.1282 ± 0.0105	0.41 ± 0.54	1.1295 ± 0.0086	-0.06 ± 0.44	1.1284 ± 0.0106	0.43 ± 0.54	1.1296 ± 0.0085	-0.03 ± 0.43
Msqm (1.309)	1.1869 ± 0.0573	-1.16 ± 2.76	1.1842 ± 0.0419	-0.83 ± 2.68	1.2110 ± 0.0573	0.06 ± 2.71	1.1861 ± 0.0415	-0.54 ± 2.00
σ_D (nm/s ²)	32.7		58.7		32.7		30.7	

6. Comparison of the results obtained with ETERNA and VAV

The estimated tidal factors, given in Tables 3 and 4, generally agree within two times the associated VAV errors.

It is clear indeed that we cannot consider the errors determined by ETERNA as reflecting the real signal to noise ratio.

As explained in section 2 we tried to rescale the errors given by the program ANALYZE. In Table 5 we propose three different solutions:

- (1) the original evaluation through eq. 2 and 3;
- (2) a “colored” evaluation after suppression of the f_o/f factor;
- (3) the white noise solution.

For ETERNA, as pointed out in section 4, the white noise evaluation is the only one able to take into account the increase of noise when the pole tide is not corrected, but this increase affects all the spectrum, while it is mainly concentrated on Sa and Ssa with VAV.

When the pole tide is subtracted i.e. when all known error sources are removed, the frequency dependent evaluation (1) gives unrealistic error estimates on Sa and Ssa and increases artificially the RMS errors for frequencies lower than Mf.

Table 5: Comparison of the error determination on the amplitude factors for station Vienna

- (1) original ETERNA formula (eq.3)
- (2) frequency independent Scale= $L(0)/L(\omega n)$
- (3) white noise

Wave (ampl.)	no pole tide corr. $L(0)=0.1928, L(\omega n)=0.1553$				pole tide corr. $L(0)=0.1898, L(\omega n)=0.0700$			
	VAV (no weight)	ETERNA			VAV (no weight)	ETERNA		
(nm/s^2)		(1)	(2)	(3)		(1)	(2)	(3)
Sa (3.164)	0.1923	1.7888	.00490	0.0395	0.0886	1.7797	0.0488	0.0180
Ssa (19.928)	0.0310	0.1436	0.0079	0.0063	0.0145	0.1414	*0.0079	0.0029
Msm (4.327)	0.0218	0.1069	0.0336	0.0271	0.0218	0.1052	0.0331	0.0122
Mm (22.624)	0.0042	0.0182	0.0066	0.0053	0.0042	0.0179	0.0065	0.0024
Mf (42.826)	0.0023	0.0055	0.0040	0.0032	0.0023	0.0054	0.0040	0.0015
Mtm (8.200)	0.0106	0.0194	0.0212	0.0171	0.0106	0.0191	0.0209	0.0077
Msqm (1.309)	0.0573	0.0931	0.1312	0.1057	0.0573	0.0916	0.1290	0.0476

* based on $L(00)=0.937$

** based on $L(00)=0.597$

VAV error determination, which is the only valid one in the LS sense, is generally comprised between the white noise estimation (1) and the estimation (2) “colored” through the ratio $L(0)/L(\omega n)$. For the waves Sa and Ssa one should prefer the estimation based on $L(00)$.

7. Local versus 3D pressure correction

The atmospheric pressure effect is composed of the attraction and elastic deformation terms. The deformation term can be modeled by the Green’s function method, using surface pressure data only. For modeling the attraction term 3D data are required in order to consider the real density distribution within the atmosphere, as the same surface pressure may correspond to different density distributions (Neumeyer et al., 2004). From the European Centre for Medium-Range Weather Forecasts (ECMWF) 3D atmospheric pressure, humidity and temperature are now available on 60 height levels up to about 60 km, at an interval of 6 hours and with a spatial resolution of $0.5^\circ \times 0.5^\circ$. Neumeyer et al. (2006) showed that a 1.5° radius is sufficient to evaluate correctly the 3D attraction term.

As the 3D pressure correction model is only available after January 2001, it was not possible to separate the pole tide signal from the Sa group. For station Vienna, the pole tide signal has been subtracted from the gravity data using the tidal factor 1.1526 computed in Ducarme et al., 2006b. The 3D results are compared with the local pressure admittance ones.

With ETERNA the error diminishes throughout the complete LP band when a 3D pressure correction is applied. (Table 6), while with VAV only the VLP groups Sa and Ssa show a diminution of the errors. This behavior has been confirmed in all the studied SG records.

