THEORETICAL DESCRIPTION OF THE EXTENSIONAL AND ROTATIONAL STRAIN TENSOR COMPONENTS

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Abstract

Equations are provided for scientists dealing with deformations of the Earth. The equations are based on spheroidal Love-Shida numbers for the case of a symmetric, non-rotating, elastic, isotropic (SNREI) Earth with a liquid core. By these means, the strain tensor components were calculated for the case of zonal, tesseral and sectorial tides both for extensional and rotation strain tensor components.

1. INTRODUCTION

The study of strains due to different geophysical phenomena has been discussed in research contributions since long. Neverthaless the equations which describe the tensor components are often consist misprints and the complete description according to authors knowledge never was publisted earlier.
The paper is using the similar way of the derivation of theoretical equations as it was done by Varga & Grafarend (1996) for the stress tensor components. A special attention is given to the rotational strain components because they can be used succesfully in study of tectonical processes, in observation of seismic waves and in earth tidal studies (Watanabe, 1962; Ozawa 1975; Voosoghi 2000).

2. THE TIDAL POTENTIAL AND TIDE GENERATED DISPLACEMENTS

The tidal potential can be written as (Varga, Grafarend, 1996)

$$W_{2i} = \frac{3}{4} GM \frac{r^2}{d^3} Y_{2i}(\Phi, \lambda) = V_2(r,d) Y_{2m}(\Phi, \lambda) \tag{1}$$

G - gravitational constant
M - the mass of the tide generating body
r - the distance from the Earth centre
d - Earth-Moon distance
Φ - northern latitude
λ - eastern longitude
i - can take the value $m = 0$ (long-periodic zonal tide)
$m = 1$ (diurnal tesseral tide)
$m = 2$ (semi-diurnal sectorial tide)

The reference system: · the vertical axis points to the centre of the Earth

· the horizontal axes are oriented to south along the meridian and to west in the first vertical

In this system in (1) the spherical components are:

$$Y_{20} = 3\left(\sin^2\Phi - \frac{1}{3}\right)\left(\sin^2\delta - \frac{1}{3}\right) \quad \text{- zonal tides}$$
$$Y_{21} = \sin 2\Phi \sin 2\delta \cos H^* \quad \text{- tesseral tides} \qquad (2)$$
$$Y_{22} = \cos^2\Phi \cos^2\delta \cos 2H^* \quad \text{- sectorial tides}$$

δ declination

H^* hour angle

For the calculation of the tidal strain components the derivati ves of Y_{2m} are needed:

a. Latitudional first and second order derivatives
b. Longitudional first and second order derivatives
c. Mixed second order derivatives

a. Latitudional derivates	b. Longitudional derivates	c. Mixed derivates
$\frac{\partial Y_{20}}{\partial\Phi} = 3\sin 2\Phi\cdot\left(\sin^2\delta - \frac{1}{3}\right)$	$\frac{\partial Y_{20}}{\partial\lambda} = 0$	
$\frac{\partial Y_{21}}{\partial\Phi} = 2\cos\Phi\cdot\sin 2\delta\cdot\cos H^*$	$\frac{\partial Y_{21}}{\partial\lambda} = \sin 2\Phi\sin 2\delta\cdot\sin H^*$	
$\frac{\partial Y_{22}}{\partial\Phi} = -\sin 2\Phi\cos^2\delta\cdot\cos 2H^*$	$\frac{\partial Y_{22}}{\partial\lambda} = 2\cos^2\Phi\cos^2\delta\cdot\sin 2H^*$	
$\frac{\partial^2 Y_{20}}{\partial\Phi^2} = 6\cos 2\Phi\cdot\left(\sin^2\delta - \frac{1}{3}\right)$	$\frac{\partial^2 Y_{20}}{\partial\lambda^2} = 0$	$\frac{\partial^2 Y_{20}}{\partial\Phi\partial\lambda} = 0$
$\frac{\partial^2 Y_{21}}{\partial\Phi^2} = -4\sin 2\Phi\cdot\sin 2\delta\cos H^*$	$\frac{\partial^2 Y_{21}}{\partial\lambda^2} = -\sin 2\Phi\sin 2\delta\cdot\cos H^*$	$\frac{\partial^2 Y_{21}}{\partial\Phi\partial\lambda} = 2\cos 2\Phi\sin 2\delta\cdot\sin H^*$
$\frac{\partial^2 Y_{22}}{\partial\Phi^2} = -2\cos 2\Phi\cos^2\delta\cos 2H^*$ Eqs (3a)	$\frac{\partial^2 Y_{22}}{\partial\lambda^2} = -4\cos^2\Phi\sin^2\delta\cdot\cos 2H^*$ Eqs (3b)	$\frac{\partial^2 Y_{22}}{\delta\Phi\partial\lambda} = -2\sin 2\Phi\cdot\cos^2\delta\sin 2H^*$ Eqs (3c)

