Parameters of the fluid core resonance estimated from superconducting gravimeter data

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Abstract

We have estimated the parameters of FCR (Free Core Resonance) based on the gravity data obtained from four SGs (Superconducting Gravimeters) at Esashi and Matsushiro in Japan, Canberra in Australia and Membach in Belgium. The corrections applied to the observed tidal factor and phase are of two for the phase delaying due to the analog filter of the SGs and for the ocean tides. We have compared the ocean tide effects computed from four global ocean tide models. They are Schwiderski model (1980) and three models of NAO99bJ (Matsumoto et al., 2000), CSR4.0 (Eans and Bettadpur, 1994) and GOT99.2b (Ray, 1999), which were derived from the TOPEX/Poseidon (T/P) altimeter data. While the three T/P models give very consistent correction values each other at each observation site, Schwiderski model clearly shows a systematic difference from them in both the amplitude and phase. We used here NAO99bJ model for the correction. The observed tidal admittances (i.e. complex tidal factors) were fitted to a dumped harmonic oscillator as a model for FCR and we have obtained the following values by stacking the four sites; 429.66+/-1.43 sidereal days, 9,350 - 10,835, -4.828E-4+/-3.4E-6, -3.0E-5+/-4.5E-6 for the eigenperiod, the Q-value and the real and imaginary parts of the resonance strength, respectively. Our results for the gravity data suggest that a systematic difference between two estimations from the gravity and the nutation, which has been shown in the previous works, is mainly caused by the inaccurate correction for the ocean tide effects.

1. Introduction

Precise observational determination of the FCR parameters is important for the study in geodynamics, because they give us useful information to constrain the physical parameters at the CMB (Core Mantle Boundary) related to the coupling between the Earth’s core and mantle.

One of the motivations of this study is to clear the reason/s of a systematic difference in FCR parameters deduced from two different kinds of observations (i.e. tide and nutation), in particular the difference in Q-values, which has been shown in the previous works (for example, Neuberg et al., 1987 and Sato et al., 1994 for the gravity tide, Gwin et al., 1986 and Defaigne et al., 1994, 1995 for the nutation). Thus, in general, it is observed a tendency that the Q-values estimated from the SG data are smaller than those estimated from the VLBI nutation data. Defaigne et al. (1994) estimated the FCR parameters by stacking two data sets of the SG and VLBI. According to their results, while the results for stacking both data sets or stacking only the VLBI data sets give a large Q-value exceeding 40,000, the results for stacking only using the SG data show a small value of about 4,000, which is about 10 times smaller than the VLBI data.

2. Data and analysis

We analyzed the data obtained at four SG sites; three of GGP-Japan network, namely, Esashi and Matsushiro in Japan and Canberra in Australia, and one is Membach in Belgium. For Membach, we used the GGP data archived at International Center for Earth Tides and distributed by the center as ‘GGP-Data CD#A2’. The reason why we chose this site from the GGP sites is that 5 years data in length are available for this site. Moreover, it is well known that the ocean tide effect in the diurnal tidal band is small in the central

Europe. Therefore, this may give us useful information to see an accuracy of the estimations of ocean tide effects. The geographical position of each site and the data length used here are listed in Table 1.

The tidal factors and phases were analyzed by means of an algorithm 'BAYTAP-G' (Tamura et al., 1991) using the 1-hour data corrected for the spikes and steps to the original data. To separate the atmospheric pressure effect on the gravity data, the local pressure data obtained at each site were taken in the analysis by means of a response term to be estimated.

