Estimation of the precision by the tidal analysis programs ETERNA and VAV

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Abstract

A comparison between the algorithms, used by the programs ETERNA and VAV for estimation of the precision is made. It is shown that the algorithm of ETERNA does not use the principles of the least squares method, because it does not use sum of squares of residuals. VAV uses sum of squares of residuals of filtered numbers in a time/frequency domain. However, the variant of the algorithm used till now does not take into account the deviation of the noise from the normal distribution. In a new version of VAV a test of normality is included and a further development of the analysis method by applying weight of the filtered numbers.

1. Introduction.

The initial intention of this paper was to present results of comparative analyses of Superconducting Gravimeters (SG) data by the programs ETERNA (Wenzel, 1994) and VAV (Venedikov et al., 2001, 2003, 2004, 2005, Ducarme et al., 2004, 2005, 2006a, b). The aim was, by using the estimates of the precision, to make conclusions about the advantages of one or another of the programs.

In this connection we have carefully studied, actually for the first time, the algorithm used by ETERNA for the computation of the root mean square errors (RMS). It was surprising to establish that the RMS of ETERNA are not Least Squares (LS) estimates, i.e. they are not root mean square errors.

In these circumstances the comparison between the RMS of ETERNA and VAV has lost its sense. Thus the paper had been reoriented to compare the algorithms of the programs used for the estimation of the precision.

In this connection we would like to recall a fundamental principle of the LS estimation of the precision.

The precision is estimated through the root mean square errors (RMS) of the data and the estimated unknowns. In all cases the RMS are to be determined through the sum of squares of residuals (SSQR) of the data. The theoretical reason is that SSQR is expected to have the Pearson χ^2 distribution. Then an expression like $x = 1.182 \pm 0.002$, where 0.002 is the RMS of *x* has the meaning of a confidence interval with confidence probability 62%. On the basis of the expression $x = 1.182 \pm 0.002$ we can conclude that the true value of *x*, actually its mathematical expectation E((*x*), will be

$$\mathrm{E}(x) \in \text{to the confidence interval } \left(1.182 - t \times 0.002,\ 1.182 + t \times 0.002\right) \tag{1}$$

where *t* is the coefficient of Student, depending on a chosen confidence probability. E.g. for probability 95% and a great number of data *t* = 1.96.

If the RMS 0.002 is not computed according to the LS rules through the SSQR, the expression $x = 1.182 \pm 0.002$ has not any concrete statistical meaning.

SSQR can be directly used in this way provided the data are charged by a white noise (WN). This is not the case of the tidal data. Both ETERNA and VAV solve the problem by using frequency dependent RMS of the data, but this is made in completely different ways.

2. Estimation of the precision by ETERNA.

We shall follow how ETERNA determines the RMS $\sigma_{\mathrm{ET}}(\delta_g)$ of the amplitude factor δ_g of a tidal group g in one of the main tidal families at f = 1, 2, 3 or 4 (cycles/day) cpd. For revealing the algorithm used we had to decipher it in the FORTRAN code of the program.

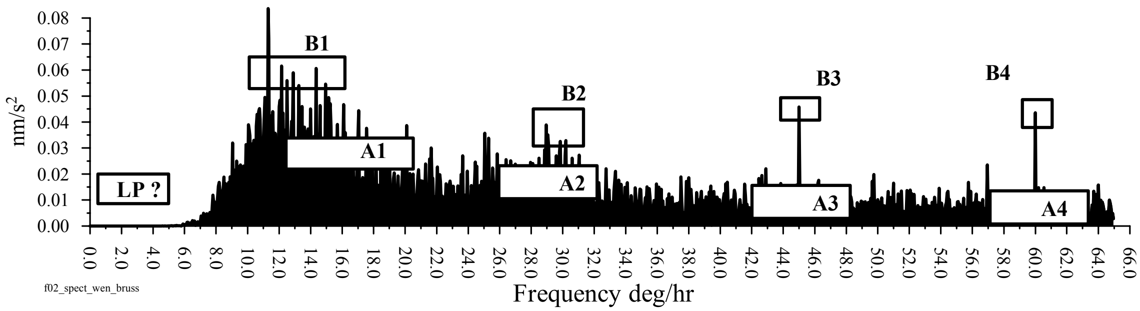

Figure 1. SG data Brussels, ETERNA spectrum of the residuals, obtained with application of the Pertsev filter. The average noise levels $L(f)$ at the frequencies f = 1, 2, 3 & 4 cpd (cycles/day) are determined as arithmetic means of the amplitudes in the intervals, given in Table 1 and indicated in Figure 1 by A1, A2, A3 & A4 respectively.

Table 1. Frequency ranges for the determination of the average noise levels $L(f)$.

