

An Introduction to the use of HICUM for Signal Analysis

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Abstract

This paper introduces a novel method for signal analysis whereby a weak signal can be detected in a noisy environment, providing the time period is known. The time period for each earth tide signal is found using the Doodson argument. Each cycle is then divided into sectors and a histogram of the data is plotted. The histograms for all the cycles are then stacked to produce the cumulative effect, hence the name for the method is HiCum . The paper uses synthetic data to demonstrate the ability of the technique to detect a signal in a noisy environment. Some of the additional benefits of the method over spectrum analysis are shown using field data from EDAS. There is also a section which gives some practical guidance on how to use HiCum with mDAS Grapher.

Keywords: Earth Tides, EDAS, Signal Analysis, Stacking.

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1 Introduction

The following is a description of HiCum (**H**istograms **C**umulation), an alternative method of signal analysis, which has been developed to work with the EDAS system by van Ruymbeke et al. (2001). HiCum, like Spectrum Analysis, analyses signal signatures based on Fourier Analysis. Whereas Spectrum Analysis is necessary when the frequency of the signal is not known, in situations where the time period is clearly defined, such as diurnal fluctuations, then the HiCum method is more accurate and is capable of extracting information that would be lost during Spectrum Analysis. It is therefore a powerful tool in monitoring the complex interactions induced by tectonic activities. Here computer generated data is used to test the accuracy of the method and to demonstrate its usefulness. Also included in the text is some practical guidance on the use of the software program, mDAS Grapher, in which HiCum has been embedded.

2 Background Theory

Fourier's Theorem states that any periodic function can be expressed as a sum of sine waves.

$$f(x) = S(a \cos rx + b \sin rx) + \frac{1}{2}c \quad (1)$$

where r takes integral values and a , b , c are constants.

It can be used as a method of determining the harmonic components of a complex periodic function. Since equation (1) is unchanged by replacing x by $x + 2kp$, where k is an integer, it necessarily represents a periodic function in x of period $2p$. Consequently in discussing series of this type it is sufficient to consider any interval of width $2p$ or 360° .

Thus if we have a signal that varies over a period of time, and we can clearly define the frequency, $w/2p$, then we can equate that to the interval $2p$ (or 360°) and equation (1) becomes:

$$f(x) = S(a \cos wt + b \sin wt) + \frac{1}{2}c \tag{2}$$

Where t is the instantaneous time.

With w clearly defined, the various harmonics of the system can be found.

HiCum has been developed to analyse data linked with tidal phenomenon (e.g. M2) where the time period can be accurately defined and is stable. HiCum has several advantages over Spectrum Analysis in extracting information where there are complex interactions in a multi parameter environment. Using HiCum the parameters of the fundamental sine wave, the harmonics and any non linearity in the signal can be detected on weak signals with high noise levels.

3 EDAS

Using EDAS, numerous readings from several diverse sensors can be taken at frequent intervals (e.g. every minute) over long periods of time (months) and stored in ASCII files. These files can be analysed at leisure using the software package mDAS Grapher, which has been specifically created for the EDAS files. To further simplify the analysis of the data the **Histograms Cumulation (HiCum)** method has been incorporated in the mDAS Grapher software package. Its objective is to put forward a graphical display of the behaviour of the non-linearities recorded by the sensors.

4 HiCum

The inspiration for HiCum came from the field of meteorology where stacking data has been used for many decades e.g. Emter et al (1985), following Bartel's work in 1938. A summary of work on a complementary method is given by Zürn and Rydelek This stacking method is the foundation of the HiCum method. A signal, which at first sight appears to be a white noise signal has its time base divided into a series of constant length time periods. The selected time period will be that which is suspected to have an influence on the parameters in question e.g. the M2-wave for gravimetric data. This time period is, by definition, equivalent to an interval of width $2p$ or 360° . For each period 360 sectors of 1° histogram are created and then the results from each time period are synchronised, normalised by the number of events in each sector and the results from the same time period each day are added, stacked, resulting in an averaging effect producing a picture of the variations, in relation to the wave selected (M2). Once the shape of the histogram for that time period is established the parameters (i.e. Phase and Amplitude) can be computed. Figure 1 is the schematic of the method.

