An overview on wavelet multi-resolution decomposition compared with traditional frequency domain filtering for continuous gravity data denoising

Panepinto Stefano[1], Greco Filippo[2], Luzio Dario[3], Ducarme Bernard[1]

[1]*Royal Observatory of Belgium, Brussels, Belgium,* [2]*Istituto Nazionale di Geofisica e Vulcanologia Sezione di Catania, Italy,* [3]*Dipartimento di chimica e Fisica della Terra, University of Palermo, Italy*

Abstract: Continuous gravity recordings in volcanic area could play a fundamental role in the monitoring of active volcanoes and in the prediction of eruptive events too. This geophysical methodology is utilized, on active volcanoes, in order to detect mass changes linked to magma transfer processes and, thus, to recognize forerunners to paroxysmal volcanic events. Spring gravimeters are still the most utilized instruments for microgravity studies because of their relatively low cost and small size, which make them easy to transport and install. Continuous gravity measurements are now increasingly performed at sites very close to active craters, where there is the greatest opportunity to detect significant gravity changes due to a volcanic activity. Unfortunately, spring gravity meters show a strong influence of meteorological parameters (i.e. pressure, temperature and humidity), especially in the adverse environmental conditions usually encountered at such places. As the gravity changes due to the volcanic activity are very small compared to other geophysical or instrumental effects we need a new mathematical tool to get reliable gravity residuals susceptible to reflect the volcanic effect. In the following we present and discuss a preliminary work about the confrontation between the traditional filtering methodology and the Wavelet transform. The overall results show that the performance of the wavelet-based filter seems better than the Fourier one. Moreover, the possibility of getting a multi-resolution analysis and study local features of the signal in the time domain makes the proposed methodology a valuable tool for gravity data processing.

Keywords: *Gravimeter, wavelet transform, filtering, volcanic monitoring*

INTRODUCTION

Studies on the effect of external influencing parameters on the behavior of gravity sensors for the monitoring of volcanic activity have revealed that temperature, pressure and humidity play a very critical role [El Wahabi et al., 1997; Warburton and Goodkind, 1997]. Several authors have demonstrated that meteorological parameters, especially atmospheric temperature, do affect continuously running spring gravimeters. In particular, in El Wahabi et al. (1997) it has been proven that, over a yearly period, temperature changes can cause up to 1 mGal instrumental effect. An admittance up to 0.2 mGal/°C, over changes with period longer than 1 month, has been evidenced in Carbone et al (2003). An important issue with the gravity data sequences is the need of separating the useful signal (i.e. the volcano-related one) from unwanted components (signal due to tides, instrumental and meteorological effects). Obviously, frequency-domain filters cannot be applied to remove the effect of these perturbations since the spectrum of each component of various origins has wide intervals of superposition.

The aim of the present work is to get a preliminary test concerning the application of the wavelet decomposition to the continuous gravity data. The wavelet transform is thus applied to a short sequence (7 months registration) acquired at Mt Etna site (see Figure 1). The continuously running station is located about 2 km NE of the summit NE crater at the "Pizzi Deneri Volcanological Observatory" (PDN; 2820 m a.s.l.). Data presented and analyzed in the following were acquired through L&R PET 1081 gravimeter with a measurement range of about 5 mGal. Besides gravity, other parameters were acquired: ground tilt in two perpendicular directions, atmospheric temperature, pressure, humidity and tension from the power system feeding the station. Data were recorded at 1datum/min sample rate (each datum is the average calculated over 60 measurements)

through a CR10X Campbell Scientific data-logger and transmitted through a wireless connection to Catania.
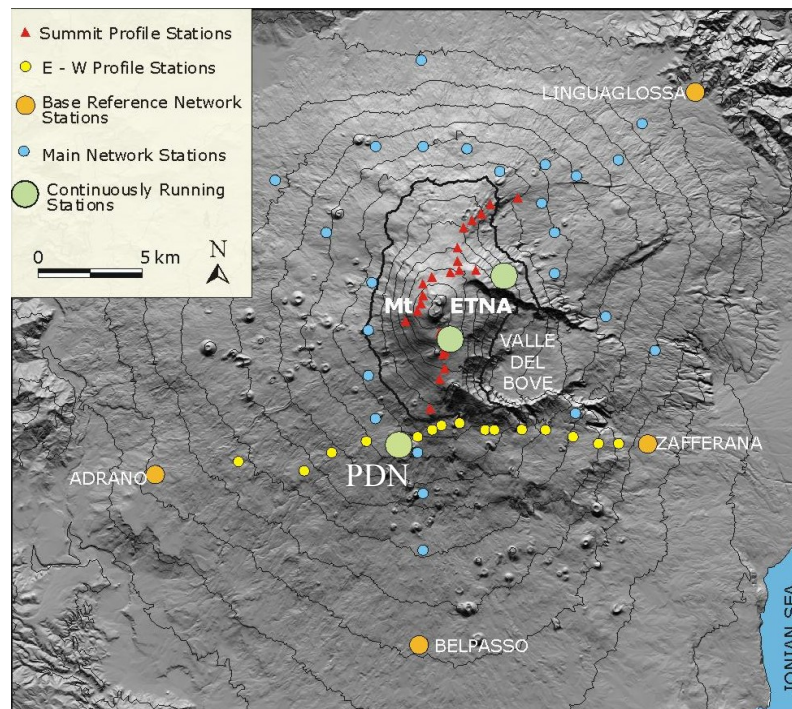
Fig. 1: Sketch map of Mt Etna showing the gravity network installed by INGV