3. Scale factors of the SGs

Figure 2 shows a comparison of the results for absolute calibration of the scale factors (mGal/Volt) to the changes in relative sensitivity those are represented using the temporal variation of the amplitude coefficients of M_2 wave. The comparison at Esashi is displayed in this figure. The amplitude coefficients of M_2 wave were obtained from successive monthly analysis of the original 1-hour data of Volt unit for the 10 years shown in Table 1. We see in Fig. 2 that the relative sensitivity of the SG is very stable, even though it is observed an annual variation for this wave or some outliers mainly caused by interruptions of the observation. The linear trends shown in Fig. 2 suggest that the change in the M2 amplitude coefficients is only -0.01 % in magnitude during the 10 years. On the other hand, the scale factors calibrated with the absolute gravimeters show a change of +0.29 % for the same observation period, which is larger than the change in relative sensitivity by about 30 times.

A similar tendency is observed in the comparison at Canberra. The Canberra SG has been calibrated with the FG5 absolute gravimeters at three times during the three years of 1998-2000 and Amalvict et al. (2001) reported the computation results for these calibrations. According to their results, the absolute scale factors show a linear change with a rate of -1.2% during the three years. On the other hand, the change in the relative sensitivity estimated from monthly variations of the M_2 amplitude coefficient is +0.054 % in magnitude.

Although the reason/s making the difference between the changes in the absolute calibrations and those in the relative sensitivities is/are not clear yet, for the SGs at Canberra and Esashi, we adopted here a weighted mean averaged over the absolute calibration values as the scale factor of each SG. We used the RMS error of each calibration as the weight. For Matsushiro and Membach, we adopted the results by Imanishi et al. (2002) and the calibration table given by Royal Observatory of Belgium (Hendrickx, personal communication, 2002), respectively. The error in the calibrations with the absolute gravimeters is estimated at a range of 0.04 % to 0.2 %.

4. Corrections

Before correcting for the effects of ocean tides, we applied a correction for the phase delaying due to the analog filter of each SG to the analysis results by BAYTAP-G. We used following values (unit: degree per cycle per day) for the correction; -0.166, -0.166, -0.168, which were estimated from a method described in Imanishi et al. (1996), and -0.1608 (Camp et al., 2000) for Esashi, Canberra, Matsushiro and Membach, respectively. Although the analog filter of the Membach SG has been changed in January 1 1998 from the TIDE filter to the GGP1 filter, we analyzed the 5 years data without dividing them into two periods, in order to avoid degradation in frequency resolution of the analysis. This has been done by being artificially shifted the time of the theoretical tide computed with BAYTAP-G by the amount corresponding to the phase difference between the TIDE and GGP1 filters, for the observation period when the GGP1 filter is used. For the other three sites, we used the data obtained by the TIDE filter.

The ocean tide effects (attraction and loading) were estimated by using a computer code called 'GOTIC2' (Matsumoto and Sato, 2001). We have compared the four global ocean tide models; namely three of NAO99bJ (Matsumoto et al., 2000), CSR4.0 (Ray, 1999) and GOT99 (Eans and Bettadpur, 1994), which were based on the TOPEX/POSEIDON (T/P) altimeter data, and Schwiderski model (1980), which has been widely used in the study for Earth tides as a conventional standard global ocean tide model. As a result, it can be pointed out that the former three models based on the T/P data give very consistent amplitude and phase at any of the four observation sites and for any of the three major tidal waves of O_1, K_1 and M_2, even though these sites are largely separated in their locations on the earth. Compared with this, Schwiderski model shows a clear systematic difference from the T/P models in both the amplitude and phase. As an example, comparison at Canberra is shown in Figure 1.

We adopted here the NAO99bJ model for the ocean tide corrections. For the minor waves, which are not available to use a global ocean tide model, we adopt a correction value that was interpolated or extrapolated from nearby main waves (Matsumoto and Sato, 2002).