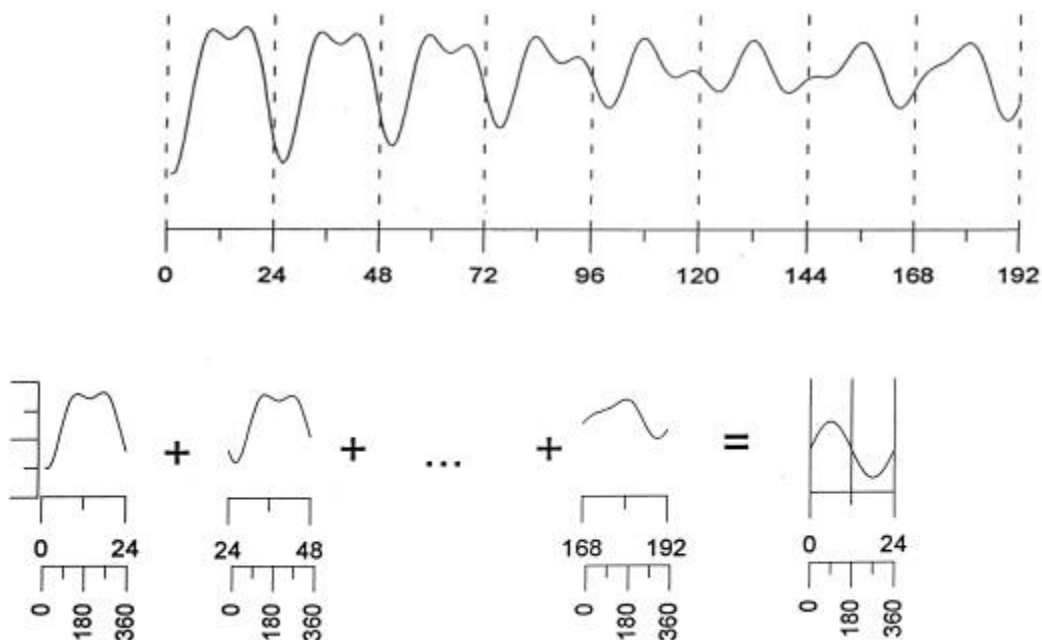


Fig. 1. Principle of HiCum Stacking applied to an eight day gravimeter signal (hourly scale).
 The series is cut in constant length time intervals corresponding to the period selected by a Doodson argument (i.e. 360° for 24 hours S1 period). The obtained histograms are simply added to obtain an average signal of the concerned component.

Once the fundamental sine wave has been detected it can then removed and the residual data checked for any non-linearity or harmonics. The program gives the option of removing up to four harmonics from the main signal, leaving only the non-linear residuals.

5 Use of Computer Generated Data to Check Accuracy of Method

In order to prove the accuracy of the method a series of known cosine waves and noise signals were computer generated, mixed and transferred to mDAS Grapher. The signals were then analysed using HiCum to see if the individual original waveforms could be detected.

5.1 The first scenario was a mixture of two cosine waves,

$$\cos\omega t + \cos\omega' t \quad \text{where } \frac{d\omega}{\omega} = 0.035$$

On entering the period for the first wave, $\cos\omega t$, HiCum should be able to detect that wave and pick up little of the second wave and conversely when the period for $\cos\omega' t$ is analysed $\cos\omega' t$ should be detected and little from the first wave.

Waves of time period S2 and M2 were used. They were constructed from 40320 data points, which is equivalent to a reading every minute for a lunar month. HiCum was then applied using first the time period for the S2 wave.

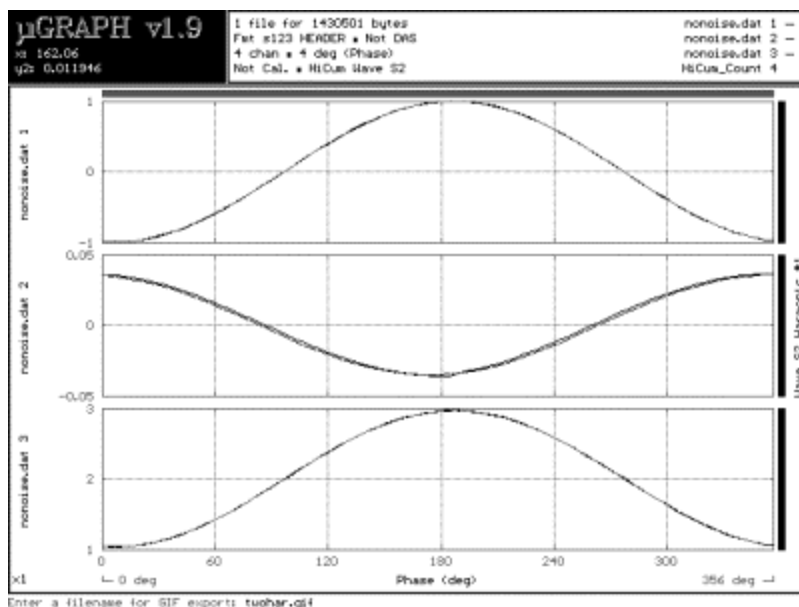


Figure 2 The top graph shows the signal detected from the pure S2 wave, the middle graph shows the signal detected from the M2 wave on the S2time period and the bottom shows the signal detected from a mixture of the S2 and M2 wave, all using a time period of S2.

Figure 2 clearly shows that the first wave, which has phase angle of -173° and an amplitude of 0.965431, is

detected from a mixture of the two signals with a phase angle of -173° and an amplitude of 0.965431, the second wave (M2) is detected with a phase angle of -006° and an amplitude of 0.0354039.