Name	Tidal family (cpd)	Angular speed (deg/hr)	
		From	To
L(1)	*f* = 1	12.0	17.9
L(2)	*f* = 2	26.0	31.9
L(3)	*f* = 3	42.0	47.9
L(4)	*f* = 4	57.0	62.9

The colored character of the noise is taken into account through the spectrum of the residuals of the hourly filtered numbers. It uses the so called "average noise levels" $L(f)$ at frequency f, which are arithmetic means of the amplitudes, as shown by Figure 1 and Table 1.

It also uses the RMS for unit weight σ_0, called in the output "Standard deviation" and computed according to LS under the condition of WN. Then ETERNA determines the so called "Average noise level" at "white noise" L(wn) through

$L(\mathrm{wn}) = \sigma_0\sqrt{\pi / n}$, where n is the number of the hourly data. (2)

It does not correspond to the description of the program ANALYZE in "Manual ETERNA3.3.hlp", where it is stated that L(wn) is "*the estimated Fourier amplitude of the white noise in the frequency band 0-6cpd*". It seems that L(wn) is intuitively computed as the square root of the energy in the basic frequency interval π/n, as σ_0^2 is the total spectral energy.

Further ETERNA determines the RMS $\sigma_{\mathrm{WN}}(\delta_g)$ of δ_g, again for the case of WN, as

$$\sigma_{\mathrm{WN}}(\delta_g) = C_g.\sigma_0 \quad (3)$$

where C_g is a coefficient, obtained through the matrix of the LS normal equations.

Finally, ETERNA determines the colored RMS of δ_g as

$$\sigma_{\mathrm{ET}}(\delta_g) = \sigma_{\mathrm{WN}}(\delta_g)\frac{L(f)}{L(\mathrm{wn})} = C_g \frac{\cancel{\sigma_0} L(f)}{\cancel{\sigma_0}\sqrt{\pi/n}} = C_g . L(f) \Big/ \sqrt{\pi/n} \tag{4}$$

With a good enough approximation $C_g \approx 1/\left(h_g \sqrt{n/2}\right)$, where h_g is the min theoretical amplitude in the group g. By using this we get

$$\sigma_{\mathrm{ET}}(\delta_g) \approx L(f)/\left(h_g \sqrt{\pi/2}\right) \tag{5}$$

In a similar way we get, for the RMS of the phase lag α_g of ETERNA

$$\sigma_{\mathrm{ET}}(\alpha_g) \approx L(f)/\left(h_g \sqrt{\pi/2}.\delta_{\mathrm{g}}^2\right) \text{ in radians} \tag{6}$$

It is strange, that σ_0, which is obtained through the SSQR, finally disappears in these expressions. Also strange is the presence of the $\sqrt{\pi}$ in the definition of L(wn), without any theoretical explanation. The expressions look as intuitive ones, which had been adapted empirically, e.g. by including the term $\sqrt{\pi} = 1.7724$, until ETERNA gets some reasonably looking values.

Further remarks to this way of estimation of the precision are the following.

(i) The RMS σ_{ET} are not LS estimates because they use arithmetic means $L(f)$ of amplitudes instead of SSQR, i.e. the RMS are actually not root mean square errors.

(ii) The levels $L(f)$ depend on how the intervals A are chosen. If we do not obey to the dogmatic definition by ETERNA of the intervals (Table 1), this can be done in many different ways. E.g., if A in Figure 1 are replaced by B, we may get different values of $L(f)$.

(iii) As shown by the example in Section 3 hereafter, in particular cases of the noise ETERNA can provide completely disorientating estimates of the precision.

(iv) The spectrum is computed with a fixed frequency step of 0.05 deg/hr, which is too coarse for long series of data. E.g. for Brussels the frequency step should be 0.0022 deg/hr, i.e. the spectrum looses 90% of the information in the residuals. Possibly, the large step 0.05 deg/hr has been chosen in order to save computer time, which was important earlier, when ETERNA has been developed.

(v) The spectrum is computed till frequencies at 4 cpd, may be again in order to save computer time. Thus it does not allow to obtain $L(f)$ values at higher frequencies, e.g. for the shallow water tides. ETERNA is not prepared for analysis of the ocean tides.

(vi) The precision of the multi-channel analysis is estimated under WN assumption only. Unlike in VAV, these coefficients are not frequency dependent.

3. Estimation of the precision by the VAV program

The main algorithm of VAV, is based on the partition of the data into N intervals $I(T)$ of equal length ΔT and central epochs $T = T_1, T_2, \ldots T_N$. As shown by Figure 2, the intervals are generally without overlapping. If there is a gap, it should remain between the corresponding neighboring $I(T)$. In the case of a gap VAV allows an overlapping of the $I(T)$ in order to avoid loosing data. When a moving filtration is used, as in the method of Chojnicki/ETERNA, every gap brings a loss of data of one length of the filter used.

