Simplistic models of vertical seismic noise above 0.1 mHz derived from local barometric pressure

by

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1 Introduction

It has long been known that long-period seismic noise in all components has at least some of its sources in the local atmosphere. At periods much higher than the gravest free oscillations of the earth a vast amount of literature exists demonstrating clearly the influence of gravitational attraction of the sensor mass by the local atmospheric mass and of the free air effect due to pressure loading of the local crust (e. g. Warburton and Goodkind 1977). Müller and Zürn (1983) noticed the effects of passing coldfronts on a gravimeter and could explain these observations also mainly by Newtonian attraction. Zürn and Widmer (1995) showed for the first time that free mode observations can be appreciably improved by applying a simple correction for the local air pressure effect as well understood at much longer periods. With this correction one can reach noise power spectral densities (at least for the best gravimeters at the best stations) lower than predicted by the New Low Noise Model of Peterson (1993) for frequencies less than 1 mHz. However, the same authors also noticed that the correction does not work for frequencies higher than 1 - 2 mHz, actually there it reduces signal-to-noise ratios (SNR). This is not a serious problem since barometric pressure could be low-pass filtered before using it for the correction. Still the question arises, why this is the case although the physical mechanisms must be similar. Since much more seismological information can be obtained above than below 1 mHz improved SNRs above 1 mHz would be highly welcome.

2 Simplistic models for higher frequencies

As one tries to understand the seismic noise at higher and higher frequencies and wavenumbers a series of complications arises: the physical processes in the atmosphere get more and more complicated , the barometric pressure fluctuations get smaller and smaller, quasistatic treatment of the elastic earth will be insufficient and so on.

The cold front model used by Müller and Zürn (1983) already took dynamics and lateral variation of the atmosphere into account to some extent, by incorporation of a typical shape and ground speed of a front and by estimating the density jump across the front from the observed jumps in pressure and temperature. The magnitudes of the observed gravity changes could be rather well explained by this model (see table 1 in Müller and Zürn 1983). However, one major shortcoming of the model was the fact, that the only physical effect taken into account was the gravitational attraction of the gravimeter mass.

A vertical accelerometer senses several forces on its mass. Some sources of deformation have a changing direct gravitational attraction on the sensor mass like the sun, moon (tides) and the atmosphere, mass redistribution in the earth leads to secondary changes in gravitation, local vertical displacements cause a free air effect and inertial effects. Table 1 shows for different phenomena the relative contributions of these effects to the total recorded signal. Note that additional contributions are possible, one exotic example being Coriolis forces from toroidal modes (Zürn et al., 2000). In the frequency range just above 1 mHz the inertial effect plays already a very strong role and must be considered. So naturally our next step must be the inclusion of this effect.

Table 1: Relative contributions to signals sensed by a vertical accelerometer. The third example corresponds to a simple vertical motion like on a shake table or calibration platform.

Signal	Frequency	Inertial effect	Free air effect	Mass redistr. effect	Gravitation
M_2-tide	22.4 μHz	0.0042	0.6	-0.45	1.00
${}_oS_2$	0.30 mHz	0.815	0.67	-0.48	0.0
—	0.28 mHz	0.500	0.500	0.0	0.0
${}_oS_{10}$	1.725 mHz	0.98	0.025	-0.008	0.0
Rayleigh-Wave	0.05 Hz	1.00	0.0	0.0	0.0
P-wave	1.0 Hz	1.00	0.0	0.0	0.0

The first new model just adds the inertial effect to the model so successful below 1 mHz. It consists of an elastic layer with thickness D and Lamé constants μ and λ over a rigid halfspace and under a laterally homogeneous atmosphere. The density in this atmosphere is either exponentially decreasing with altitude z and a scale height H (isothermal model $\rho(z) = \rho_o \cdot exp(-z/H)$) or constant with height ($\rho(z) = \rho_o$) up to H and zero above. When pressure varies harmonically with angular frequency ω the admittance between pressure Δp and vertical acceleration Δg for both atmospheric models is:

$$\frac{\Delta g}{\Delta p}_B = -\frac{2 \cdot \pi \cdot G}{g} + \frac{(\lambda + \mu) \cdot D}{\mu \cdot (3\lambda + 2\mu)} \cdot (\omega^2 + |\frac{\delta g}{\delta z}|) \qquad (1)$$

where G is the gravitational constant and g is the gravitational acceleration at the surface. The first term is very simple and results in -4.27 nm/s^2. The second and third term depend on the elastic layer and represent the inertial and free air effects, respectively, due to the vertical displacement induced by the pressure load. This model does not take into account, that the spatial scale of atmospheric density variations decreases with increasing frequency, so eventually at some higher frequency it must fail for this reason.

The second model is the one already described by Neumann and Zürn (1999): a pressure wave propagates horizontally with phase velocity c_h, angular frequency ω and horizontal wavenumber k_h over an elastic halfspace with Lamé constants as above. Density in the air decays exponentially with height z and with a scale height H as in an isothermal atmosphere.

$$\Delta p = p_o \cdot exp(i \cdot (k_h \cdot x - \omega \cdot t)) \qquad (2)$$

$$\Delta \rho = \frac{p_o}{c^2} \cdot exp(i \cdot (k_h \cdot x - \omega \cdot t)) \cdot exp(-z/H) \qquad (3)$$

This is a perfect model for Lamb waves. For acoustic-gravity waves with real vertical wavenumbers (Lamb waves have imaginary vertical wave number $k_v = i \cdot 1/H$) the loading effects will be well modeled, but not the direct gravitation (e. g. Gossard and Hooke 1975). For a vertical accelerometer on the surface of the halfspace we have the pressure admittance:

$$\frac{\Delta g}{\Delta p}_W = -\frac{2 \cdot \pi \cdot G \cdot H}{c^2 \cdot (1 + k_h \cdot H)} + \frac{\lambda + 2 \cdot \mu}{2 \cdot \mu \cdot (\lambda + \mu) \cdot k_h} \cdot (\omega^2 + |\frac{\delta g}{\delta z}|) \qquad (4)$$

with c the velocity of sound, $k_h = \omega / c_h$ and $H = c^2/g$. For infinite horizontal wavelength $k_h = 0$ and the first term reduces to $-2 \cdot \pi \cdot G/g$ as for the first model. Note that c_h can assume values from 10 m/s up to the sound velocity c (330 m/s).

These two admittances are now frequency dependent and they can be used to compute (in the frequency domain) from an observed time series of local atmospheric pressure p(t) the pressure induced "signal" (noise) deterministically as a time series (pressure seismogram) or statistically in the form of noise power spectra. The spectrum of the "pressure seismogram" is:

$$w_z(\omega) = \frac{1}{i \cdot \omega} \cdot H_v(\omega) \cdot \frac{\Delta g}{\Delta p}(\omega) \cdot \Delta p(\omega) \qquad (5)$$

where H_v is the transfer function of the accelerometer for output w_z (in volts or counts) w. r. t. ground velocity, as normally used in seismology for broadband seismometers.

Inspection of the equations for the admittances in both models shows that an angular frequency ω_o can be found for which the admittance vanishes, because the effects compensate each other. This is certainly a property of the simplistic models, in reality this will not be true and in any case this "hole" will be filled by noise from other sources (eventually the instrumental noise). and the background free oscillations of the earth, e. g. Ekström, 2001). The Bouguer plate model has only one such conditions while the wave model has a second one at very long periods, where the inertial effect is negligible, but the free air effect gets so big that it compensates the gravitational effect. This is an unrealistic feature, because of the halfspace approximation.