The strain field defined on a spherical surface (i.e. on the surface of the Earth $r = a$ (or on any spherical surface within it) can be decomposed into a spheroidal and a toroidal component. Because in this paper we concentrating on phenomena related to the tidal potential (Eq.(1)) only spheroidal strain is considered and the SNREI (symmetric non – rotating, elastic, isotropic) Earth with a liquid outer cove are taken into consideration.

In this case the tide generated displacement (Grafarend, 1986):

$$d(r,\lambda,\Phi) = (d_r, d_\lambda, d_\Phi)$$
$$d_{r_m}(r,\lambda,\Phi) = H(r)\cdot g(a)^{-1} Y_{nm}(\Phi,\lambda) V_n(r,d)$$
$$d_{\Phi_m}(r,\lambda,\Phi) = T(r)(rg(a))^{-1} \frac{\partial Y_{nm}(\Phi,\lambda)}{\partial \Phi} V_n(r,d)$$
$$d_{\lambda_m}(r,\lambda,\Phi) = T(r)(r\cos\Phi\cdot g(a))^{-1} \frac{\partial Y_{nm}(\Phi,\lambda)}{\partial \lambda} V_n(r,d) \qquad (4a)$$

The corresponding derivatives of the displacement vector are:

$$\frac{\partial d_{r_m}(r,\lambda,\Phi)}{\partial r} = \frac{\partial (H(r) V_2(r,d))}{\partial r} g(a)^{-1} Y_{nm}(\Phi,\lambda)$$
$$\frac{\partial d_{\Phi_m}(r,\lambda,\Phi)}{\partial \Phi} = T(r)(g(a)\cdot r)^{-1} \frac{\partial^2 Y_{nm}(\Phi,\lambda)}{\partial \Phi^2} V_n(r,d)$$
$$\frac{\partial d_{\lambda_m}(r,\lambda,\Phi)}{\partial \lambda} = T(r)(g(a)\cdot r\cdot \cos\Phi)^{-1} \frac{\partial^2 Y_{nm}(\Phi,\lambda)}{\partial \lambda^2} V_n(r,d) \qquad (4b)$$

3. EQUATION OF MOTION

In Eqs. (4) the auxiliary functions $H(r)$ and $T(r)$ are for characterisation of the radial and lateral displacement along the radius and at the surface $(r=a)$ they are the Love $h = H_{(a)}$ and Shida $\ell = T_{(a)}$ numbers respectively.

The Love number k is not needed in this study of the lunisolar strain tensor components It is:

$$k = R(a) - 1$$

The functions $H(r)$, $T(r)$ and $R(r)$ for different orders n can be obtained from the solution of the equation system of motion.

Let us introduce the following auxiliary functions (Molodensky, 1953):

$$M_n(r) = r^2 \mu(r)\left(\frac{\partial T_n(r)}{\partial r} + H_n(r) - \frac{2}{r} T_n(r)\right) \qquad (5)$$
$$N_n(r) = (\lambda^*(r) + 2\mu(r))\frac{\partial H_n(r)}{\partial r} + \lambda(r)\left[\frac{2}{r} H_n(r) - \frac{n(n+1)}{r^2} T_n(r)\right] \qquad (6)$$
$$L_n(r) = r^2\left(\frac{\partial R_n(r)}{\partial r} - 4\pi G\rho(r) H_n(r)\right) \qquad (7)$$