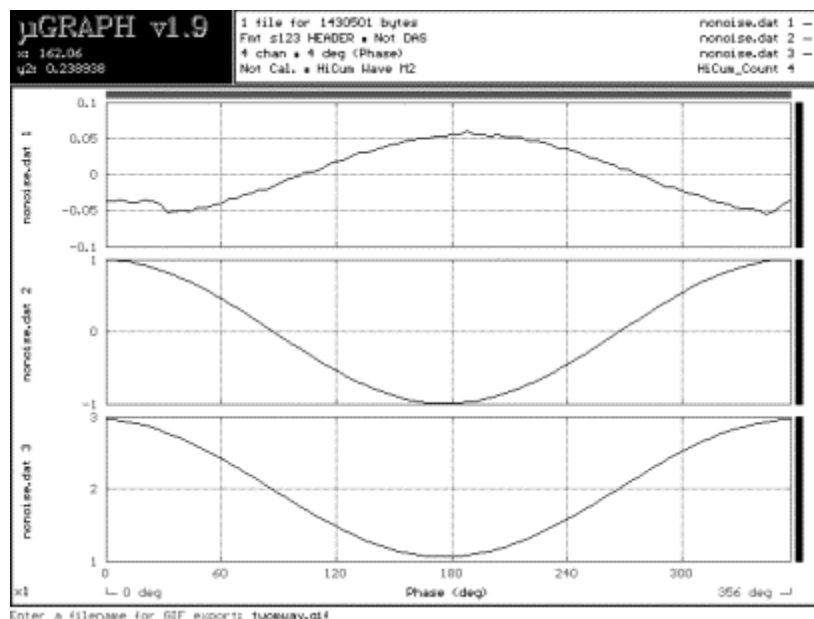


Figure 3 The top graph shows the signal detected from the S2 wave on the M2 period, the middle graph shows the signal from the M2 wave and the bottom shows the signal detected from a mixture of the S2 and M2 wave using a time period of M2.

Figure 3 shows that with M2 time period applied to the same data, the second wave, which has phase angle of -003° and an amplitude of 0.997707, is detected with a phase angle of -003° and an amplitude of 0.946315 and the S2 wave is detected with a phase angle of -170° and an amplitude of 0.052728. This demonstrates that HiCum is able to detect a wave phase difference to a high degree of precision when the time period can be clearly defined.

5.2 A second scenario exploring the effect of noise and quantity of data on the quality of results.

This was aimed at determining how the accuracy of detection of a waveform is affected by both the size of the noise to signal ratio, k , and the number of readings taken, N .

The values of k used were 0, 1, 10, 100 and 1000 (0 = no noise). The time periods were 10, 100 and 1000 days. The phase and amplitude of the S2 wave for each scenario is given in the table below.

	N	10 days		100 days		1000 days	
k		Phase	Amplitude	Phase	Amplitude	Phase	Amplitude
0		-173°	0.999802	-173°	0.9998	-173°	0.999793
1		-174°	0.998864	-173°	1.00031	-173°	0.999956
10		-170°	1.00116	-173°	0.990286	-173°	0.998895
100		-145°	1.13387	-169°	0.906448	-174°	0.990887
1000		-094°	5.45362	-088°	0.59966	-179°	0.918826

Table1 A table showing how the ability to detect a wave's amplitude is affected by the noise and the number of readings taken.

As is expected, we can see from Table 1 that as the noise to signal ratio increases, the ability of HiCum to detect the signal diminishes. However, we can also see that if the number of readings taken is increased, then HiCum is able to detect the signal in a highly noisy environment. This is a direct result of the methodology being based on the summation of repeated weak periodic signals, resulting in a strongly identifiable signal.

6 Example of information from HiCum using field data

The following section shows an example of the analysis of data records from a super conducting gravimeter by van Ruymbeke et al (2001). Using the HiCum method, a constant length time period M2-wave was selected. The fundamental sine wave was then removed and the residual data checked for any non-linearity or harmonics present.