5. Estimation of FCR parameters

The FCR parameters were estimated by fitting the observed complex tidal admittances to a dumped harmonic oscillator as a model for the resonance. In order to reduce a possible effect on our analysis due to the calibration errors for the scale factor of SGs, we used the admittances normalized with the O_1 wave as previously Cummins and Wahr (1993) or Sato et al. (1994) adopted. Thus the model used here is;

$$G(wi) = F(wi)/F(w1) - 1 = B((wi-w1)/(w0-wi))$$
$$\text{with } w0 = 2pf_0 (1+jQ^{-1}/2).$$

Here wi, w1 and w0 are the angular frequencies of the i-th tidal wave, O_1 wave and FCR, respectively. F(wi) is the observed complex tidal admittance of the i-th wave, F(w1) that of the O_1 wave, B a complex coefficient representing the strength of resonance (Br for the real part and Bi for the imaginary part) , f_0 the real part of the eigenfrequency of FCR, Q^{-1} the inverse of Quality factor, and j the imaginary unit. The method used for fitting is a modification of ' Marquandt method', so that the variation range of unknown parameters to be fitted is banded by means of a dynamical biweight method (Nakagawa and Oyanagi, 1982). We searched the optimum combination of the initial values for fitting by applying this binding condition, but the final solutions were solved using the obtained optimum initial values without applying any binding conditions for all unknown parameters.

The final parameter values obtained from stacking of the four sites are 429.66+/-1.43 sidereal days (s.d.), 9,350 to 10,835, -4.828E-4+/-3.4E-6, -3.0E-5+/-4.5E-6 for the eigenperiod ($T_0 = 1/f_0$), Q-value, Br and Bi, respectively. Figure 3 shows the comparison between the observed admittances and the FCR curve computed using the parameters shown above. The waves near the resonance are displayed.

6. Discussions

6.1. Sensitivity change of SG

At Esashi, during the 10 years shown in Fig. 2, the superconducting sphere has been dropped many times by the effect of large earthquakes or to exercise the wintering members of Japanese Antarctic Research Expedition. Whenever the sphere was dropped at a frequency of once or twice a year, it has been levitated by adjusting the superconducting currents within a range of 4-5 A.

Although the adjustment of the superconducting currents of the Esashi SG has been done many times, as shown in Fig. 2, the mean relative sensitivity inferred from the M_2 amplitude coefficients is stable at a degree of about 0.01 % during the 10 years. This suggests that the sensitivity change of the SG itself is very small, insofar as we maintain the value of superconducting currents carefully. Related to this, it may be worth to note that, at both the Canberra and Esashi sites, the same analog/digital converters have been used through the observation periods shown in Table 1. On the other hand, judged from the temporal changes of the absolute calibrations and relative sensitivities shown in Fig. 2, we may also say that the long-term change in the observed amplitude of M_2 wave including the ocean tide effects is also small at the similar order shown in Fig. 2.

6.2. Reliability of the obtained FCR parameters

Based on the Esashi data, we have investigated the reliability of our FCR parameters by two methods. One is the Monte Calro method and other is a sensitivity test to the phase error in the Y_1 wave.

In the former test, the distribution of the solutions for each parameter was examined from 5000 data sets, which were generated by artificially adding Gaussian random numbers to the observed admittances of each wave. Variances of the random numbers are similar in magnitude to the RMS errors for the observed admittances. Although the distributions of solutions obtained from this test clearly show the correlations between f_0 and Br and between Q^{-1} and Bi as expected from the correlation matrix, we obtained the values of 428.607+/-0.137 s.d. and 14,436 to 15,076 for T_0 and Q-value as the mean values over the 5000 data sets. In this connection, by the fitting only using the observed Esashi admittances without stacking, we obtained 428.49+/-5.50 s.d. and 62,814 to 264,550 for the eigenperiod and the Q-value, respectively.

On the other hand, the sensitivity test was carried out by artificially changing the Y_1 phase within a range of +/-1.5 deg (about 3 times of the observation error for this wave). The amplitude of this wave is small but the estimated FCR parameters are sensitive to the Y_1phase in particular the eigenperiod and the Q-value, because this wave has a period very close to the eigenperiod of FCR. The results are displayed in Fig. 4. The rectangular areas shown in this figure indicate the error range of the obtained parameter values, thus the magnitude of vertical side of each rectangular indicates the error range of the estimated parameters, which is expected from the error for the Y_1 phase shown by the horizontal side of each rectangular.