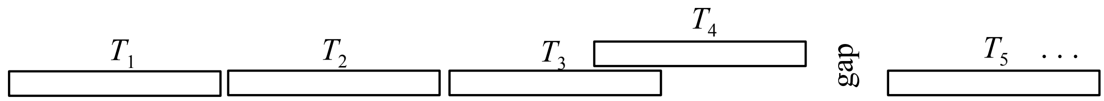

Figure 2. Scheme of the organizing of the intervals $I(T)$ used by VAV.

Figure 3 shows details of an $I(T)$.

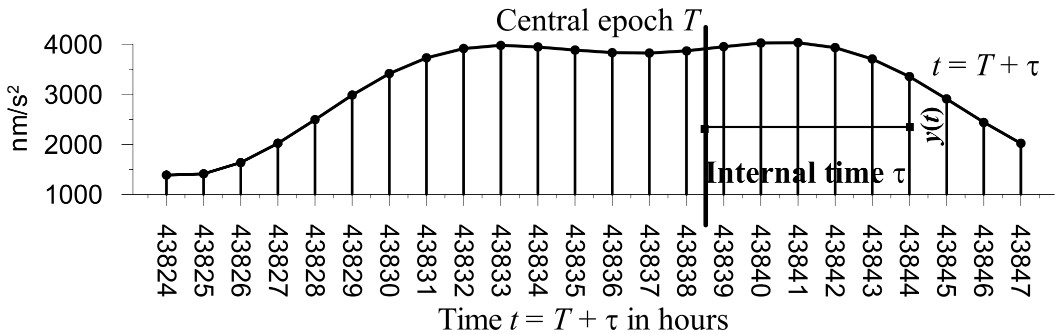

Figure 3. Example of an interval $I(T)$ with $\Delta T = 24$ h.

The drift is represented independently in all $I(T)$ by polynomials of power K_d, i.e. by polynomials having arbitrarily different coefficients in the different $I(T)$.

Our recent experiments have shown that the case of $K_d = 0$ is a very efficient one, especially for $\Delta T = 24^h$, provided the analysis includes the determination of the LP tides in parallel with the shorter D, SD, … tides (Ducarme et al., 2004). The variant $K_d = 0$ means that the drift is represented by a stepwise function, remaining a constant in every given $I(T)$, which is allowed to freely change between the neighboring $I(T)$.

In this section we shall consider only the case of $K_d = 0$ and $\Delta T = 24^h$ & 48^h , because the general case of these parameters needs sophisticated explanations.

In a first stage of VAV the hourly data $y(t)$ in every $I(T)$ are transformed through filtration into even and odd filtered numbers (u,v), as shown by (6):

$$\left(u_f(T), v_f(T)\right) = \sum_{\tau=-\theta}^{\theta} F_f(\tau) y(T+\tau), \text{ where } \theta = \left(\Delta T - 1\right)/2, \text{ i.e. } 11.5 \text{ or } 23.5 \qquad (7)$$

The filtration is made by using a set of orthogonal filters $F_f(\tau)$, which are complex functions of the internal time τ. For the case considered here

$$F_f(\tau) = \sqrt{2/\Delta T}.\text{Exp}\left(\frac{2\pi f}{24^h}\tau\right) \text{ which amplifies the frequency } f \text{ cpd} \qquad (8)$$

The filters $F_f(\tau)$ eliminate an arbitrary constant, i.e. a polynomial of power $K_d = 0$.

We have a full set of (u,v) quantities, when the (u,v) used are

$$\begin{aligned} &\text{For } \Delta T = 24: \left(u_f(T), v_f(T)\right) \text{at frequencies } f = 1, 2, \ldots \mu = 12 \text{ cpd} \\ &\text{For } \Delta T = 48: \left(u_f(T), v_f(T)\right) \text{at frequencies } f = 0.5, 1.0, 1.5, 2, \ldots 11.5, \mu = 12.0 \text{ cpd} \end{aligned} \qquad (9)$$

Due to the orthogonality of $F_f(\tau)$ the full set of (*u,v*) in (9) is a legal transformation of the hourly data *y*(*t*) from the time domain in a time/frequency domain in the following sense. Let LS is applied on *y*(*t*) with corresponding observation equations. We shall get identical results through the application of LS on (*u,v*) by using the same observation equations, modulated by the filters. The transformation remains practically legal, if some of the frequencies, at which useful signals do not exist, are omitted. E.g., if some high frequencies are omitted, i.e. if the highest frequency is $\mu < 12$ cpd.

Following this possibility, VAV applies LS on (*u,v*) as if (*u,v*) are the observations. As a result it provides the estimates of the unknowns, in which we are interested, the adjusted $(\tilde{u},\tilde{v})$ of the observed (u,v) and the residuals

$$\Delta u_f(T) = u_f(T) - \tilde{u}_f(T) \ \&\ \Delta v_f(T) = v_f(T) - \tilde{v}_f(T) \text{ for all values of } T \text{ and } f. \qquad (10)$$

If we had data with WN, all u_f & $v_f, f = 1,...\mu$ would have one and the same standard deviation. Then it would be estimated by the RMS for unit weight

$$\sigma_0 = \sqrt{\sum_{f=1}^{\mu}\left(\sum_T \Delta u_f^2(T) + \sum_T \Delta v_f^2(T)\right)\Big/\left(2N\mu - m\right)} \qquad (11)$$

where *m* is the number of the unknowns.