With the use of Eqs.(5) – (7) the equations of motion can be written as

$$-\frac{\partial M_n(r)}{\partial r} = \rho(r)\cdot r^2\left(R_n(r)+\frac{\partial W(r)}{\partial r}H_n(r)\right)+N_n(r)r^2+$$
$$+2\mu(r)\left[H_n(r)-\left(n^2+n-1\right)T_n(r)-\frac{\partial H_n(r)}{\partial r}r^2\right] \tag{8}$$

$$-\frac{\partial N_n(r)}{\partial r} = \rho(r)\left[\frac{L_n(r)}{r^2}-4\frac{\partial W(r)}{\partial r}\frac{H_n(r)}{r}+\frac{n(n+1)}{r^2}T_n(r)\frac{\partial W(r)}{\partial r}\right]+$$
$$+\frac{2\mu(r)}{r}\left[2\frac{\partial H_n(r)}{\partial r}-\frac{2H_n(r)}{r}+\frac{n(n+1)}{r^2}T_n(r)\right]-\frac{n(n+1)}{r^n}M_n(r) \tag{9}$$

$$\frac{\partial L_n(r)}{\partial r} = n(n+1)\left(R_n(r)-4\pi G\rho(r)T_n(r)\right) \tag{10}$$

where $\mu=\mu(r)$ and $\lambda^*=\lambda^*(r)$ are the Lamé constans, $\rho(r)$ is the density function and $W=W(r)$ denotes the geopotential.

To describe the normal (radial) and horizontal (lateral) strain components the Eqs 5, 6 are needed which provide the tangential (M_m) and normal (N_n) stresses. The auxiliary functions H_n and T_n are also needed for the radial and the tangential displacements.

If a system of dimensionless units is introduced (the radius of the Earth is $a=1$, $g(a)=1$ is the mean acceleration of the gravity at the surface, the unit of the density is the mean density of the Earth and the gravitional constant is $G=\frac{3}{4}\pi$) the boundary conditions at the Earth's surface $(r=a=1)$ can be given as (Varga, 1983):

$$N_n(a) = -\frac{2n+1}{3}\cdot\alpha_N$$
$$M_n(a) = \frac{2n+1}{3n(n+1)}\cdot\alpha_M$$
$$L_n(a) = (2n+1)\cdot\alpha_L \tag{11}$$

α_N, α_M and α_L may take only the values 0 or 1.

1. In the case of earth tides

$$\alpha_N=0 \quad \alpha_M=0 \quad \alpha_L=1$$

and we get with Eqs. (5) –(11) the Love-Shida numbers h_n, k_n and ℓ_n.

2. If the normal load acting at the Earth's surface

$$\alpha_N=1 \quad \alpha_M=0 \quad \alpha_L=1$$

and we get the load numbers $h_n^{'}, k_n^{'}$ and $\ell_n^{'}$.

3. If the stress which acts on the Earth is not connected with masses, consequently the load is potential free, the so called potential free Love numbers can be obtained

$\alpha_N=1 \quad \alpha_M=0 \quad \alpha_L=0$ for the normal stress

$\alpha_N=0 \quad \alpha_M=1 \quad \alpha_L=0$ for the tangential stress

The two triplets of potential free Love numbers can be denoted as $h_n^{''}, k_n^{''}, \ell_n^{''}$ and $h_n^{'''}, k_n^{'''}, \ell_n^{'''}$.

4. THE STRAIN TENSOR, SURFACE AND VOLUME DILATATION

The linear strain tensor has the following components:

$$e_{rr_m} = \frac{\partial d_{r_m}}{\partial r} \tag{12}$$

$$e_{\Phi\Phi_m} = \frac{1}{r}\frac{\partial d_{\Phi_m}}{\partial \Phi} + \frac{d_{r_m}}{r} \tag{13}$$

$$e_{\lambda\lambda_m} = \frac{1}{r\cdot\cos\Phi}\frac{\partial d_{\lambda m}}{\partial \lambda} + \frac{dr_m}{r} + \operatorname{tg}\Phi\frac{d_{\Phi_m}}{dr} \tag{14}$$