DISCRETE WAVELET TRASFORM AND MULTI-RESOLUTION ANALYSIS

Wavelets are mathematical tools for analyzing time series. As the name suggests, a wavelet is a "small wave". A small wave grows and decays essentially in a limited time period. There are two main classes of wavelets. The first one is the continuous wavelet transform (CWT) which is designed to work with time series defined over the entire real axis; the second, is the discrete wavelet transform (DWT), which deals with series defined over a range of integer values [Percival et al., 2002].
The continuous wavelet transform of a function $f(t)$ with finite energy is defined by

$$\widetilde{W}(a,b) = \frac{1}{\sqrt{a}} \int f(t) \psi\left(\frac{t-b}{a}\right) dt \tag{1}$$

where $b \in$ and $a \in R^+$, with R^+ being the set of positive real numbers. Every wavelet is obtained by a translation and dilation of a 'mother wavelet' $\psi \equiv {}^2(R)$, where $L^2(R)$ denotes the Hilbert space of square integrable functions. The function ψ has unit energy and zero mean although higher order moments may also be zero; it is chosen with a compact support to obtain localization in space.
For practical applications the scale parameter a and location parameter b need to be discretized. With the choice $a = 2^l$ and $b = ka$ with $l,k \in$ (Z is the set of integers), the discrete family of orthonormal wavelets,

$$\psi_{l,k}(t) = \frac{1}{\sqrt{2^l}} \psi\left(\frac{ - 2^l}{2^l}\right) = \frac{1}{\sqrt{2^l}} \psi\left(t - \right) \tag{2}$$

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is obtained by dilating or contracting and translating the function $\psi\ ') \equiv \gamma_{,0}(t)$.

In this case, the discrete wavelet transform (DWT) is

$$\widetilde{w}_{l,k} = \int\limits_{-\infty}^{\infty} f(t)\,\psi_{,,\dot{t}}(t)\,dt \tag{3}$$

and the inverse wavelet transform is

$$f(t) = \sum_{l=\infty}^{+\infty}\sum_{k=\infty}^{+\infty}\widetilde{w}_{l,k}\psi_{,,}(t) \tag{4}$$

in the L^2-sense.

The framework of multi-resolution analysis, developed by Mallat (1992) and Meyer (1993), allows a signal to be approximated at certain scales or resolutions (Foufoula-Georgiou and Kumar, 1994). At a fixed level l of resolution 2^{-l} samples per unit length, the approximation of the signal is given by its orthogonal projection, $S_l f = \sum\limits_{k=\infty}^{\infty} f,\phi_{,,}\rangle\phi_{,,}(t)^1$, onto the vector space V_l included in $L^2(R)$. The vector space V_l represents the space of all possible approximated signals at resolution 2^{-l}, and the orthonormal basis $\phi_{,,}$ derives from a function ϕ defined as the scale function or 'father wavelet'. The details of the signal at any scale 2^l characterize the difference between the process carried out at two different scales 2^l and 2^{l-1}, or equivalently at two different resolutions. If $\psi_{,,,}$ is the orthonormal basis in the vector space O_l (orthogonal complement of V_l in V_{l-1}), the details are given by the orthogonal projection, $D_l f = \sum\limits_{k=\infty}^{\infty} f,\psi_{,,}\rangle\psi_{,,}(t)$, of the original signal on to the vector space O_l, so O_l is orthogonal to V_l, i.e.

$$V_{l-} = ?_l \oplus\ _{,\cdot}. \tag{5}$$

If $f(t)$ is defined by a set of m discrete values, the wavelet expansion consists of the coefficients,

$$\widetilde{w}_{l,k}\ \text{with}\ l \in\ _{,-}(-\ ?)_{,-}^{-};\ k \in\ ),2^{|l|}-\ ], \tag{6}$$

where $L = \log_2 m$ (we pick values that are separated by multiples of 2^l).

Using this notation, the finest and the coarsest levels correspond to $l = -(L-1)$ and $l=0$, respectively.