In fact, the data are charged by “Colored Noise” (ColN), i.e. the standard deviation of u_f & $v_f, f = 1,...\mu$ is dependent on the frequency *f*, usually increasing with decreasing *f* (red noise). In the same time we have established, that the sequence

$$\left(u_f(T_1), v_f(T_1)\right), \left(u_f(T_2), v_f(T_2)\right), ...\left(u_f(T_N), v_f(T_N)\right) \text{ for a fixed frequency } f \qquad (12)$$

can be reasonably accepted as non-correlated.

On the basis of this VAV solves the problem of the ColN by using separately the residuals $(\Delta u_f, \Delta v_f)$ for getting the RMS $\sigma_v(u,v)$, as an estimate of the standard deviation of (u_f, v_f). Namely, we use the RMS of the data at frequency *f* computed through

$$\sigma_f(u,v) = \sqrt{\left(\sum_T \Delta u_f^2(T) + \sum_T \Delta v_f^2(T)\right)\Big/(2N - m_f)} \qquad (13)$$

where m_f is the number of unknowns, whose determination is mostly depending on $\left(u_f, v_f\right)$.

4. An extreme case of a parallel application of ETERNA and VAV programs.

As said above, the quantities (*u,v*) are a legal transformation of the data *y*(*t*). If this is neglected and (*u,v*) are examined at only one fixed frequency *f*, it seems that VAV uses a decimation of the data with a time step ΔT. Thus for $\Delta T = 48^{\text{h}}$, earlier used by MV66 (Venedikov, 1966), the Nyquist frequency becomes 0.25 cpd, which is an extremely large frequency. Then, if e.g. the tide O1 with frequency ν(O1) is considered, the waves

$$\begin{aligned} &\text{A1 with frequency } f(\text{O1}) - 0.5 \text{ cpd} \\ \&\ &\text{A2 with frequency } f(\text{O1}) + 0.5 \text{ cpd} \end{aligned} \qquad (14)$$

are aliases of O1 which cannot be separated from O1 and which may affect the results for O1.

Under such assumptions, Schüller (1978) has shown that if the waves A1 and A2 are added to a series of data the MV66 results for O1 are strongly affected.

It has been shown (Venedikov, 1979) that A1 and A2 can be separated from O1 and thus they are not aliases of O1. Nevertheless, on the basis of the result of Schüller, Wenzel (1997) has declared that MV66 (and thus VAV now) "violates the sampling theorem".
It is strange that this severe critic has been made on the basis of the effect of A1 & A2 on MV66 only, but it has not been shown, what is the effect of the same A1 & A2 on other programs, in particular on ETERNA.

In order to avoid confusions we have made a similar experiment, somewhat extended (Table 2), but this time testing both VAV and ETERNA.

Table 2. Variants of testing ETERNA and VAV. The amplitudes of all waves are 38 μgal.

<table>
<tr><td colspan="2">Variant 0: Original SG data Brussels without added waves</td></tr>
<tr><td colspan="2">Variant 1: SG data Brussels 1987 – 2000 with added A waves:</td></tr>
<tr><td>A1 with frequency f(O1) – 0.5 cpd</td><td rowspan="2">The "aliases" used by Schüller</td></tr>
<tr><td>A2 with frequency f(O1) + 0.5 cpd</td></tr>
<tr><td>A3 with frequency f(O1) – 0.25 cpd</td><td rowspan="2">Two more waves, related with the supposed Nyquist frequency</td></tr>
<tr><td>A4 with frequency f(O1) + 0.25 cpd</td></tr>
<tr><td colspan="2">Variant 2: SG data Brussels 1982 – 2000 with added A waves:</td></tr>
<tr><td>A1 with frequency f(O1) – 0.5 cpd</td><td rowspan="2">The "aliases" used by of Schüller</td></tr>
<tr><td>A2 with frequency f(O1) + 0.5 cpd</td></tr>
<tr><td>A3 with frequency f(O1) + 1.0 cpd</td><td rowspan="2">Two more "aliases"</td></tr>
<tr><td>A4 with frequency f(O1) + 1.5 cpd</td></tr>
<tr><td>A5 with period 7 hours</td><td>A wave, suggested by W. Zürn</td></tr>
</table>

VAV has been used with $K_d = 0$ & $\Delta T = 48^{\mathrm{h}}$ and nearly full set of (u,v) in (9) with highest frequency $\mu = 11.5$ cpd.

Let δ_0 is the amplitude factor of a given tide, obtained by the analysis of the data in variant 0 (without A waves) and δ_A is the amplitude factor obtained by the analysis of the data, after the waves A are added. Then the effect of the waves A on the amplitude is naturally measured by the difference $\Delta = |\delta_A - \delta_0|$.