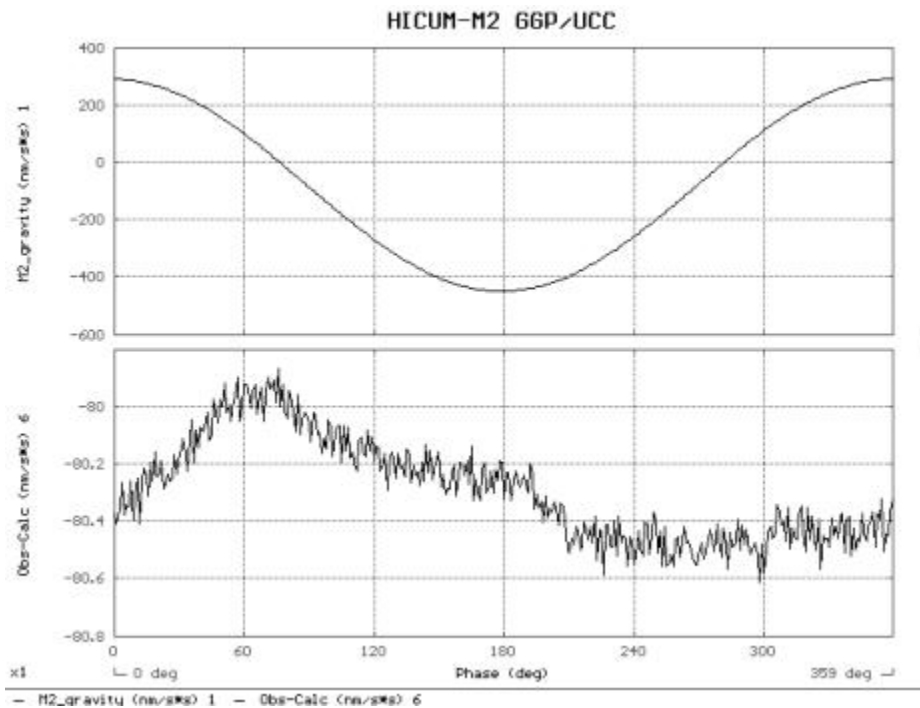


Fig. 4. Top: An amplitude-phase plot of the HiCum-M2 for the gravity versus the HiCum-M2 for the theoretical earth tide.
Bottom: The same plot with the sinusoidal trend removed and a more sensitive scale on the y-axis

In order to determine the exact nature of the signal the detected signal is plotted against M2 and from the top graph in Figure 4 we can see near perfect linearity with an amplitude of around 350 nm/s². In the lower graph the pure sinusoidal signal is then removed and the sensitivity of the scale is increased to reveal the non-linear behaviour of M2 component with a residue of less than 0.3nm/s². This residue represents a small lag between the two signals, and a non-linear behaviour with an amplitude of lower than 30 nanoGal, i.e. close to 0.1% of the amplitude as recorded by the gravimeter. This detection of non-linear hysteresis on raw data would not be possible with Spectrum Analysis and so demonstrates the usefulness HiCum in situations where the time period is known. The high level of precision on a weak signal was possible, because records were taken repeatedly over years and the results from the same time period each day were added, stacked, resulting in an averaging effect producing a detailed picture of the daily variations.

7 Using HiCum

μ DAS Grapher can be downloaded from the internet. Below is a detailed description of how μ DAS Grapher was used to prove the accuracy of the HiCum method for analysing the type of data expected from the Gravitational Balance GB02, which is currently under development at the Observatoire Royal de Belgique. As above a series of known cosine waves and noise signals were computer generated. In order to simulate the

signature of the Gravitational Balance GB02, the selected cosines were of time periods 720s and 750s representing the oscillations of the masses and the eccentric cam respectively. Noise signals were also generated one with a noise to signal ratio of 1:1 and the other 10:1. The signals were constructed from 3000 data points created at 6s time intervals representing 18000s or 5h of data. This data was then transferred to μ DAS Grapher.

NB The files should be opened in MGR with /ic to interpret all commas as decimal points and /i:s to put a time base on the data, each command must be separated by a space.

Once the file had been loaded into mDAS Grapher The data function was used to mix the signals using the formula option. *Select Alt A then F8, you will then be prompted for your required formula.* Figure 3 shows the various scenarios that were created. *To add channels simply enter the number or the letter of the channel as a sum e.g. 1+2. If you need to enter a numerical function then the number should be enclosed in brackets <> to distinguish it from a channel number e.g. 3.<0.5> multiplies channel 3 by 0.5.*