At a fixed level $j,$ the function $f(t)$ is represented by

$$f(t) = S_j f(t) + \sum_{l=(L-1)}^{j} D_l f(t). \tag{7}$$

A discrete signal $f(t)$ can thus be expressed as a wavelet expansion by orthonormal basis

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[1] $\langle f,\phi_{,,,}\rangle$ is the inner product: $\langle f,\phi_{l,k}\rangle = \int\limits_{-\infty}^{\infty} f(t)\phi_{l,k}(t)\,dt$

functions derived from the two related functions: scaling function ϕ) and the wavelet ψ '). The above expansion (*Eq.* 7) means that scaling and translating ϕ) and ψ ') leads to an orthogonal basis for the analysis of signal. In such way *f(t)* is decomposed into a smoothed version $S_j f(t)$ obtained by applying the low-pass filter ϕ and a sum of the details (D_j) obtained by the high-pass filter ψ.

The relationship between the representation of *f(t)* at level *j* and that at level (*j* - 1) is summarized by

$$S_{j-} f(t) = S_j f(t) + D_j f(t). \tag{8}$$

Figure 2 shows the recursive scheme, usually called a pyramid algorithm, used to calculate the wavelet coefficients from the finest level to a chosen level *j*.

The multi-resolution analysis is obtained by applying the inverse wavelet transform to the coefficients of each level.
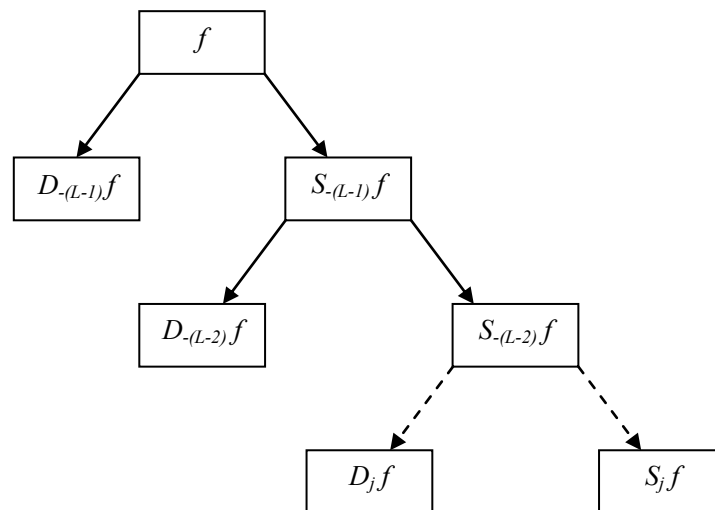
Fig. 2: Recursive scheme used to calculate the wavelet coefficients of a function *f* from the finest level -*L*+1 to a chosen level *j*

The advantage of analysing a signal with wavelets is that it enables local features of the signal to be studied with a detail matching their characteristic scale. In the temporal domain, such a property allows transient signals to be represented effectively [M.T. Carrozzo et al., 2002]. Thus it can be said that the DWT enables a *multi-resolution analysis* of a signal to be made.

CHOICE OF THE WAVELET BASIS

Different wavelet basis are obtained by varying the support width of the wavelet. Changes in the wavelet support in general affect the final frequency characteristics of the wavelet transform. Usually the amplitudes of the coefficients change and, consequently, the scale where the signal and noise separate also changes.

The choice of a wavelet basis could represent an open problem for filtering. Since it was impossible to test the filter effectiveness with statistical significance for all the wavelets available in the Matlab® software package, some criteria were used to make a preliminary selection of a set of wavelet bases.

Deighan and Watts (1997) and Grubb and Walden (1997) considered the compactness of the wavelet functions in the time and frequency domains as a useful parameter for choosing the best wavelet basis to solve a specific problem.

A criterion to evaluate the suitability of a wavelet of a specific shape to represent the signal is to determine how concise and localized its wavelet representation is.

The "maximum compactness" or "minimum entropy" criterion, which Coifman and Wickerhauser (1993) and Fedi and Quarta (1998) applied for data compression and data analysis, respectively, was used here in an attempt to optimize the choice of the wavelet basis.

In particular, this criterion was applied to the sequence available for this study, which spans from June to December 2005 and was acquired at one of the Etna's continuous gravity stations (PDN). To find the most appropriated support for our case the transform is repeated using different basis and Shannon's entropy of the analysed gravity sequence was then calculated. As underlined by the test carried out over 39 wavelet basis, it seems that the range of relative variability of the values calculated is more restricted only for a narrow number of wavelet basis and leads to the selection of a set of six wavelets: Symmlet 8, Bior 6.8, Daubechies 5-7-9, Rbio 2.8. The averages of the entropy values are shown in Table 1 for each tested basis of this small group.

Tab. 1: Table of the executed test showing the averages of the entropy values for each tested wavelet basis

Wavelet basis	Entropy
Symmlet 8	4.02
Bior 6.8	4.13
Daubechies 5	4.21
Rbio 2.8	4.22
Daubechies 7	4.27
Daubechies 9	4.37

DATA PROCESSING, COMPARATIVE TECHNIQUE AND RESULTS

Previous to the filtering analysis of the sequence at hand, acquired by LaCoste & Romberg PET 1081 at Mt Etna site, its spikes, gaps steps and large amplitude oscillations due to earthquakes events are corrected using T-soft, a graphical and interactive software developed by Van Camp and Vauterin (2005).