From the above two tests, we can say that (1) our solutions may have not a large systematic error due to the algorithm used in the fitting and (2) the estimation errors caused by the effects of the error in the observed Y_1 phase are at the orders of +/-1 day and -20,000 to 8333 for T_0 and Q-value, respectively. As pointed out by Zurn and Rydelek (1991), we have a case that the obtained Q-value shows a large negative value, even though the FCR cannot possess a negative Q. But it shows a possibility that the analysis for a system having a large Q-value brings a negative Q-value, when a combination of parameters in the form of f_0 /Q is used for fitting and the analysis is performed in the frequency domain. Such situation appears in the results for the sensitivity test to the Y_1 phase. However, this and the magnitude of Q-value obtained from the stacking indicate that the gravity data also give a large Q-value exceeding about 10,000 as well as that estimated from the nutation data.

6.3. Comparisons with theory

Many of previous analyses results obtained from various observation means indicate that the observed eigenperiod shows the significant sift from the value inferred from an elastic, rotating and oceanless earth model (i.e. about 460 sidereal days, for example, Wahr, 1981). Our result obtained here (429.66+/-1.43 s.d.) supports this too. Based on two models, namely a PREM hydrostatic model (Dziewonski and Anderson, 1981) and a nonhydrostatic inelastic model modified from the PREM model, Dehant et al. (1999a) give 456.98 s.d. and 431.37 s.d. as the eigenperiods for the former model and the later one, respectively. Our value is consistent with their non-hydrostatic inelastic value rather than the elastic-hydrostatic one and it is also consistent with the value obtained from the nutation data, for example, 433.5+/-0.3 s.d. to 433.9+/-0.5 s.d. which were obtained from the stacking the nutation data (Defraigne, et al., 1995).

Judged from the systematic difference in the ocean tide effects estimated from the Schwiderski model and from those from the T/P models (see Fig. 1), this study suggests that a systematic difference between the gravity tide and the nutation in particular in that for the Q-value is mainly due to inaccurate ocean tide correction in the previous studies, because the ocean tide effect on the nutation observation is much smaller than that on the tidal gravity observation (for example, Hass and Schuh, 1996, Dehant et al., 1999b). The Q-value obtained here indicates that, at the diurnal band, the effect of coupling at the CMB is weak as suggested from the nutation observations.

In this study, we have only used the NAO99bJ ocean model for the ocean tide correction. But it is needed to compare with the FCR parameters estimated from the admittances corrected using other ocean models, in order to constrain them much tightly. This and the comparison of the observed resonance parameters with the theory including the effect of phase shift due to the Earth's inelastic property (for example, Mathews, 2001) are remained for further study.

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Table 1. Gravimeter data used in this study

Site	Latitude	Longitude	Height (m)	SG	Period
Membach	50.609 N	6.007 E	250.00	CT#021	95/08/03 - 99/06/30
Esashi	39.148 N	141.335 E	393.00	T#007	92/01/26 - 01/12/31
Matsushiro	36.544 N	138.203 E	451.10	T#011	96/09/10 - 00/03/31
Canberra	35.321 S	149.008 E	724.00	CT#031	97/01/27 - 02/01/18