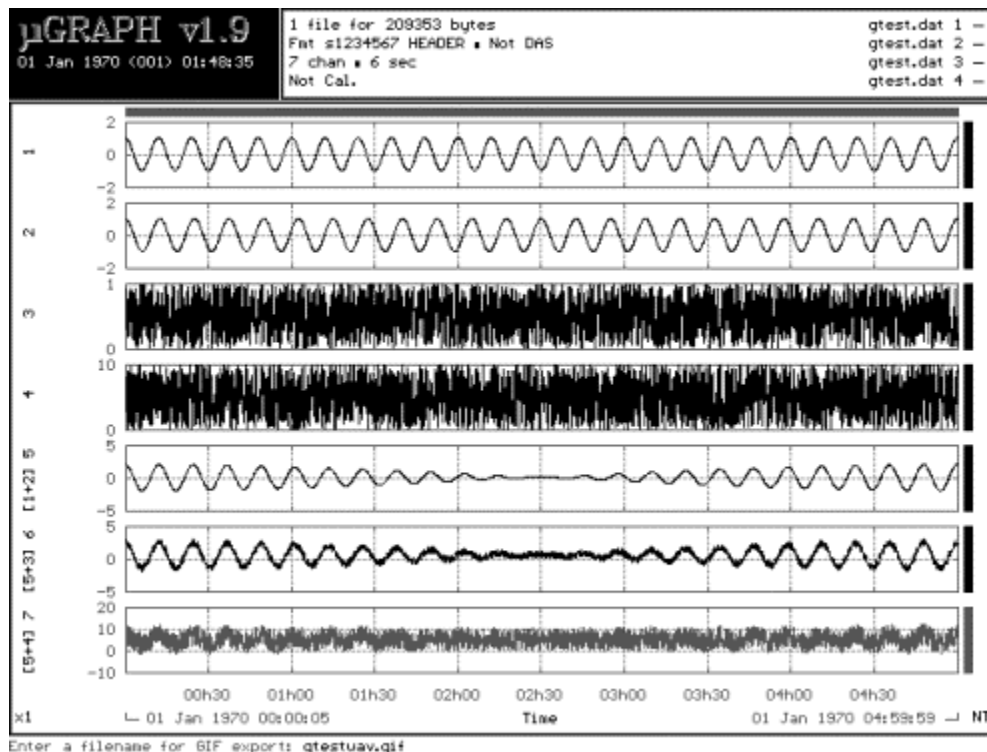


Figure5 Graph 1 $\cos(2\pi/720)t$, Graph 2 $\cos(2\pi/750)t$, Graph3 noise 0-1, Graph 4 noise 0-10, Graph 5 the addition of the two cosine waves, Graph 6 two cosine waves with noise 0-1, Graph 7 two cosine waves with noise 0-10.

The signals were then analysed using HiCum to see if the individual original waveforms could be detected. *To do this you need to enter shift F8 and you will be prompted to enter the time period for the first wave Our first wave had a time period of 720s but we needed to enter 120 (720/6) since our choice for the generated wave had been data at 6s intervals.* When the time period 720s is applied HiCum should be able to detect that wave and pick up little of the second wave, conversely when the time period 750s is analysed, the second wave should be detected and little from the first wave. The results are shown below.

Figure 6 is a graphical demonstration of how the various scenarios shown in figure 5 compare with a wave of time period 720s. For each of the synthetic signals we can see the summation of the histograms created from the signal when it has been divided into lengths of 720s and how this compares with a wave of time period 720s. As expected there is an exact correlation for the 720s wave with no noise (graph 1), we can also see a good correlation when the waves are mixed and noise is added (graphs 5-7). *To obtain the comparisons press F7 the third strike gives the 1st harmonic, as shown below in figure 6. The first strike will give the mean*

value, the second the linear regression, the fourth the 2nd harmonic, the fifth the 3rd harmonic and the sixth the 4th harmonic a further strike brings you back to nothing being superimposed, no examples of these options are given in this text.

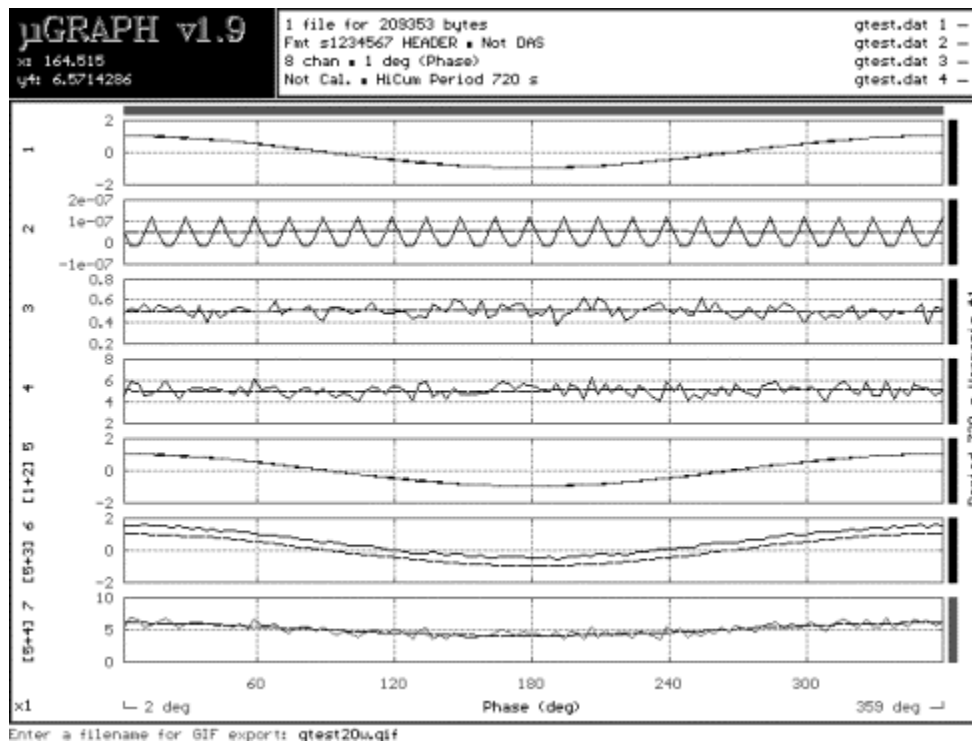


Figure 6 A comparison of a wave of period 720s with the scenarios shown in figure 5.