The result of the comparison is shown by Figure 4.

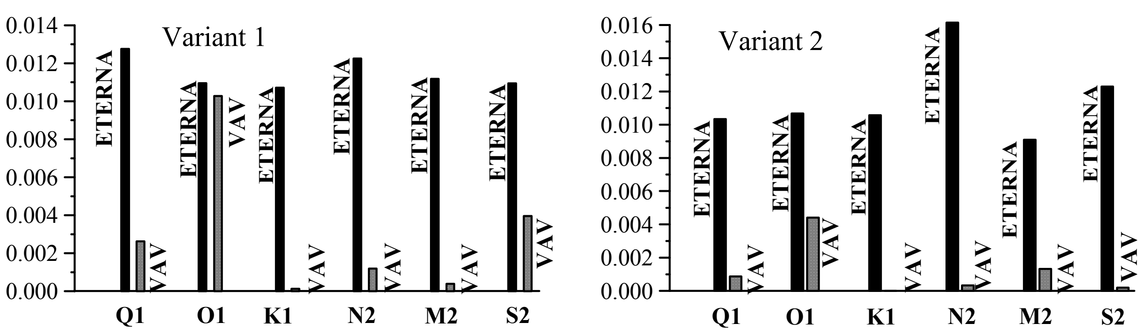

Figure 4. The effect $\Delta = |\delta_A - \delta_0|$ of the added waves A, variants 1 & 2.

Surprisingly or not, the effect of the A waves is, first, considerably higher for ETERNA and, second, it is expanded on all D and SD waves. There is a “third” item, also not in favor of ETERNA, concerning the effect of the A waves on the estimates of the precision.

Figure 5 shows that the effect of the A waves, variant 1, is the addition of a noise, completely deforming the tidal curve. Same is the situation with the data, obtained in variant 2. Obviously, a reasonable result should show a considerably lower precision, i.e. considerable higher RMS, when the data in variant 1 and 2 are analyzed.

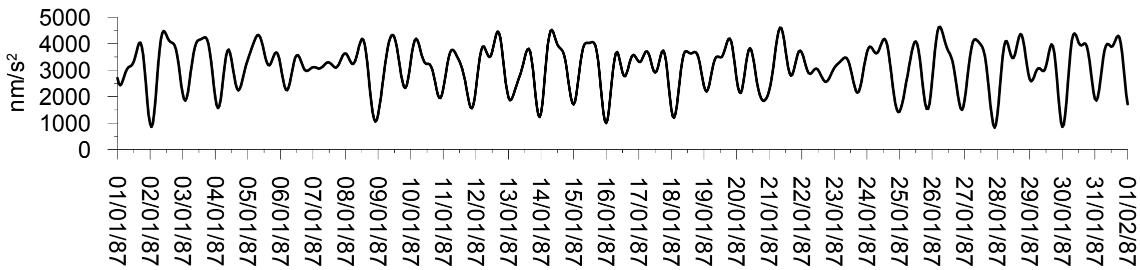

Figure 5. Sample of the data after adding the A waves, variant 1.

Indeed, as shown by Tables 3 and 4, the VAV analyses show considerably lower precision after adding the A waves. E.g. the RMS of δ(M2) in Table 3 is increased from ±.00005 to ±.00619, i.e. more than 120 times.

Table 3. RMS σ of the δ factor of the main D & SD tides of the original data (variant 0) and the data with added A waves (variant 1 in Table 2); SG data Brussels 1987 – 2000.

	Q1	O1	K1	N2	M2	S2
VAV, RMS σ_{VAV}						
Variant 0, No added waves	±.00043	±.00008	±.00006	±.00025	±.00005	±.00011
Variant 1 of added waves	±.03727	±.00713	±.00505	±.03214	±.00619	±.01374
ETERNA, RMS σ_{ET}						
Variant 0, no added waves	±.00037	±.00007	±.00005	±.00022	±.00004	±.00009
Variant 1 of added waves	±.00147	±.00029	±.00021	±.00022	±.00004	±.00009

Table 4. RMS σ of the δ factor of the main D & SD tides of the original data (variant 0) and the data with added A waves (variant 2 in Table 2); SG data Brussels 1982 – 2000.

	Q1	O1	K1	N2	M2	S2
VAV, RMS σ_{VAV}						
Variant 0, No added waves	±.00040	±.00008	±.00005	±.00023	±.00004	±.00010
Variant 2 of added waves	±.01086	±.00207	±.00148	±.00819	±.00157	±.00346
ETERNA, RMS σ_{ET}						
Variant 0, No added waves	±.00035	±.00007	±.00005	±.00022	±.00004	±.00009
Variant 2 of added waves	±.00041	±.00008	±.00006	±.01168	±.00226	±.00493

ETERNA in Table 3 shows only a weak increase of the RMS for the D tides, but absolutely NO changes in the RMS for the SD tides. In Table 4 ETERNA shows more important increase of the RMS for the SD tides, but almost no changes in the RMS of the D tides.