Table 6: Comparison of the results based on the local pressure correction and on the 3D model of Neumeyer et al., 2006. Station Vienna from 2001.01.01 to 2004.12.31
Pressure admittance $-2.755 \text{ nms}^{-2}/\text{hPa}$ (ETERNA), $-3.255 \text{ nms}^{-2}/\text{hPa}$ (VAV diurnal)

a) amplitude factors

LP groups	SA	SSA	MSM	MM	MF
ETERNA*					
Local AP correction	3.2230	1.2121	1.1736	1.1407	1.1494
	± 0.0600	± 0.0099	± 0.0459	± 0.0087	± 0.0037
3D AP Correction	3.4551	1.2071	1.21476	1.1372	1.1469
	± 0.0483	± 0.0081	± 0.0375	± 0.0071	± 0.0030
VAV					
Local AP Correction	3.17539	1.09390	1.16094	1.15182	1.15062
	± 0.08330	± 0.01342	± 0.04305	± 0.00746	± 0.00186
3D AP correction	4.05966	1.09216	1.19928	1.14678	1.14984
	± 0.08104	± 0.01245	± 0.04830	± 0.00865	± 0.00247

b) phase differences

LP groups	SA	SSA	MSM	MM	MF
ETERNA*					
Local AP correction	70.307	-4.227	0.566	0.047	0.360
	± 1.033	± 0.454	± 2.245	± 0.437	± 0.183
3D AP Correction	49.604	-4.492	1.439	-0.080	0.319
	± 0.792	± 0.374	± 1.775	± 0.358	± 0.150
VAV					
Local AP Correction	28.023	-4.570	0.343	-0.038	0.337
	± 1.452	± 0.692	± 2.124	± 0.371	± 0.093
3D AP correction	14.519	-5.211	2.339	0.292	0.233
	± 1.061	± 0.643	± 2.309	± 0.432	± 0.123

* the RMS errors are computed without introduction of the $1/f$ dependence.

To check how the spectrum of the residues is improved by the 3D pressure correction scheme we compared the amplitude spectra obtained with ETERNA (Table 7a), computing mean values on sliding frequency intervals.

It is clear that the diminution of the noise reaches its maximum around the frequency $6^\circ/\text{h}$ (2.5day), between LP and D tides, and covers the entire LP spectrum. For period shorter than 1.5day the local pressure correction is better. Inside the LP spectrum the averaged amplitude spectra show fluctuations and no clear tendency is appearing in the ratio 3D/local (Table 7b). For Sa the frequency range is probably too narrow to get significant results. We can thus consider that the noise reduction using 3D pressure correction is effective for all the LP groups.

The behaviour of ETERNA is thus normal. As VAV computes the LP tides with frequencies higher than Ssa together with the diurnal waves, the background noise is computed through σ_1 i.e. mainly in the frequency range of L(1). From table 7a it can be seen that there is no improvement in this band. It will thus be necessary to revise the scheme used by VAV for the computation these LP tides.

Table 7: Mean spectral amplitudes in nm/s^2 for different frequency bands

- a) The spectral amplitudes are computed by ANALYZE with a fixed increment of 0.0033cpd ($0.05^\circ/\text{hr}$).

Average Noise level	Frequency band $^\circ/\text{h}$		Local pressure correction	3D pressure correction	Ratio 3D/local
L0	0.1	2.9	0.3731	0.3278	0.879
	2.0	6.0	0.1317	0.0925	0.702
	4.0	8.0	0.0885	0.0570	0.644
	6.0	10.0	0.0615	0.0469	0.762
	8.0	12.0	0.0451	0.0405	0.898
L1	10.0	14.0	0.0362	0.0386	1.066
	12.0	17.9	0.0323	0.0450	1.393

- b) The spectral amplitudes are computed using Tsoft (Van Camp and Vauterin, 2005) and averaged in the frequency range indicated in columns 2 and 3.

Average Noise level	Frequency band $^\circ/\text{h}$		Local pressure correction	3D pressure correction	Ratio 3D/local
Sa	0.0375	0.0615	2.3128	2.3365	1.010
Ssa	0.0615	0.3135	0.7560	0.6880	0.910
Msm	0.3135	0.4890	0.3564	0.3265	0.916
Mm	0.4890	0.8205	0.2722	0.2476	0.762
Msf	0.8205	1.0380	0.1997	0.1604	0.898
Mf	1.0380	1.3695	0.1507	0.1251	0.830
Mstm	1.3695	1.5825	0.1262	0.0878	0.696
Mtm	1.5825	1.8420	0.1070	0.0951	0.889
Msqm	1.8420	2.1750	0.1303	0.1128	0.866
Mqm	2.1750	2.7450	0.1114	0.0871	0.782

6. Conclusions

To get correct estimations of S_a and S_{sa} it is very important to eliminate or estimate the pole tide and build a correct polynomial representation of the non harmonic part of the signal.

As already pointed out in Ducarme et al., 2006c, the error estimation by ETERNA is not valid in the Least Squares sense. Moreover the error evaluation by ETERNA is biased by the noise outside the frequency band averaged by $L(0)$, e.g. the pole tide signal, when it is not eliminated.

The users should suppress the $1/f$ dependence in the error estimation for LP tides according to the factors f_0/f given in Table 2 in order to define “colored” errors in a way similar to the short period tides. The true RMS errors are probably comprised between the white noise estimates and the “colored” ones. However, for the VLP tides S_a and S_{sa} $L(0)$ is underestimating the noise level. The VAV analysis program should be preferred for its correct evaluation of the RMS errors.

The 3D pressure correction scheme is diminishing the background noise inside the entire LP spectrum and down to a period of 1.5 day. However the VAV computing scheme is not reflecting this improvement for LP tides with period lower than 6 months.

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