Figure 7 gives the numerical comparisons of phase and amplitude for the various signals. *To obtain this information press F2 twice, pressing Esc will remove the data.* From figure 7 we can see that the original wave has a phase of -001° and this is clearly detected from a situation where the two signals are mixed and also when the signals are mixed with a signal, having a noise to signal ratio of 1:1. On mixing the signals with a noise to signal ratio of 10:1 the accuracy is reduced giving a phase of -003° .

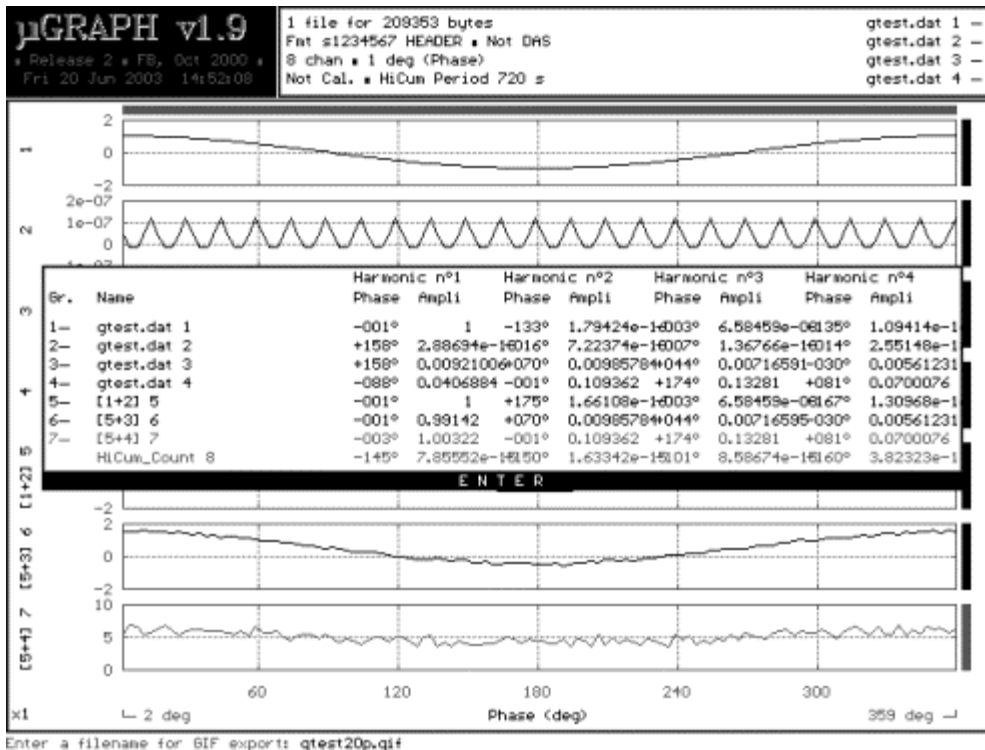


Figure 7 A numerical analysis of the scenarios shown in figure 6

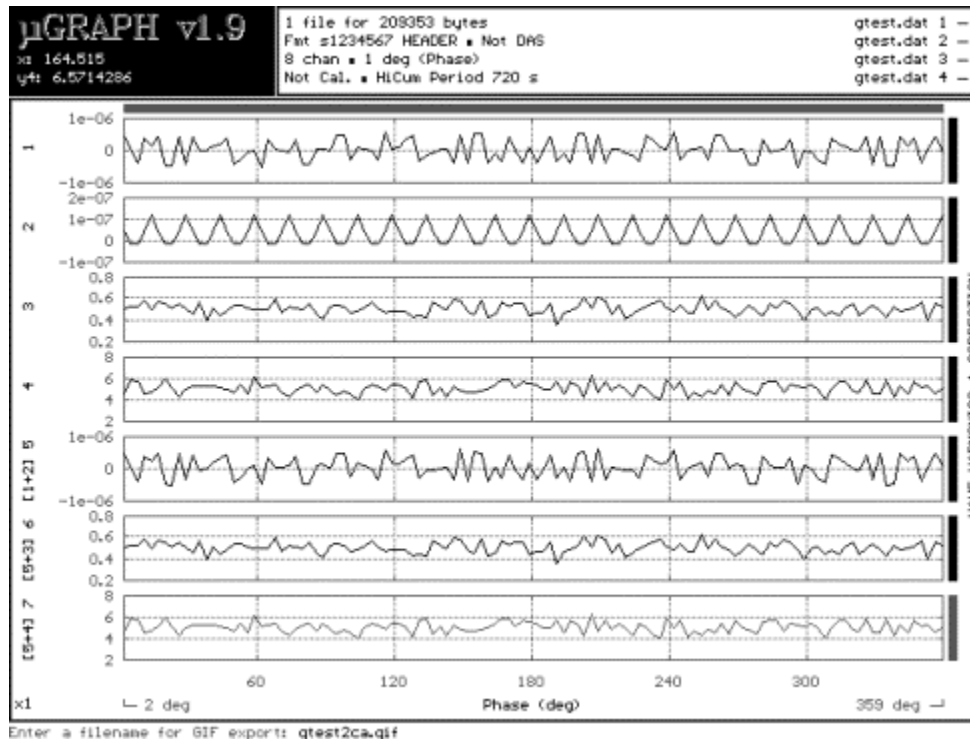


Figure 8 The residual signals for each scenario once a signal of time period 720s has been removed.