3 Examples

Two 10-day pressure time series and seismograms from an STS-1 vertical seismometer from BFO (Richter et al. 1995) were selected in order to check these models, one from July 2000 with strong pressure disturbances, the other from May 2002 with less strong pressure variations. The tides were removed from the seismograms by subtracting a few tidal lines. Power spectral densities in acceleration were computed, for the seismic data these were corrected for the instrument response, for the pressure data they were multiplied by the acceleration-pressure admittances given by eqns. (1) and (4) in order to make them comparable. In all cases $\lambda = \mu = 75GPa$ for the basement rocks of the area around BFO was assumed, corresponding to S- and P-velocities of 5.0 and 8.6 km/s for a density of 3000 kg/m^3. These are certainly on the high side. For the Bouguer model D = 20 km was adopted. Figs. 1 to 3 present these power spectral densities for acceleration (PSD) as functions of frequency. For comparison the New Low Noise Model (NLNM) of Peterson (1993) is also shown. It is clear that any seismically produced power in the PSDs of the seismic data can never be modeled by air pressure variations. The quieter the station, the more seismic signals one will detect in a time series and days without them cannot really be found.

Fig. 1 shows the results for the time series from July 1, 12:00 to July 9, 12:00, Fig. 2 for May 1, 12:00 to May 9, 12:00, Fig. 3 for May 3, 0:00 to May 6, 12:00; all times are UTC. For the July seismogram the PSD is more than a factor of ten above the NLNM for most frequencies, reflecting strong pressure variations at long periods and seimic activity at the high frequencies. It is The Bouger and Wave models explain the PSD well up to frequencies of 2 and 0.7 mHz, respectively. This part is dominated by the gravitation effect which cannot be modified by the elastic part of the models. Both models have deep "holes" where the admittances go through zero as discussed above. The wave model starts dropping off at the low frequency end because of the second zero also mentioned above. The Bouguer model clearly overpredicts the observed noise at frequencies above a few mHz, not unexpectedly, because the atmosphere is assumed to stretch from infinity to infinity horizontally in two dimensions. This result, of course, depends on the thickness of the elastic layer and the shear modulus assumed. The horizontal phase velocity c_h for the wave model was chosen such (200 m/s), that the