The signal from any continuous running gravity meter is affected by the tidal acceleration (Torge, 1989). In order to remove the Earth tides effect, the raw data are decimated to 1-h ordinates for a classical tidal analysis using the ETERNA3.4 software (ANALYZE, Wenzel, 1994). A RMS error on unit weight of 12 nm/s^2 was obtained, implying tidal residuals on the gravity signal up to 1-2 µGal peak-to-peak over the most relevant tidal families (diurnal and semidiurnal).

Continuously recording spring gravimeters are affected by aperiodic temporal variations in the display of the zero position called instrumental drift (Torge, 1989). To correct the data for the main effect of instrumental drift a best linear fit was removed from the sequence.

The gravity signal perturbations, due to the local pressure changes consist of two main effects: the upwards attraction of the air masses on the sensor of gravimeter, and the loading effect, due to the vertical displacement of the ground and the associated changes of potential. The linear pressure correction often leads to an obvious decrease of the RMS error of tidal gravity factors. Global pressure changes may also make contributions to the annual gravity signals, but these effects are difficult to model [Richter et al., 2004]. Moreover, the instrumental reactions to meteorological effects are very complex and can trigger large annual variations in the drift.

Through the optimization of an automatic one-dimensional algorithm, that enables the wavelet decomposition to be made, a multi-resolution analysis of the signal is obtained in Matlab[®] development environment.
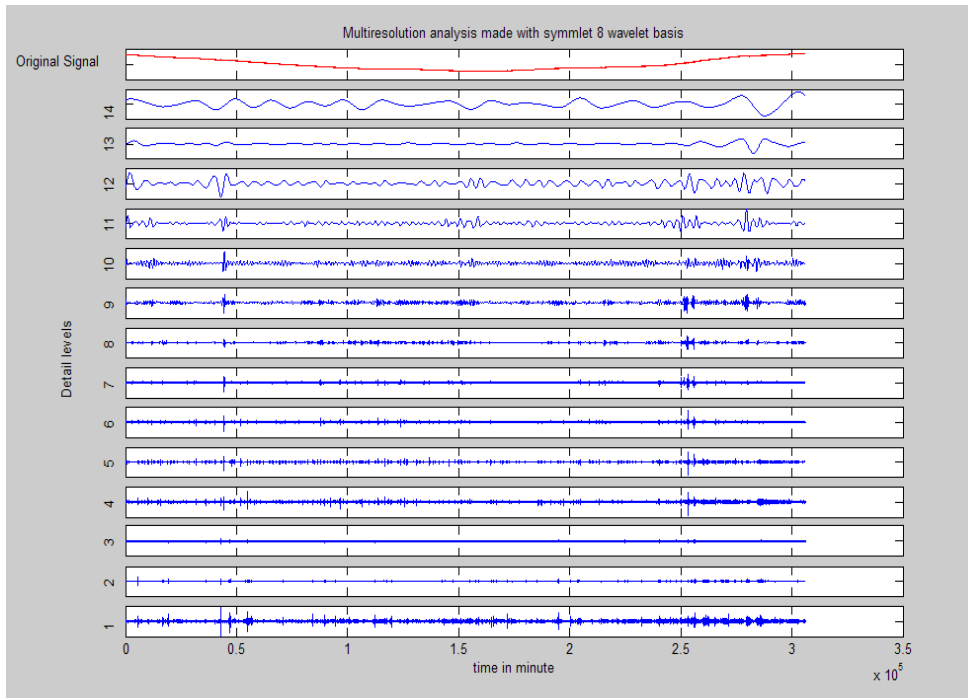
Fig. 3: The picture shows the multi-resolution analysis of the gravity signal made with Sym8 wavelet basis. The input signal is displayed at the top (red one).

Figure 3 shows an example of multi-resolution analysis of the continuous gravity sequence under discussion after the elimination of the tide and linear drift. It is performed using the wavelet basis Symmlet 8 (Daubechies, 1992). Since the gravity sequence consist of 308160 samples and the length of Symmlet 8 basis is 16, using the general criterion:

$$lev = (log(lx/(lw-1))/log(2))$$

where:
lev is the maximum allowed level decomposition;
*lx* is the length of the signal;
lw is the length of the wavelet basis;
the multi-resolution analysis contain 14 scales each containing 308160 samples. In such way the first scale ($l = 1$) has 16384 coefficients, the next ($l = 2$) contains 8192 coefficients and so on…; this adds up to 308160 numbers including the mean value of f (t). The original signal is thus obtained (as showed in the pyramid algorithm) with a summation of all the detail levels plus the approximation level (S_j) that is the long period content of the signal.