$$e_{r\Phi_m} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial d_{r_m}}{\partial \Phi} + \frac{\partial d_{\Phi m}}{\partial r} - \frac{d_{\Phi m}}{r}\right) \tag{15}$$

$$e_{r\lambda_m} = \frac{1}{2}\left(\frac{\partial d_{\lambda m}}{\partial r} + \frac{1}{r\cdot\cos\Phi}\frac{\partial d_{rm}}{\partial \lambda} - \frac{d_{\lambda m}}{r}\right) \tag{16}$$

$$e_{\Phi\lambda_m} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial d_{\lambda m}}{\partial \Phi} + \frac{1}{r\cos\Phi}\frac{\partial d_{\Phi m}}{\partial \lambda} - \operatorname{tg}\Phi\frac{d_{\lambda_m}}{r}\right) \tag{17}$$

To describe the surface deformation and the dilatation

$e_{\Phi\Phi} + e_{\lambda\lambda}$ and $e_{\Phi\Phi} + e_{\lambda\lambda} + e_{rr}$

the following auxiliary functions can be introduced:

$$S_n(r) = \frac{1}{r}\left[2H_n(r) - n(n+1)T_n(r)\right] \tag{18}$$

$$F_n(r) = \frac{\partial H_n(r)}{\partial r} + \frac{2}{r}H_n(r) - \frac{n(n+1)}{r^2}T_n(r) \tag{19}$$

Eqs.(12) – (17) render possible, together with Eqs. (4) – (10) the determination of the strain-tensor components for the elastic Earth mantle:

$$e_{rr}(r) = \frac{1}{\lambda(r) + 2\mu(r)}\{\lambda(r)[n(n+1)T_n(r) - 2H_n(r)] + N_n(r)\}V_n(r,d)Y_{n,m}(\Phi,\lambda) \tag{20}$$

$$e_{\Phi\Phi}(r) = \left[\frac{T_n(r)}{r}\frac{\partial^2 Y_{n,m}(\Phi,\lambda)}{\partial \Phi^2} + \frac{H_n(r)}{r}Y_{n,m}(\Phi,\lambda)\right]V_n(r,d) \tag{21}$$

$$e_{\lambda\lambda}(r) = \left[\frac{T_n(r)}{r}\left(\operatorname{tg}\Phi\frac{\partial Y_{n,m}(\Phi,\lambda)}{\partial \Phi} + \frac{1}{\cos^2\Phi}\frac{\partial^2 Y_{n,m}(\Phi,\lambda)}{\partial \lambda^2}\right) + \frac{H_n(r)}{r}Y_{n,m}(\Phi,\lambda)\right]V_n(r,d) \tag{22}$$

$$e_{\Phi\lambda}(r) = \frac{T_n(r)}{r\cos\Phi}\left(2\frac{\partial^2 Y_{n,m}(\Phi,\lambda)}{\partial \Phi\partial\lambda} - \operatorname{tg}\Phi\frac{\partial Y_{n,m}(\Phi,\lambda)}{\partial \Phi}\right)V_n(r,d) \tag{23}$$

$$e_{r\Phi}(r) = \frac{M_n(r)}{\mu(r)} \frac{\partial Y_{n,m}(\Phi,\lambda)}{\partial \Phi} V_n(r,d) \qquad (24)$$

$$e_{r\lambda}(r) = \frac{M_n(r)}{\mu(r)} \frac{1}{\cos\Phi} \frac{\partial Y_{n,m}(\Phi,\lambda)}{\partial \lambda} V_n(r,d) \qquad (25)$$

Surface deformation and dilation:

$$e_{\Phi\Phi}(r) + e_{\lambda\lambda}(r) = \frac{1}{r}\left[2H_n(r) - n(n+1)T_n(r)\right] Y_{n,m}(\Phi,\lambda) V_n(r,d) \qquad (26)$$

$$e_{\Phi\Phi}(r) + e_{\lambda\lambda}(r) + e_{rr}(r) = \\ = \left\{\frac{1}{r}\left(2H_n(r) - n(n+1)T_n(r)\right)\left(\frac{2\mu(r)}{\lambda(r)+2\mu(r)}\right) + \frac{N_n(r)}{\lambda(r)+2\mu(r)}\right\} Y_{n,m}(\Phi,\lambda) V_n(r,d) \qquad (27)$$

· In case of lunisolar effect there is no normal and lateral stress on the Earth's surface $(N(a) = M(a) = 0)$. Therefore if $r = a$ the r.h.s. of Eqs. (24) – (25) are equal to zero, in case of Eq. (20) the 2^{nd} term of r.h.s. $is = 0$.