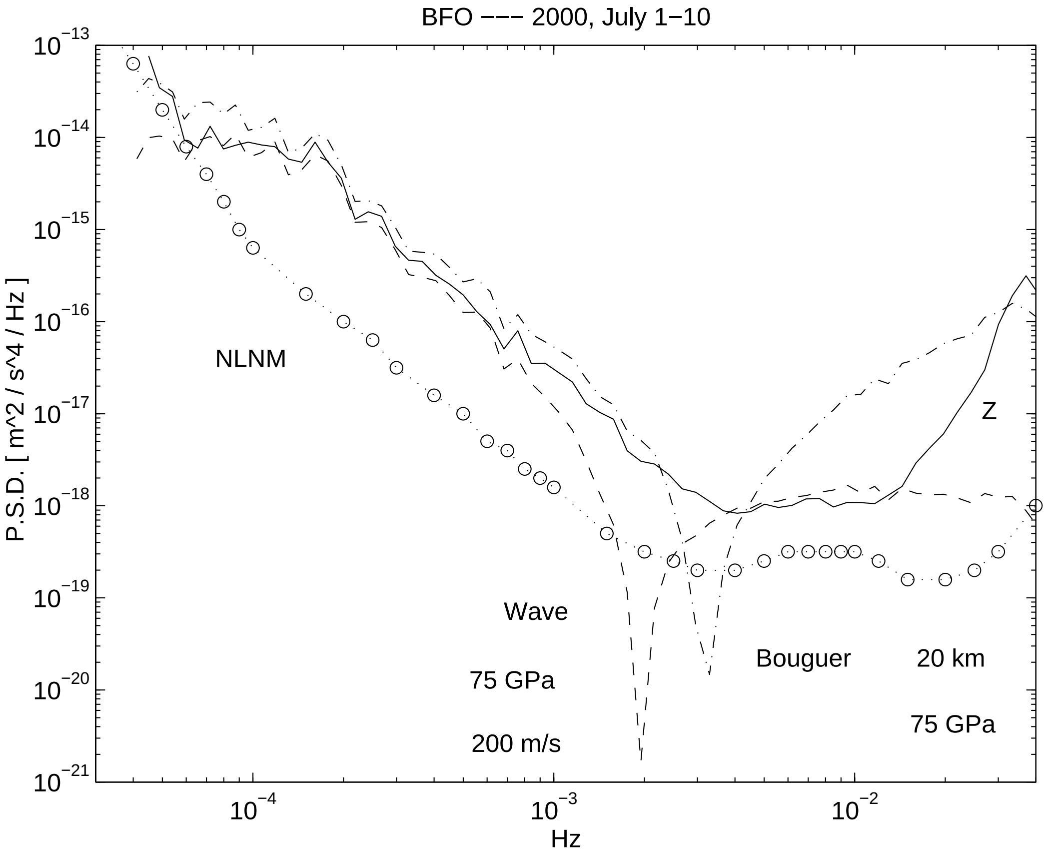

Figure 1: Acceleration power spectral densities against frequency for a 9-day (July 1, 12:00 to July 9, 12:00, 2000) vertical seismogram (data, solid line) from the STS-1 seismometer at BFO. The New Low Noise Model of Peterson (1993) is given for comparison (NLNM, dotted line with open circles). The Bouguer plate and horizontal wave models for vertical noise computed from local barometric pressure variations can be compared to the simultaneously observed noise. Model parameters are given in the figure and text.

predicted noise nearly equals the observed noise between 3 and 10 mHz. This plateau can easily be moved up and down by raising and lowering this value at constant μ, the "hole" then moves to lower and higher frequency.

For the 9 days in May 2002 the observed PSD is closer to the NLNM. The only model parameter different from Fig. 1 is $c_h = 300$ m/s, close to the speed of sound, because only by raising this value the PSD for the wave model at least in a small frequency range gets close to the observed. At low frequencies the wave model underpredicts the observed noise (because the "holes" approached each other) and it also underpredicts by far the high frequency noise, but there real seismic noise comes in to raise the observed level as demonstrated below. The Bouguer model predicts the observed noise over a very wide range of frequencies, with the exception of th vicinity of the "hole". Again we show below that this is only a fortuitous result.