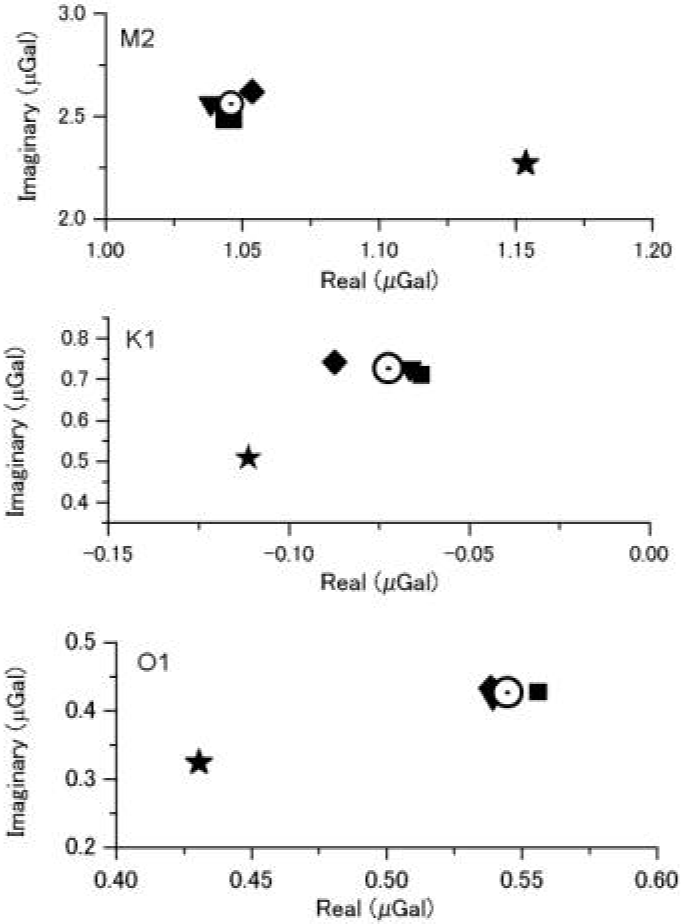

Fig. 1. Comparison of the ocean tide effects at Canberra computed from the four ocean tide models. The ocean models used are shown with the following symbols; Square: NAO99bJ, Reverse triangle: CSR4.0, Diamond: GOT99, and Star: Schwiderski. Dotted Circle shows the mean value over the three models of NAO99bJ, CSR4.0 and GOT99. A loading Green's function for the PREM earth model was used for the computation.

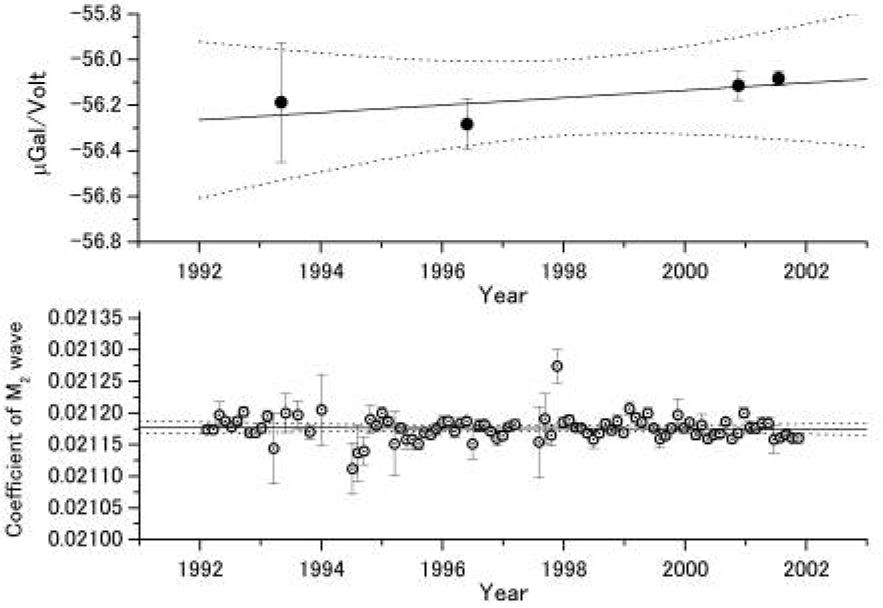

Fig. 2. Comparison between the absolute calibration and the relative sensitivity for the Esashi SG. Top: Results for the absolute calibrations and Bottom: Monthly changes in the M2 amplitude coefficient, which were computed from the data of volt unit. The solid lines and the dotted ones show the result for fitting to a linear model and the 95 % confidence intervals, respectively.