· If $r = a$ than $H_n(a) = h_n$ and $T_n(a) = \ell_n$.

Evidently different equations are valid for zonal $(m = 0)$, tesseral $(m = 1)$ and sectorial $(m = 2)$ tides. With the use of (20) – (27) together with Eqs. (2) – (4) for the strain tensor components the following expressions can be obtained.

a. Zonal strain-tensor

$$e_{rr} = \frac{3\lambda(a)}{\lambda(a)+2\mu(a)}(6\ell_2 - 2h_2)\left(\sin^2\Phi - \frac{1}{3}\right)\left(\sin^2\delta - \frac{1}{3}\right)V_2(a,d)$$

$$e_{\Phi\Phi} = \left[6\ell_2 \cos 2\Phi + 3h_2\left(\sin^2\Phi - \frac{1}{3}\right)\right]\left(\sin^2\delta - \frac{1}{3}\right)V_2(a,d)$$

$$e_{\lambda\lambda} = 3\left[(2\ell_2 + h_2)\sin^2\Phi - \frac{h_2}{3}\right]\left(\sin^2\delta - \frac{1}{3}\right)V_2(a,d)$$

$$e_{\Phi\lambda} = 6\ell_2 \frac{\sin\Phi}{\cos^2\Phi}\left(\sin^2\delta - \frac{1}{3}\right)V_2(a,d)$$

$$e_{r\Phi} = 0$$

$$e_{r\lambda} = 0$$

b. Tesseral strain-tensor

$$e_{rr} = \frac{\lambda(a)}{\lambda(a)+2\mu(a)}(6\ell_2 - 2h_2)\sin 2\Phi \sin 2\delta \cos H^* V_2(a,d)$$
$$e_{\Phi\Phi} = (h_2 - 4\ell_2)\sin 2\Phi \sin 2\delta \cos H^* V_2(a,d)$$
$$e_{\lambda\lambda} = [2\ell_2 \operatorname{tg}\Phi(2\cos 2\Phi - 1) + h_2 \sin 2\Phi]\sin 2\delta \cos H^* V_2(a,d)$$
$$e_{\Phi\lambda} = \frac{2\ell_2}{\cos\Phi}\cos 2\Phi(2\sin H^* - \operatorname{tg}\Phi \cos H^*)\sin 2\delta V_2(a,d)$$
$$e_{r\Phi} = 0$$
$$e_{r\lambda} = 0$$

c. Sectorial strain-tensor

$$e_{r\Phi} = 0$$
$$e_{r\lambda} = 0$$

$$e_{rr} = \frac{\lambda(a)}{\lambda(a)+2\mu(a)}(6\ell_2 - 2h_2)\cos^2\Phi \cos^2\delta \cos 2H^* V_2(a,d)$$
$$e_{\Phi\Phi} = (h_2 \cos^2\Phi - 2\ell_2 \cos 2\Phi)\cos^2\delta \cos 2H^* V_2(a,d)$$
$$e_{\lambda\lambda} = [h_2 \cos^2\Phi \cos^2\delta - 2\ell_2(\sin^2\Phi - 2)\sin^2\delta]\cos 2H^* V_2(a,d)$$
$$e_{\Phi,\lambda} = \frac{\ell_2}{\cos\Phi}\sin 2\Phi(\operatorname{tg}\Phi - 4)\cos^2\delta \sin 2H^* V_2(a,d)$$