Using HiCum the fundamental wave can be subtracted from each of the scenarios. This is done by pressing enter, when prompted to, after typing F7 the required number of times. The residual signal is then detected, this is shown in figure 8. As might be expected the levels of these residuals vary enormously, and is clearly demonstrated by the different scales required for the y-axis. Before analysing the data for the second time period this data should be saved by pressing F1. To save any images created in mDAS Grapher you first

need to change the image from colour to black with a white background by pressing F9 twice. Then to save press shift F1 and give your file a title with a gif extension e.g. spring.gif

Figures 9,10 and 11 show the corresponding results from entering a time period of 750s.

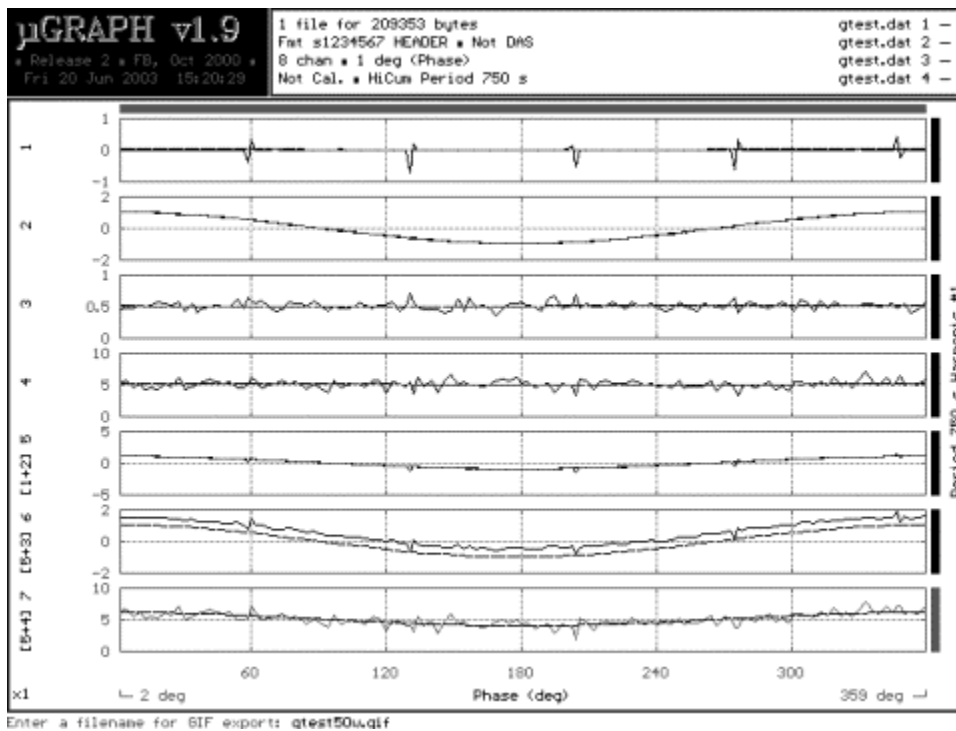


Figure 9 A comparison of a wave of period 750s with the scenarios shown in figure 5.