The multi-resolution analysis made with both detail and approximation levels allow local features of the signal to be studied in time domain and transient signals to be represented effectively. This is an important point since the continuous gravity signal could include some volcanic effects, which in time span from minute to years. The only way to check them is the correlation, in the time domain, with other geophysical or geochemical signal. On the other hand the approximation levels representation of the signal enables the separation of the long period component from the high frequency ones. In order to get a confrontation with the classical filtering method, the same component is thus obtained with both Chebyshev and Butterworth frequency filters with different cut off frequencies and polynomial orders. The figure 4 is an over-plot between the original signal and the two filtering methods. The wavelet filter is the only one fitting perfectly the general behaviour of the curve.

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a)

b)

c)
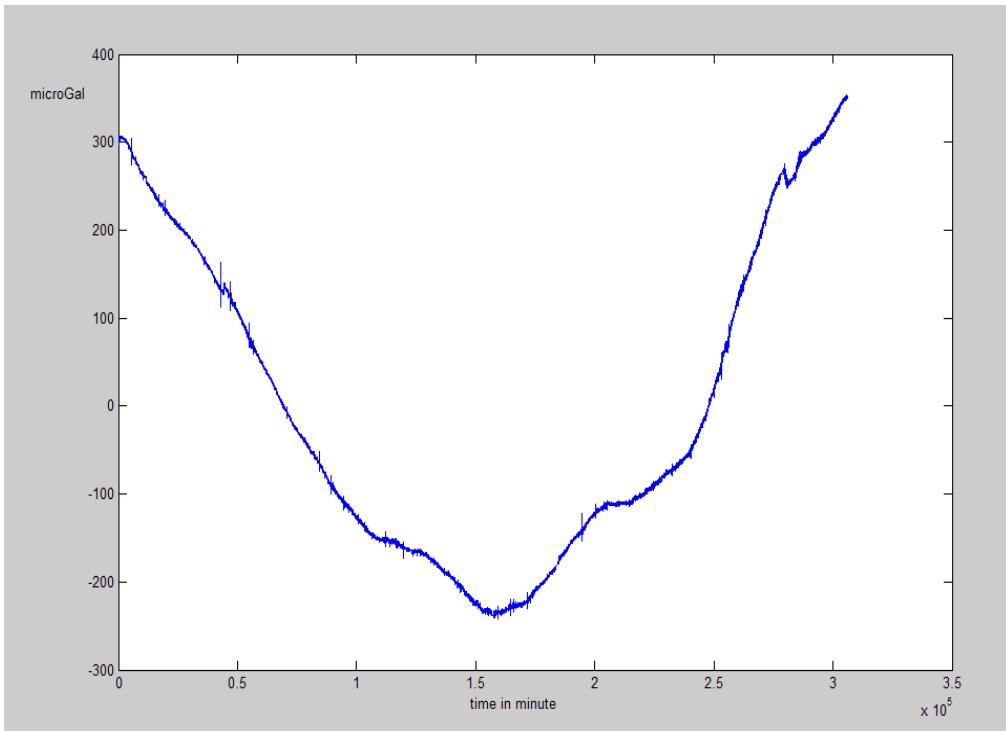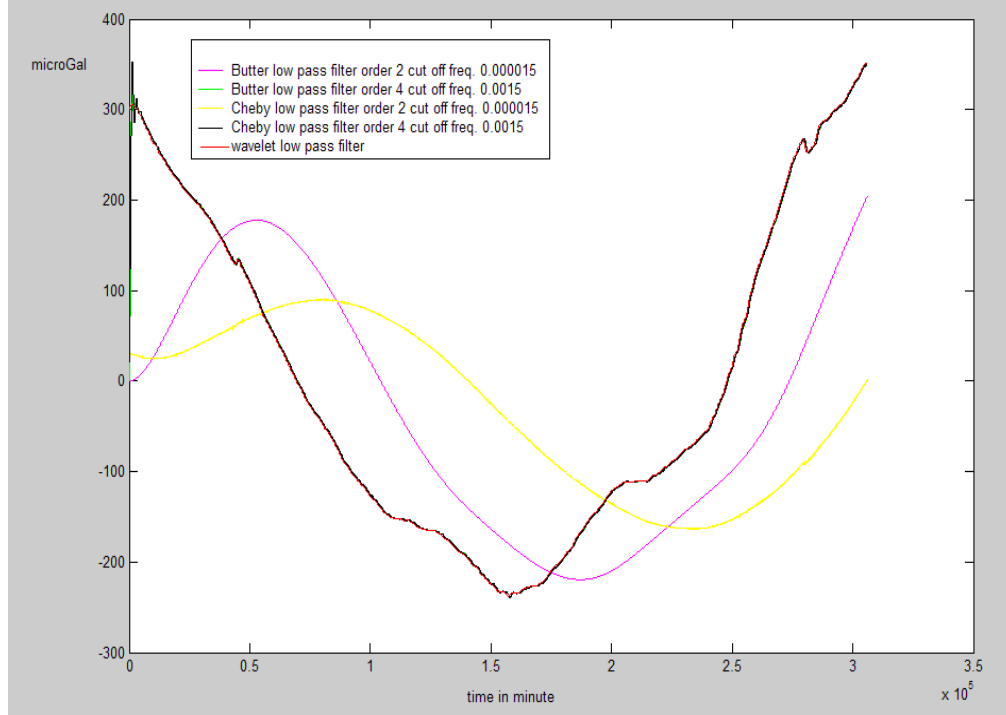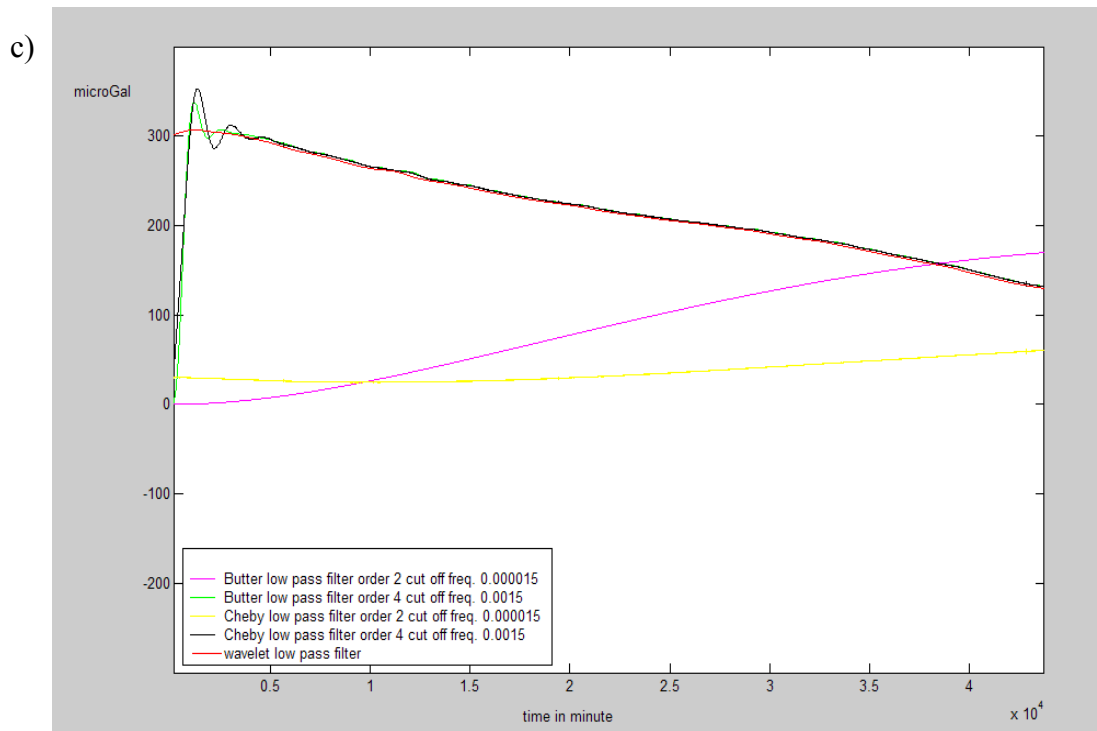
Fig. 4: Confrontation between wavelet filter and two types of filters with different order and cut off frequencies: a) on the top is displayed the original signal; b) global view of the different filtering method; c) zoom showing the edge effects;.