For the surface deformation and dilation we obtained:

a. Zonal tides

$$e_{\Phi\Phi} + e_{\lambda\lambda} = (2h_2 - 6\ell_2)\left(3\sin^2\Phi - \frac{1}{3}\right)\left(\sin^2\delta - \frac{1}{3}\right)V_2(a,d)$$
$$e_{rr} + e_{\Phi\Phi} + e_{\lambda\lambda} = \frac{2\mu(a)}{\lambda(a)+2\mu(a)}(2h_2 - 6\ell_2)\left(3\sin^2\Phi - \frac{1}{3}\right)\left(\sin^2\delta - \frac{1}{3}\right)V_2(a,d)$$

b. Tesseral tides

$$e_{\Phi\Phi} + e_{\lambda\lambda} = (2h_2 - 6\ell_2)\sin 2\Phi \sin 2\delta \cos H^* V_2(a,d)$$
$$e_{rr} + e_{\Phi\Phi} + e_{\lambda\lambda} = \frac{2\mu(a)}{\lambda(a)+2\mu(a)}(2h_2 - 6\ell_2)\sin 2\Phi \sin 2\delta \cos H^* V_2(a,d)$$

c. Sectorial tides

$$e_{\Phi\Phi} + e_{\lambda\lambda} = (2h_2 - 6\ell_2)\cos^2\Phi \cos^2\delta \cos 2H^* V_2(a,d)$$
$$e_{rr} + e_{\Phi\Phi} + e_{\lambda\lambda} = \frac{2\mu(a)}{\lambda(a)+2\mu(a)}(2h_2 - 6\ell_2)\cos^2\Phi \cos^2\delta \cos 2H^* V_2(a,d)$$

5. THE ROTATIONAL STRAIN TENSOR COMPONENTS

To describe the deformations beside the strains the rotation vector also has to be used. They three components can be related to the three shear components of the strain tensor.
Eqs. (15) – (17) can be written in the following form:

$$2e_{\Phi\lambda_m}(r) = \frac{1}{r\cos\Phi}\left[\frac{\partial}{\partial\Phi}\left(d_{\lambda_m}\cdot\cos\Phi\right) + \frac{\partial d_{\Phi_m}}{\partial\lambda}\right] \tag{28}$$

$$2e_{\lambda r_m}(r) = \frac{1}{r\cdot\cos\Phi}\frac{\partial d_{rm}}{d\lambda} + \frac{1}{r}\frac{\partial}{\partial r}\left(rd_{\lambda_m}\right) = 0 \tag{29}$$

$$2e_{\Phi r_m}(r) = \frac{1}{r}\frac{\partial}{\partial r}\left(rd_{\Phi_m}\right) + \frac{1}{r}\frac{\partial d_{rm}}{\partial\lambda} = 0 \tag{30}$$

The rotational components around axes normal to the plane of corresponding shear strains are

$$2\rho_{r_m}(r) = \frac{1}{r\cos\Phi}\left[\frac{\partial}{\partial\Phi}\left(d_{\lambda_m}\cos\Phi\right) - \frac{\partial d_{\Phi_m}}{\partial\lambda}\right] = 0 \tag{31}$$

$$2\rho_{\Phi_m}(r) = \frac{1}{\cos\Phi}\frac{\partial d_{\lambda_m}}{\partial\lambda} - \frac{1}{r}\frac{\partial}{\partial r}\left(rd_{\lambda_m}\right) \tag{32}$$

$$2\rho_{\lambda_m}(r) = \frac{1}{r}\frac{\partial}{\partial r}\left(rd_{\Phi_m}\right) - \frac{1}{r}\frac{\partial d_{r_m}}{\partial\lambda} \tag{33}$$

These three equations with Eqs. (4) – (10) give the rotational components in the form:

$$\rho_{r_m}(r) = \frac{M(r)}{\mu(r)\cos\Phi}\frac{\partial Y_{nm}(\Phi,\lambda)}{\partial\lambda}V_n(r,d) = 0$$

$$\rho_{\Phi_m}(r) = \left[\frac{2}{r}\left(H_n(r) - T_n(r) - \frac{M_n(r)}{\mu(r)}\right)\right]\frac{1}{\cos\Phi}\frac{\partial Y_{n,m}(\Phi,\lambda)}{\partial\lambda}V_n(r,d)$$

$$\rho_{\lambda_m}(r) = \left[\frac{2}{r}\left(H_n(r) - T_n(r) - \frac{M_n(r)}{\mu(r)}\right)\right]\frac{\partial Y_{n,m}(\Phi,\lambda)}{\partial\Phi}V_n(r,d)$$