Thus generally, with the exception of the SD case in Table 4, ETERNA failed to notice the huge noise, added to the data.

The explanation of the anomalous estimates of the precision by ETERNA is the dependence of these estimates on the intervals in Figure 1 and Table 1. In the case their frequencies fall out of the intervals, their effect remains inappreciable. This is an exaggerated case but, anyway, it shows the dependence of the estimates of ETERNA on the choice of the intervals.

An attempt to escape from the situation has been made by returning to the option of ETERNA (Table 5) based on the assumption of WN, although this is a non-standard and non-recommended procedure for the D and SD tides. In such a way we have got indeed a considerable increase of the RMS for all tides, i.e. the strong noise became visible. However, the new RMS of ETERNA are still considerably different from VAV and they are strongly dependent on the filters used, which is somewhat strange.

Table 5. An attempt to find more reasonable estimates of the precision by applying ETERNA with the non-standard procedure under the assumption for WN, for variant 2 in Table 2.

	Q1	O1	K1	N2	M2	S2
ETERNA, Assumption of white noise, filter N60M60M2, 167 coefficients						
Variant 2 of added waves	±.03002	±.00633	±.00468	±.03239	±.00633	±.01350
ETERNA, Assumption of white noise, filter PERTSEV59, 51 coefficients						
Variant 2 of added waves	±.02603	±.00508	±.00358	±.02606	±.00504	±.01101

There are very few comparative analyses by ETERNA and VAV. Many of them (but not all of them) show a numerical agreement between σ_{VAV} and σ_{ET}, σ_{ET} always being lower than σ_{VAV} by some 10 to 20%. Such is the case of the RMS for the unperturbed data in Tables 3 & 4. The closeness, which appeared in most cases, should not be considered as universal and thus as a justification of the estimates of ETERNA. The results with the added waves are cases when the similitude completely disappears.

Something more! Let us accept that the similitude of $\sigma_{VAV}(Q1) = 0.00043$ and $\sigma_{ET}(Q1) = 0.00037$ means that $\sigma_{ET}(Q1) = 0.00037$ is a reasonable RMS. Then, since $\sigma_{ET}(Q1) < \sigma_{VAV}(Q1)$, we have also to accept that Q1 is better determined by ETERNA. Actually, since $\sigma_{ET}(Q1)$ is not an LS estimate, from the comparison of σ_{ET} and σ_{VAV} cannot be made anyone of the conclusions: neither “Q1 is better determined by ETERNA”, nor “Q1 is better determined by VAV”. Same is the situation if we had $\sigma_{VAV}(Q1) < \sigma_{ET}(Q1)$. The reality is that due to the uncertainty on $\sigma_{ET}(Q1)$ we do not know how well Q1 is estimated.

5. A new approach by the variant 2006 of the VAV program

The algorithm, described in Section 3 is used till now and is implemented in (Venedikov et al., 2004). Now we have introduced a further development of this algorithm, described hereafter.

In a discussion Dr. W. Zürn made the reasonable critical remark that VAV implies the assumption that the filtered numbers have a normal Gaussian distribution (ND) and this is actually not proven.

Indeed, the technique considered above is based on the assumption that the set of data $u_f(T), v_f(T), T = T_1, T_2, \ldots T_N$ at given frequency f has a ND. In this connection VAV has included a test of normality, based on the χ^2 criterion of Pearson.

VAV builds the observed or empirical distribution of the residuals $\Delta u_f(T), \Delta v_f(T), T = T_1, T_2, ...T_N$ for everyone of the frequencies f and the corresponding theoretical N(0, σ), i.e. central ND with standard deviation σ. The difference between the distributions is measured by the observed χ^2_{Obs} quantity. In the examples here the distributions are determined over 301 intervals with equal expected probability, so that χ^2_{Obs} has 300 degrees of freedom. The value of σ in N(0, σ) is found by making $\chi^2_{\mathrm{Obs}} = \mathrm{Min}$. The critical $\chi^2_{\mathrm{Crit}} = 342.5$ of Pearson for testing the hypothesis for ND is corresponding to confidence probability 95%. The examples are obtained by the processing of the SG data Vienna 01.07.1997 – 31.12.2002

Figure 6A shows the distribution of the (*u,v*) at $f = 1$ cpd. In practice, such pictures are often accepted as a satisfactory approximation to a ND. However, according to χ^2_{Crit}, we have NOT an ND, because χ^2_{Obs} exceeds considerably χ^2_{Crit}.