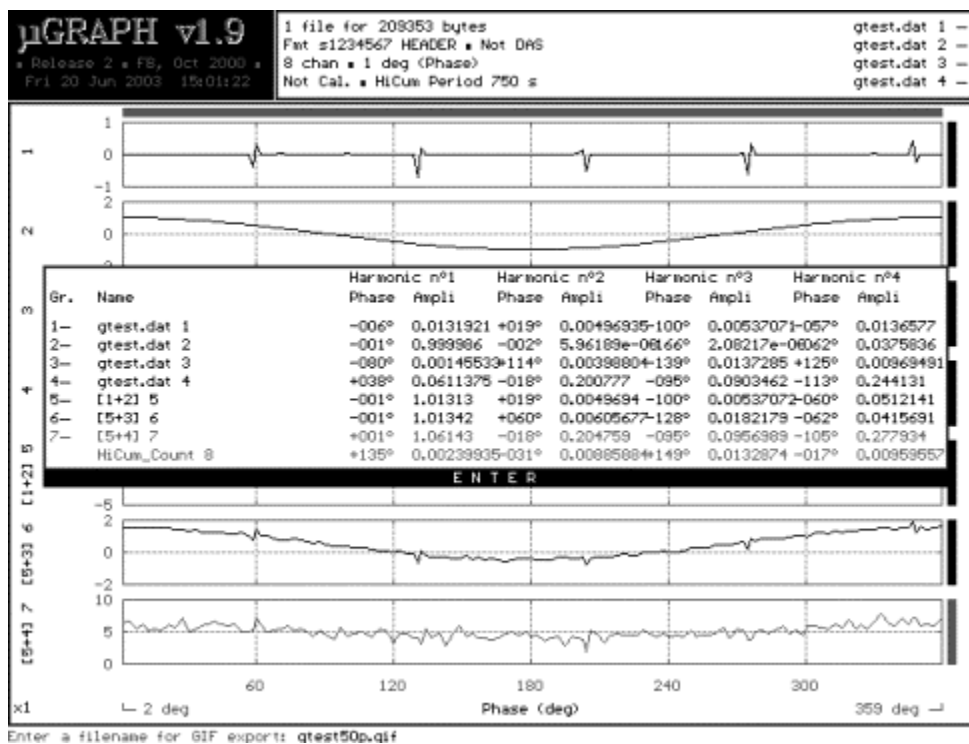


Figure 10 A numerical analysis of the scenarios shown in figure 9.

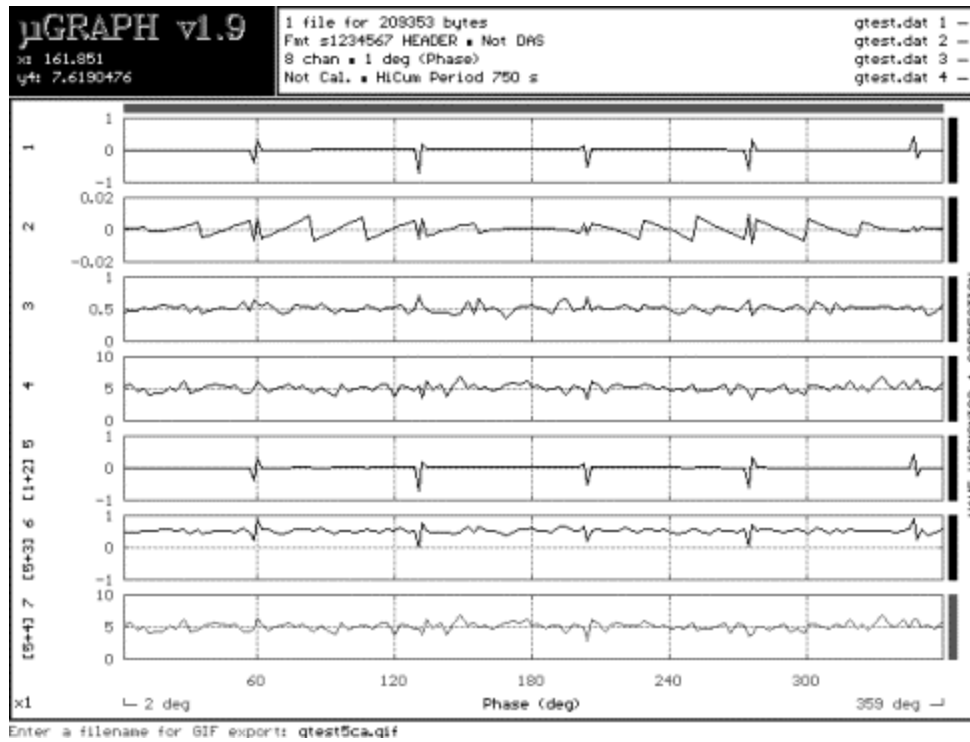


Figure 11 The residual signals for each scenario once a signal of time period 750s has been removed.

From Figure 9 we can see that once again the selected wave has been detected with a noise level of 1:1. Even with a noise level of 10:1 the selected wave is detectable. From this example we can see that with only 3000 data points two signals have been accurately detected from data containing a mixture of these signals and noise. In addition the residual values have been determined.

8 Conclusion

The use of computer-generated signals has demonstrated the power of the HiCum method in detecting signals in a noisy environment when the time period is known. The accuracy of the method has also been shown to be extremely powerful in the analysis of field data.

HiCum is a successful tool because a weak signal has been recorded repeatedly and its cumulative result produces the equivalent of a strong signal with a wealth of detailed information. HiCum has been developed to analyse data from the EDAS system but is applicable to any situation where numerous readings can be taken during each cycle and over several cycles.

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