Similarly to the strain-tensor for the surface of the Earth we have:

a. Zonal rotational components

$$\rho_\Phi = 0$$

$$\rho_\lambda = 6(h_2 - \ell_2)\sin 2\Phi\left(\sin^2\delta - \frac{1}{3}\right)V_2(a,d)$$

b. Tesseral rotational components

$$\rho_\Phi = 4(h_2 - \ell_2)\sin\Phi\sin 2\delta\sin H^* V_2(a,d)$$

$$\rho_\lambda = 4(h_2 - \ell_2)\cos 2\Phi\sin 2\delta\sin H^* V_2(a,d)$$

c. Sectorial rotational components

$$\rho_\Phi = 4(h_2 - \ell_2)\cos\Phi\cos^2\delta\sin 2H^*V_2(a,d)$$
$$\rho_\lambda = -4(h_2 - \ell_2)\cos\Phi\sin\Phi\cos^2\delta\sin 2H^*V_2(a,d)$$

Why is it interesting to observe the rotational strain?

a. Tidal observations (magnitude $\sim 10^{-8}$).
b. The rotational strain about the vertical axis responds merely to SH waves. Seismic waves S and Love waves can be recorded without any disturbance caused by P waves.
c. Tectonical rotational strain can be observed ($n \cdot 10^{-8} rad \cdot y^{-1}$, where $n \leq 5$) (Voosoghi, 2000).
d. Rotational motions in the earthquake source area.

Realization of the rotational strainmeter

1. WATANABE H. 1962
2. OZAWA I. 1966

· The sensitivity of the instruments was at that time ~ 10^{-9}. Today this value is by one or two order lower
· The drift and the relatively low resolution was due to galvanometric recording of the signals generated by electrodynamic transducers or due to the use a horizontal pendulum to transform differential variations into tilt.

6. CONCLUSIONS

The rotation strain components can be observed both by means of rotational strainmeters similar to proposed by Watanabe (1962) and Ozawa (1975) long time ago or by the possibilities offored by present day methods of space geodesy.

The rotational strain about the vertical axis responds merely to SH waves. Seismic S and Love waves with rotational strainmeters can be recorded without any disturbance caused by P waves.

Tectonicalrotational strain can be observed by VLBI and GPS observations. Their magnitude is $10^{-7} - 10^{-8} rad \cdot year^{-1}$ (Voosoghi, 2000).

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References

Grafarend E., 1986: Three-dimensional deformation analysis: global vector spherical harmonic and local finite element representation, Tectonophysics, 130, 337-359.

Melchior P., 1983: The tides of the planet Earth. Second Edition. Pergamon Press.

Molodensky S. M., 1953: Elastic tides, free nutations and some questions concerning the inner structure of the Earth. Trudi Geofis. Inst-Akad. Nauk SSSR, 19(146), 3-42 (in Russian).

Ozava I., 1975: Rotational strainmeter and the observation of the shear strain of the Earth tide with this intrumnt, Bulletin d'Information Marees Terrestres No. 52, 2398-2409.

Varga P., 1983: Potential free Love numbers. Manuscripta Geodetica, 8, 85-92.

Varga P., Grafarend E., 1996: Distribution of the lunisolar tidal elastic stress tensor components within the Earth's mantle. Physics of the Earth and Planetary Interiors, 93, 285-297.

Watabene H., 1962: A rotational strain seismometer. Disaster Prevention Research Institute Kyoto University, Bulletin No. 58, 1-15.

Voosoghi B., 2000: Intrinsic deformation analysis of the Earth surface based on 3 dimensional displacement fields derived from space geodetic measurenments. Technical Reports Department of Geodesy and Geoinformatics (Universitaet Stuttgart) Report Nr. 2000.3.