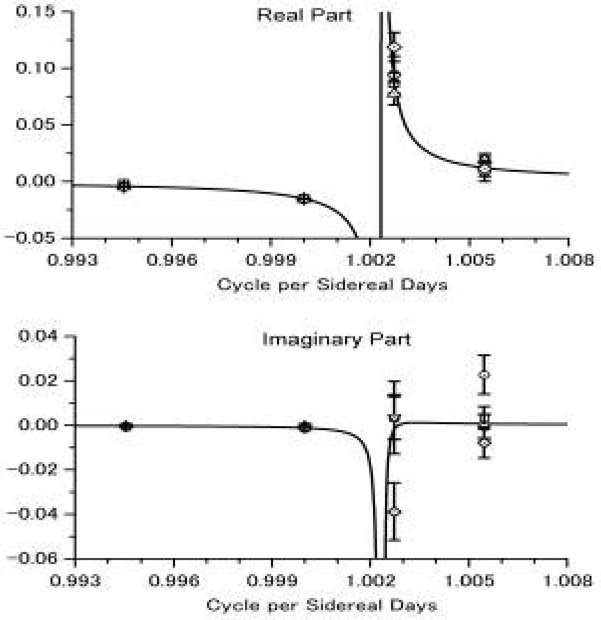

Fig. 3. Comparison between the observed admittances and the best fitted curves. Symbols stand for; Circle: Esashi, Square: Canberra, Triangle: Matsushiro, and Diamond: Membach.

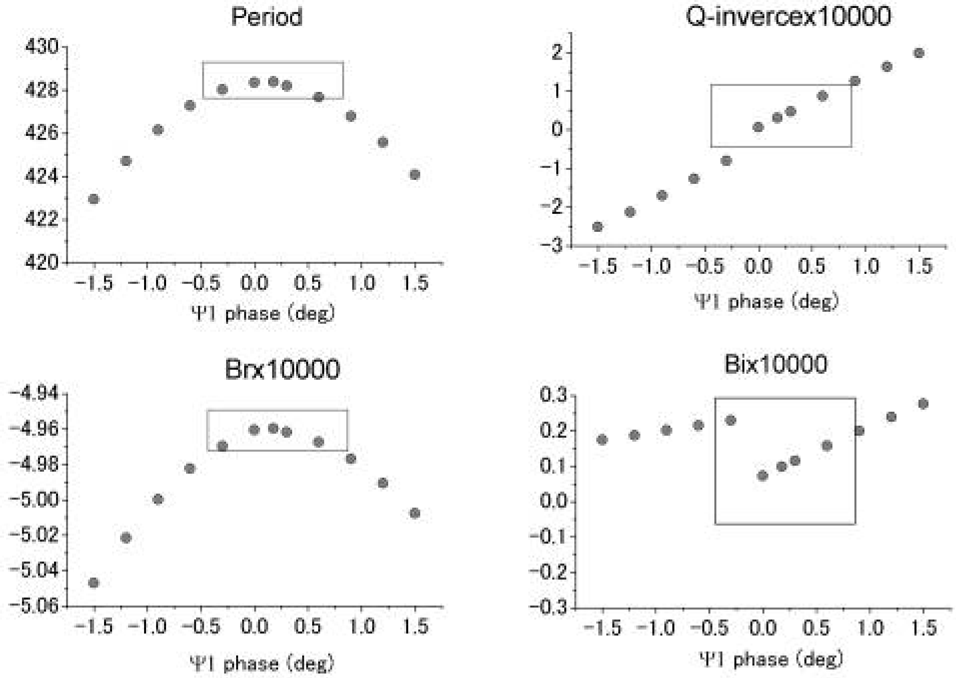

Fig. 4. Effect of the phase of Y_1 wave on the estimated FCR parameters. Rectangular boxes show the error range that would be expected from the observed RMS error of the Y_1 phase.