For Fig. 3 the 3.5 days of the May 2002 time series was selected which had the

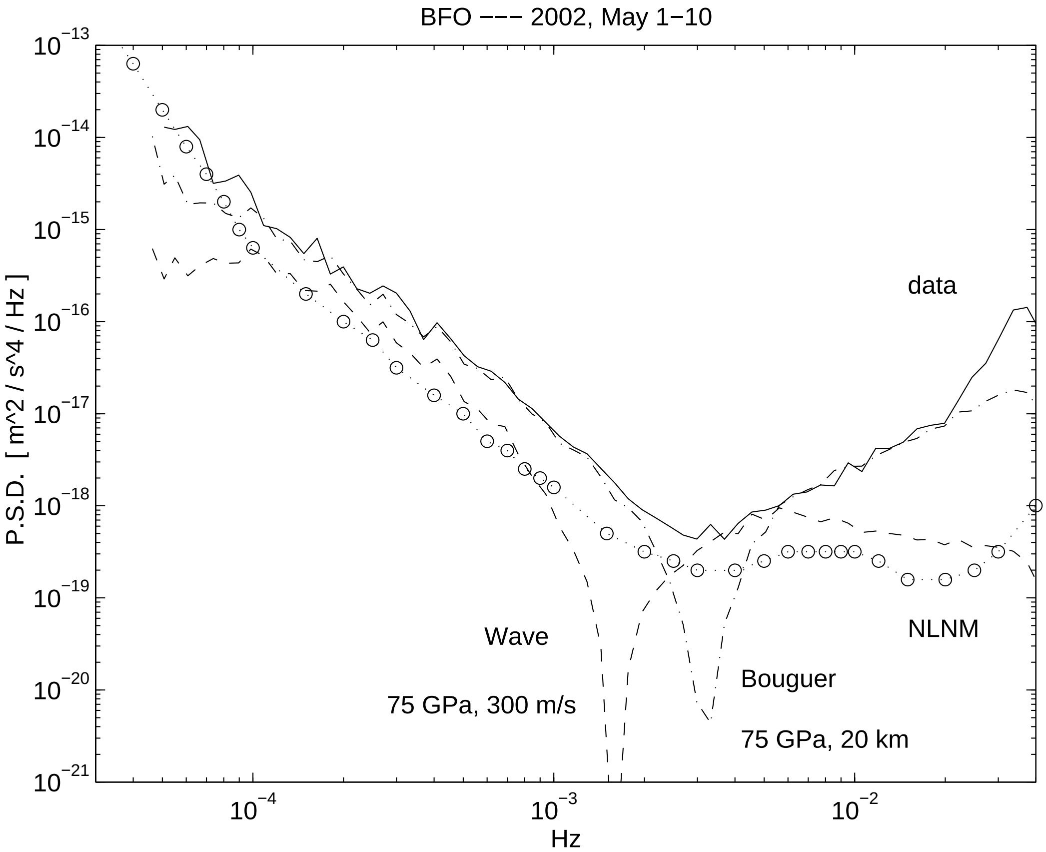

Figure 2: Same as figure 1 for the 9 days from May 1, 12:00 to May 9, 12:00, 2002.

least seismic activity in it. The models are both identical to the ones used for Fig. 2. The results clearly show firstly that the observed noise level is much lower above 5 mHz, and secondly that the same Bouguer model now overpredicts the observed noise as for July 2000. The wave model performs similarly poorly as for the full series.

The following conclusions can safely be drawn. Both (purely physical) models are predicting the general noise levels fairly well. This is not surprising below 1 mHz, because they are not significant modifications to the classical modal used by the tidal gravity community. According to Zürn and Widmer (1995) the NLNM can clearly be improved on by simple correction for local atmospheric pressure with factors very close to the first term in eq. (1). The Bouguer model tends to overpredict the higher frequency PSDs because it does not take decreasing scale of atmospheric cells for higher frequencies into account. Both (and any other) models have deep minima at frequencies depending on the chosen parameters, but generally in the range of frequencies, where the NLNM has its flat minimum. This property of the atmospheric effects has with high probability an effect on the shape of the NLNM, because it makes the contribution of the atmosphere to the noise even smaller than the dropping barometer PSD alone. Since 1998 (Nawa et al., 1998, see Ekström 2001 for more references) it is clear that in the minimum of the NLNM between 2 and 7 mHz one can see the incessantly excited free oscillations of the earth with the global atmosphere as