It seems that frequency domain filtering does not always work well because:
- Depending on both cut off frequency and filter order it also can introduce edge effects and distortions of the original signal;
- The frequency filters often leave some high frequency in the low pass filter, if we want to get a separation of the long period waves from the short ones;
- It globally removes frequencies causing a generalized smoothing effect that substantially broadens features of interest.

The long period signal obtained with the low pass filter performed by wavelet transform is more smoothed and without any significant edge effects. The same technique is then applied to the pressure, temperature and humidity signals, acquired at the same station, and the correlation analysis between the corresponding low frequencies component of the signals shows a strong anticorrelation with temperature (-0.7735) while the correlation with pressure and humidity are -0.5209 and -0.0896 respectively. Moreover, temperature and humidity signals show a correlation index of 0.5152. It seems that this long period component of the gravity sequence is a part of the annual oscillation due to the mainly influence of the seasonal temperature changes. Additionally, a cross-correlation analysis shows a time lag of about 40 days of the gravity signal with respect to the temperature changes (Fig. 5).
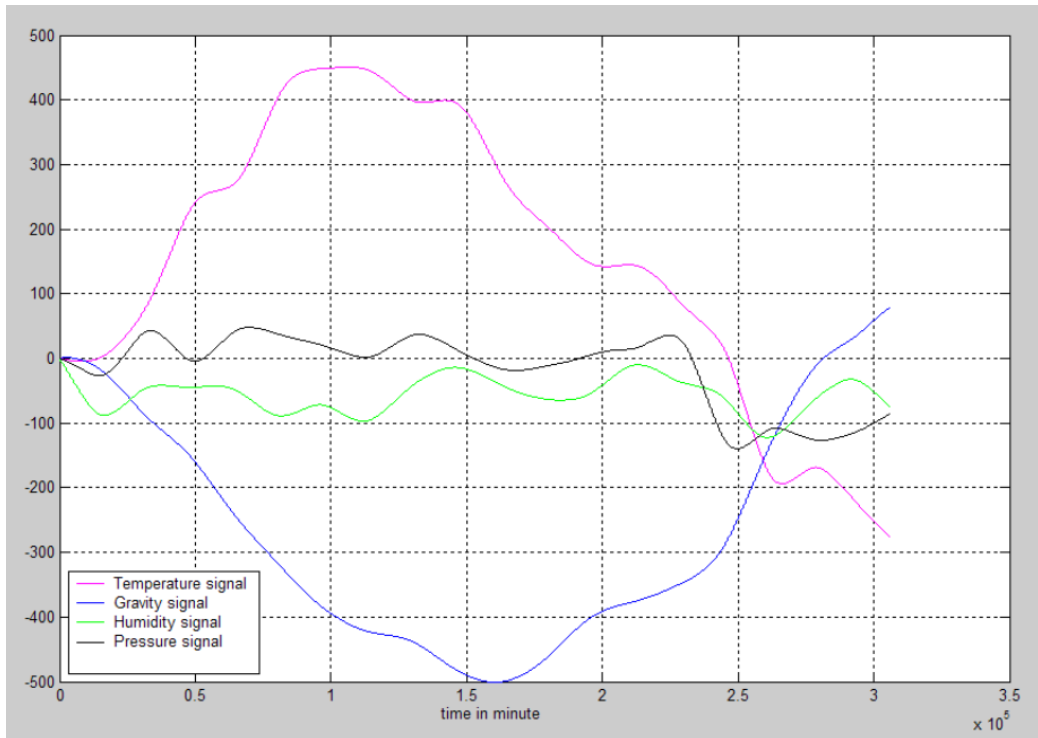
Fig. 5: Over plot between low frequency components of gravity, temperature, pressure and humidity signals. The figure shows the strong correlation and the time lag between temperature and gravity signal. In order to get the same scale, the temperature and pressure signals are multiplied by an amplification factor of 10.

The figure 6 shows the spectra of all signals obtained with the wavelet low pass filter. It shows how the wavelet filter works well since we have only low frequencies without high frequency signal. This kind of analysis clearly shows that the spectrum of each signal contain the same main components. The main peak around 200 days is the harmonic of annual period which is not resolved properly due to the insufficient time span.

It is thus more difficult to detect changes due to volcanic processes in the long period component of the signal, as this one is largely affected by the annual oscillations due to the meteorological parameters. Consequently, it is more suitable to analyze separately the low and high frequency parts of the signal.

Once the useful signal for our purposes has been suitably separated, the residuals can be used to evidence, over the time-interval considered, (i) recurrences in time (i.e. cyclic processes) and (ii) microgravity anomalies correlated with the ensuing volcanic activity (Fig. 7).