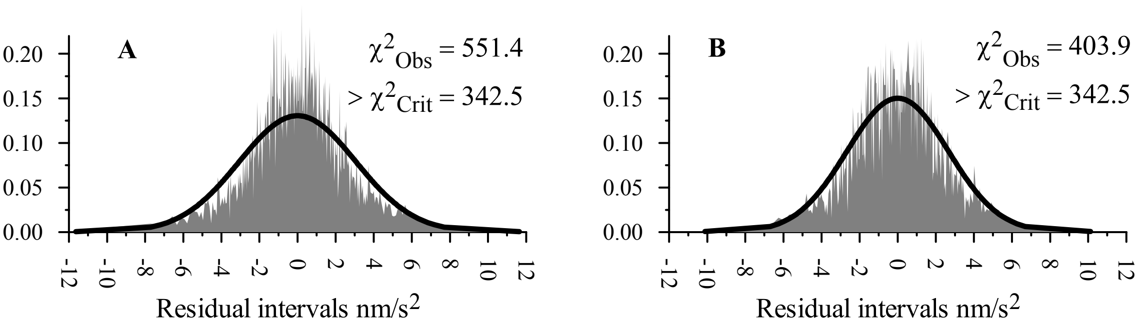

Figure 6. Observed density distribution of the residuals at $f = 1$ cpd (filed grey curve) and the corresponding ND (thick line). A: all residuals; B: eliminated large residuals (7.5% data).

Such is the case of most of the SG data we have tested. It appeared that one of the reasons for the deviations from the ND is that there is proportionally too many large residuals. Indeed, as shown by Figure 6B, when the option (Venedikov et al., 2004) of automatic elimination of data with big residual is applied, the difference between χ^2_{Obs} and χ^2_{Crit} is considerably reduced, although strictly considered, the inference is that the distribution remains not ND.

We came considerably closer to ND in the following new way of applying VAV.

VAV is run in two general loops A & B. In A VAV follows Section 3. In addition, it provides a separate determination of the RMS of u_f & v_f, instead of the common RMS (13). This is made through

$$\sigma_f(u) = \sqrt{\sum_T \Delta u_f^2(T) \Big/ \left(N - m_f\right)} \quad \& \quad \sigma_f(v) = \sqrt{\sum_T \Delta v_f^2(T) \Big/ \left(N - m_f\right)} \ . \qquad (15)$$

The idea is to use these RMS in order to apply weights to the filtered (*u,v*) and thus make them all with (i) nearly normal distribution and (ii) nearly equal variances.

By accepting that a quantity with unit weight should have an RMS equal to the RMS for the WN σ_0, obtained by (11), the weights to be applied become

$$\text{for } u_f : (\sigma_0 / \sigma_f(u))^2 \text{ and for } v_f : (\sigma_0 / \sigma_f(v))^2 \qquad (16)$$

Practically, these weights are introduced through the replacement of the filtered numbers $u_f(T)$ & $v_f(T)$ by what we call weighed filtered numbers

$$\bar{u}_f(T) = \sigma_0 . u_f(T) / \sigma_f(u) \;\; \& \;\; \bar{v}_f(T) = \sigma_0 . v_f(T) / \sigma_f(v) \qquad (17)$$

Theoretically, for perfectly well determined RMS, the weighted quantities (17) become observations with equal precision and RMS approximately equal to σ_0.

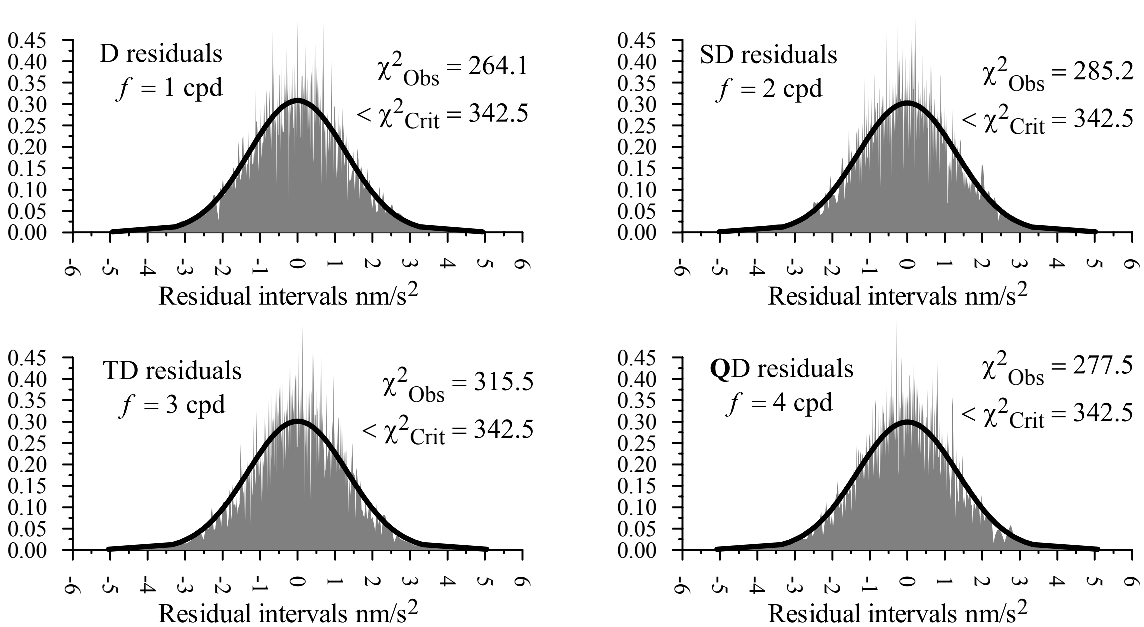

Figure 7 Observed density distribution of the residuals WITH APPLIED WEIGHTS and eliminated large residuals (6.2% of all data) with the corresponding ND.