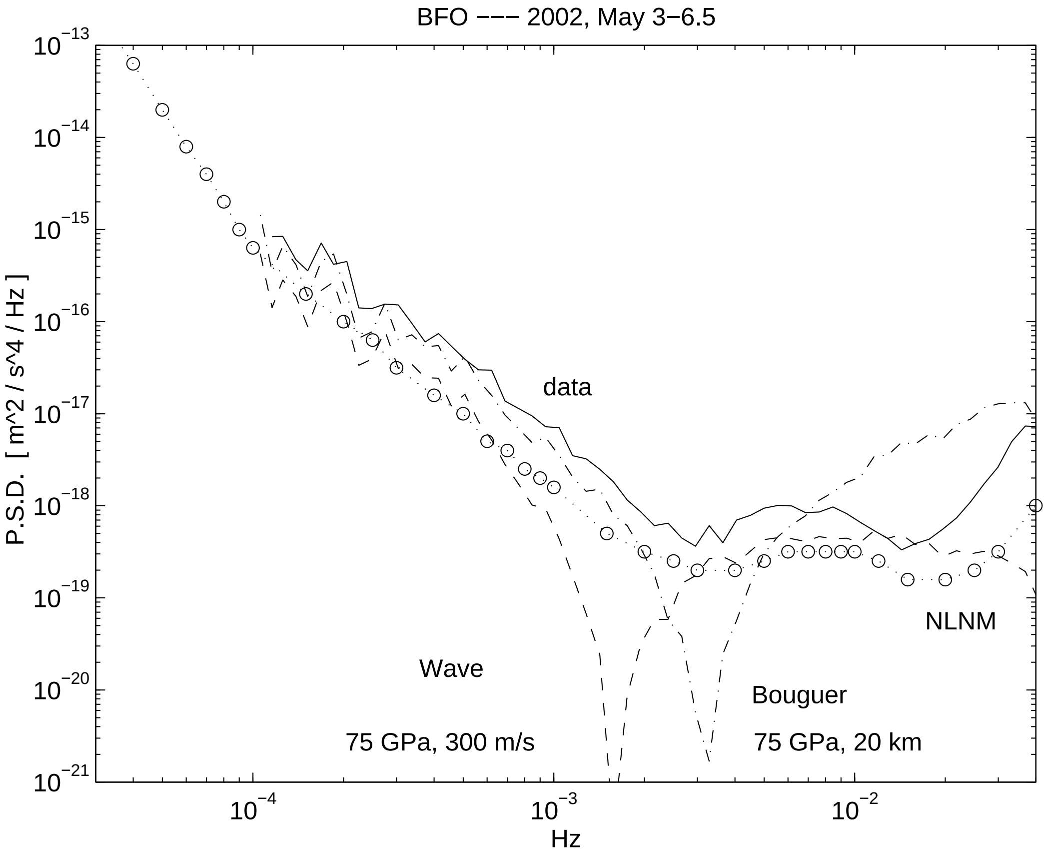

Figure 3: Same as Figure 2 but for only the part of the time series with reduced seismic activity from May 3, 0:00 to May 6, 12:00, 2002

the favored source. Since these modes cannot be seen in a single 24 hr-window, another noise source is competing strongly in this range (see Widmer-Schnidrig, this volume) which is probably of instrumental origin. Since not every station reaches values near the NLNM outside of earthquakes, some station noise could also contribute at the very good stations (Agnew and Berger, 1978). At frequencies much higher than the ones discussed here Sorrells (1971) considered propagating elastic waves as the important physical effect for producing noise, this has completely be neglected in the simplistic models tested in the work here.

References

Agnew, D. C., Berger, J. (1978). Vertical Seismic Noise at Very Low Frequencies. *J. geophys. Res.*, **83**: 5420 - 5424.

Ekström, G. (2001). Time domain analysis of Earth's long-period background seismic radiation. *J. geophys. Res.*, **106**: 26,483 - 26493.

Gossard, E. E., Hooke, W. H. (1975). *Waves in the Atmosphere.* Elsevier, Amsterdam, 456 pp.

Müller, T. and W. Zürn (1983). Observation of gravity changes during the passage of cold fronts. *J. Geophys.*, **53**: 155 - 162.

Nawa, K., Suda, N., Fukao, Y., Sato, T., Aoyama, Y., Shibuya, K. (1998). Incessant excitation of the Earth's free oscillations. *Earth Planets Space*, **50**: 3 - 8.

Neumann, U., Zürn, W. (1999). Gravity signals from atmospheric waves and their modeling. *Bull. Inf. Marées Terrestres* **131**: 10139 - 10152.

Peterson, J. (1993). Observations and Modeling of Seismic Background Noise. U. S. Geol. Surv., Open-File Rep. 93-322, 1 - 45.

Richter, B., H.-G. Wenzel, W. Zürn and F. Klopping (1995). From Chandler wobble to free oscillations: comparison of cryogenic gravimeters and other instruments in a wide period range. *Phys. Earth planet. Inter.*, **91**, 131 - 148.

Sorrells, G. G. (1971). A preliminary investigation into the relationship between long-period seismic noise and local fluctuations in the atmospheric pressure field. *Geophys. J. R. astr. Soc.*, **26**: 71 - 82.

Warburton, R. J., Goodkind, J. M. (1977). The influence of barometric pressure variations on gravity. *Geophys. J. R. astr. Soc.*, **48**: 281 - 292.

Zürn, W., Laske, G., Widmer-Schnidrig, R., Gilbert, F. (2000) Observation of Coriolis coupled modes below 1 mHz. *Geophys. J. Int.* **143**: 113 - 118.

Zürn, W. and R. Widmer (1995). On noise reduction in vertical seismic records below 2 mHz using local barometric pressure. *Geophys. Res. Lett.* **22**: 3537 - 3540.