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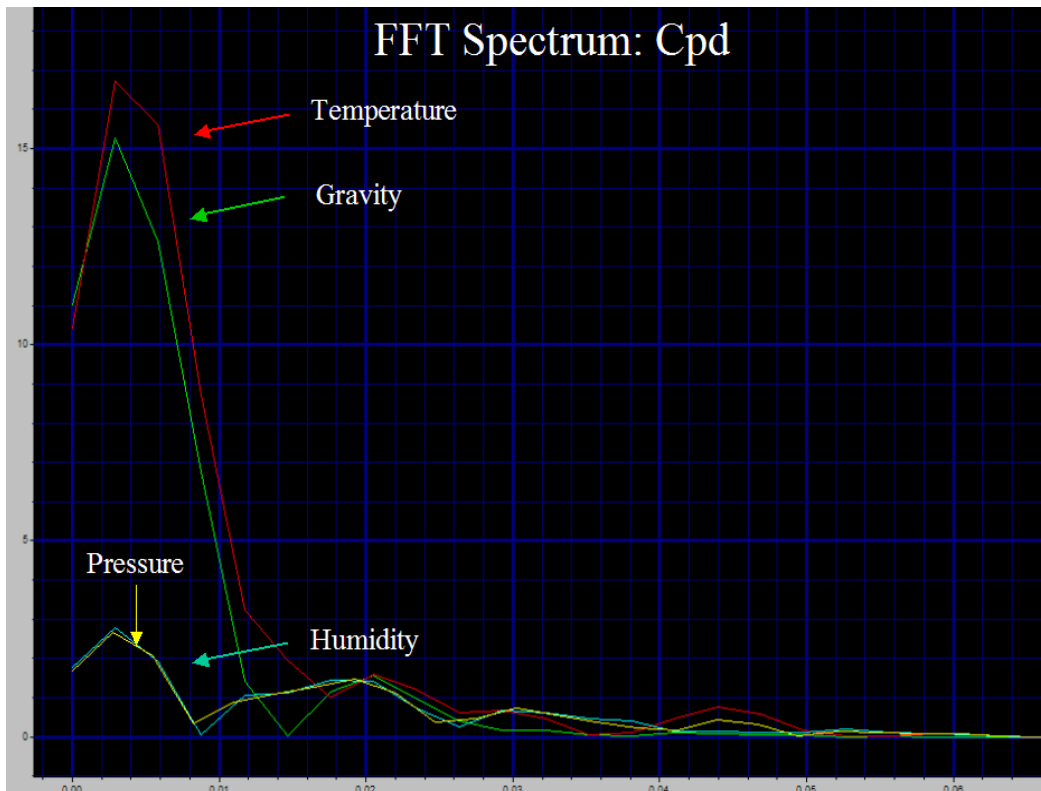
Fig. 6: Spectra (Hz vs cycles per day) of the long period components of all signals, obtained with the wavelet low pass filter. The figure shows that the filtered components contain only low frequencies.
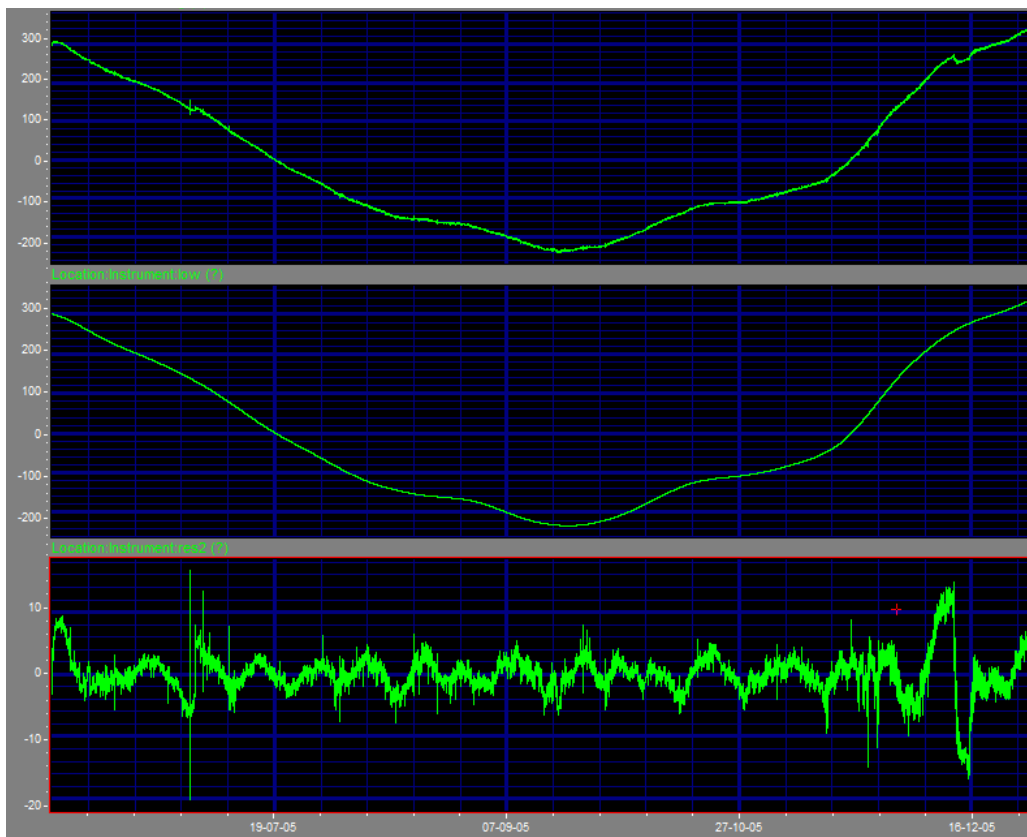
Fig. 7: Residues of the gravity signal under discussion. On the top the original signal, on the middle the low frequency component and on the bottom the high frequency one.

CONCLUSIONS

A new promising technique, for continuous gravity data noise-filtering based on "Wavelet transform", was proposed. The aim of this preliminary work is only an overview about wavelet transform with an application on the continuous gravity data.

The results obtained with wavelet decomposition seem to be better than the frequency domain filtering. The perturbations caused by the simultaneous action of different parameters (e.g. tide, instrumental drift, pressure, temperature and humidity) cannot be separated with frequency filters, since the spectrum of each component of various origins has wide intervals of superposition. The wavelet decomposition could be suitable for the construction of a non-linear multi-regression model that could enable a simultaneous correction of the main perturbing processes. Moreover, frequency domain filtering does not always work well because: (i) it globally removes frequencies causing a generalized smoothing effect that substantially broadens features of interest; (ii) depending on both cut off frequency and filter order it also could introduce edge effects and distortions of the original signal; (iii) it does not allow to study local features of the signal in the time domain.

Furthermore, the wavelet based filter does not require any tuning of parameters to produce results as good as the best achieved with frequency domain filters. This makes the process faster and more convenient. Finally, this paper stresses that the wavelet transform has others advantages over Fourier transform: multi-resolution decomposition and time-space localization, two important characteristics for denoising problems.

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