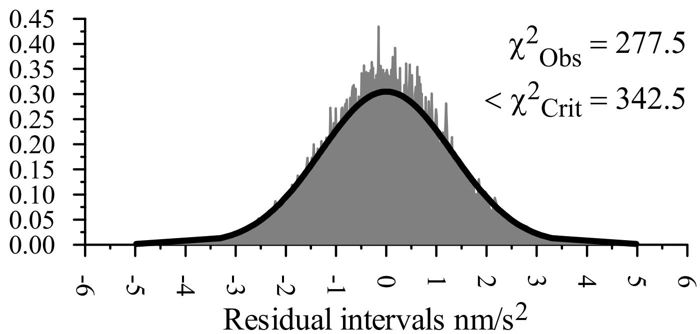

Figure 8 Distribution of the residuals $(\Delta\bar{u}_\nu, \Delta\bar{v}_\nu)$ for all $\nu = 1,2,3$ & 4 cpd WITH APPLIED WEIGHTS and eliminated large residuals (6.2% of all data).

Figure 7 shows the effect of the weights on the distribution of the residuals of the weighed quantities, separately for the main frequencies. In all cases we get a rather acceptable approximation to ND with χ^2_{Obs} clearly under the χ^2_{Crit}.

Very important is that the approximation of the ND of the residuals of all main frequencies together, as shown by Figure 8 is also satisfactory.

Unfortunately, the positive results in Figures 7 & 8 are obtained by applying an elimination of a considerable quantity of data (6.2%) with too high residuals. One of the sources of these big residuals may be due to the preprocessing of the data. ETERNA is based on a moving filtration of the data, which does not support a great number or gaps. Due to this the authors of the data are obliged to keep in the data obviously anomalous data, as well as to apply massive data reparations and interpolations, which operations introduce noise of very bad properties.

Conclusions.

We hope that this paper can attract the attention of the tidal community to the importance of the estimation of the precision. Actually, the result about a parameter x has to be represented by two numbers: the value of x and its RMS. The value without the RMS is as useless, as the RMS without the value. No need to say, that the RMS has to be determined according to the rules of the mathematical statistics, i.e. according to the LS rules. Otherwise, an incorrect RMS is equivalent to or worse than a missing RMS.

The analysis of the tidal data has to solve a difficult problem. In principle, now is firmly accepted, that the LS is perfectly suitable general method. The problem is that the data are correlated, i.e. they are charged by a colored noise. In the same time we do not dispose a priori by the covariance matrix, in order to correctly apply the MLS. The compromise, accepted by both ETERNA and VAV is to estimate the unknowns by creating the observation equations under the condition of WN. This does not affect seriously the values of the unknowns, because we always deal with very great number of data.

In the same time, since the colored noise has a frequency dependent effect, the colored character of the noise is taken into account through the estimation of the precision by frequency dependent RMS. Nevertheless, the RMS should observe the general LS principle to be determined through sums of squares of residuals, in order to be able to apply fundamental statistical tools, like the confidence intervals of Student.

It has been demonstrated that the RMS of ETERNA are, unfortunately, not LS estimates and they depend on some intuitive assumptions. Something more, there are cases in which these RMS can be completely disorientating. In addition to the extreme case, considered in Section 3, such is the doubtful case of the RMS of the parameters of the auxiliary channels in the multi-channel analysis. The case of LP waves is even more complicated and is treated in a separate paper (Ducarme et al. 2006c).

The paper has discussed the estimates of the precision, proposed by VAV. They are certainly not the ideal solution but, at least, they follow the LS principle to use the squares of the residuals. In the same time, as shown in Section 5, we have found a weak element in the RMS estimation, applied until now. Namely, it appeared that the distribution of the residuals, rigorously checked, deviates from the ND of Gauss. We have not hesitated to introduce corresponding improvement in VAV, which has brought the residuals nearly to the ND. In addition, it seems that the solution of the problem will generally improve the precision of the analysis results.

We believe that the decision to immediately modify VAV when a weak element is found is a good example of a scientific approach. In our opinion the researchers, tightly devoted to the use of ETERNA, should follow this example and critically reconsider the method of preprocessing and analysis they are currently using, especially in its options for the estimation of the precision.

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