Using Atmosphere-Gravity Correlation to Derive a Time-Dependent Admittance

by

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Abstract

The purpose of this paper is to investigate the role of the correlation between atmospheric pressure and gravity in the context of a single scalar admittance. We consider two GGP data sets from Boulder and Strasbourg, both of which we processed from the raw data before decimating to 1minute files. The cross correlation and scalar admittance between gravity and pressure were determined for 3 averaging windows of 1 hour, 6 hours and 1 day. The data clearly show that cross correlation and admittance are related; furthermore, the shorter the averaging window the more the scatter in both quantities. We found that when the correlation was high the admittance tended towards a value that was higher for shorter windows and higher frequencies. Our attempts to use the correlation to improve the traditional assumption of a single scalar factor (e.g. -0.3 μgal $mbar^{-1}$) were unsuccessful, probably due to the inherent noise in the gravity residuals.

Introduction

The use of a single scalar admittance has been well established in computing the influence of atmospheric pressure fluctuations on gravity (e.g. Warburton and Goodkind, 1977; Merriam, 1992; Crossley et al. 1995). The assumption is that when atmospheric pressure p (mbar) is recorded with relative or absolute gravity g (μgal) at a single station, the gravity can be corrected by using the relation

$$g_c = g - \alpha\,(p - p_0), \qquad (1)$$

where α is taken to be either a nominal - 0.3 μgal $mbar^{-1}$ or determined by a least squares fit of p to g, thus minimising the residual gravity g_c. The effectiveness and simplicity of this method has led to its widespread use in gravity studies for many purposes. This loading correction can amount to 10 μgal or more during extreme weather (e.g. Rabbel and Zschau, 1985) and typically accounts for some 90% of the total atmospheric effect. It has been known for a long time, however, that this local correction can be improved, especially for monthly and seasonal periods, by including global atmospheric data available through the worldwide atmospheric data services, (e.g. Boy et al., 1998). Unfortunately this computation requires a fair amount of work to collect the data and convert it into a useful time series for each station at a certain epoch. Although the use of global atmospheric pressure is gradually gaining popularity for high precision studies, a single admittance still predominates most residual gravity computations.

Several approaches to using the local pressure more effectively have been attempted, particularly in using a frequency dependent admittance (e.g. Crossley et al. 1995; Neumeyer et al., 1997, 1998; Kroner, 1998). These studies show that improvements, in the form of lower residuals, are possible with additional work, but the methods have never achieved regular use. It has also been noticed that the admittance shows some variation with time (e.g. Richter, 1987; Van Dam and Francis, 1998), usually on seasonal time scales. Our concern here is with the possible variations of α on time scales of hours to days. The reason for this is that the atmosphere is certainly variable on these short time scales, and local weather systems can move rapidly over a station in a few hours (Müller and Zürn, 1983). There is no guarantee that the correlation implied by (1) is satisfied over all length and time scales.

In addition to the admittance α, we also need to define the general cross-correlation between pressure p_i and gravity g_i

$$r_{pg} = \sum_i [p_i g_i] \Big/ \sqrt{\sum_i p_i^2 \sum_i g_i^2} \qquad (2)$$

The summation in (2) is over a subset of the whole data, called here the averaging window; it is varied from 1 hour to 1 day. We also note in both (1) and (2) that the local mean values have been subtracted from both subsets before the calculations. It should be obvious that the correlation in (2) is computed from the same subset of the data as (1).

We used two superconducting gravimeter (SG) data sets in which we analysed the cross correlation between pressure and gravity, as well as the admittance. We will show there is a strong connection between these two quantities that implies the frequency dependence of α noted above. Secondly, we will attempt to use this connection to improve the standard admittance correction based on (1). Our motivation is clearly to reduce the residual gravity even further, assuming the correction is related to pressure, in gravity studies. Noting that the local effect also dominates the global corrections, any improvement in the former is worthwhile.

Admittance for Two Data Sets

We begin with 1 year of data (days 96001 - 96366) from the Boulder GGP station at Table Mountain Gravity Observatory, Colorado. It is important for this study that all (most) known signals, other than pressure, are first subtracted from gravity, because the calculation of (1)

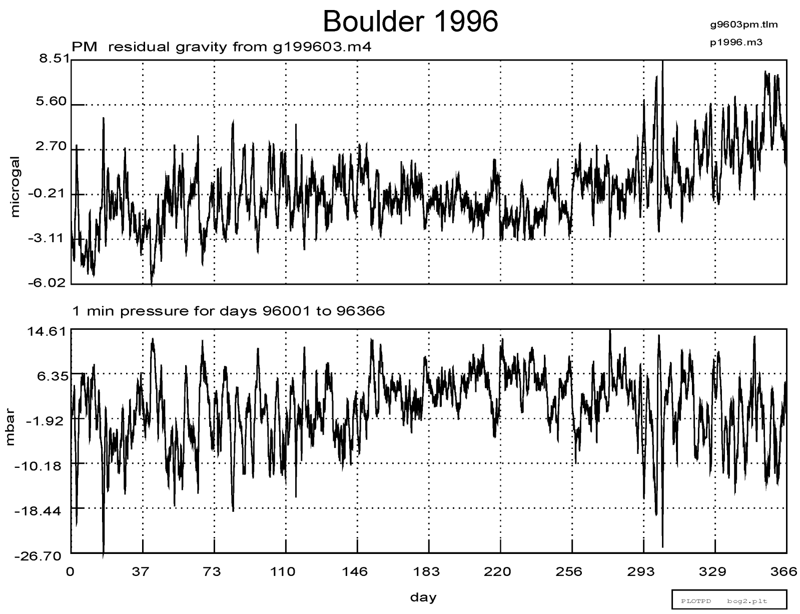

Figure 1. Residual gravity and pressure for BO.

and (2) is significantly affected by noisy data. We therefore fixed all obvious problems (gaps, spikes, offsets, earthquakes) in the pressure and gravity files, using the raw 5 second gravity and 1 minute pressure files. The gravity was then decimated to 1 minute and a local tide was subtracted using predetermined local tidal (delta, kappa) factors (see Crossley and Xu, 1998). Finally, we subtracted the IERS polar motion from the gravity. We show in Figure 1 the gravity residuals and pressure signals; it is clear that there is a large anti-correlation between the two series. This correlation can be seen more clearly in Figure 2 which shows the first 10 days of 1996.

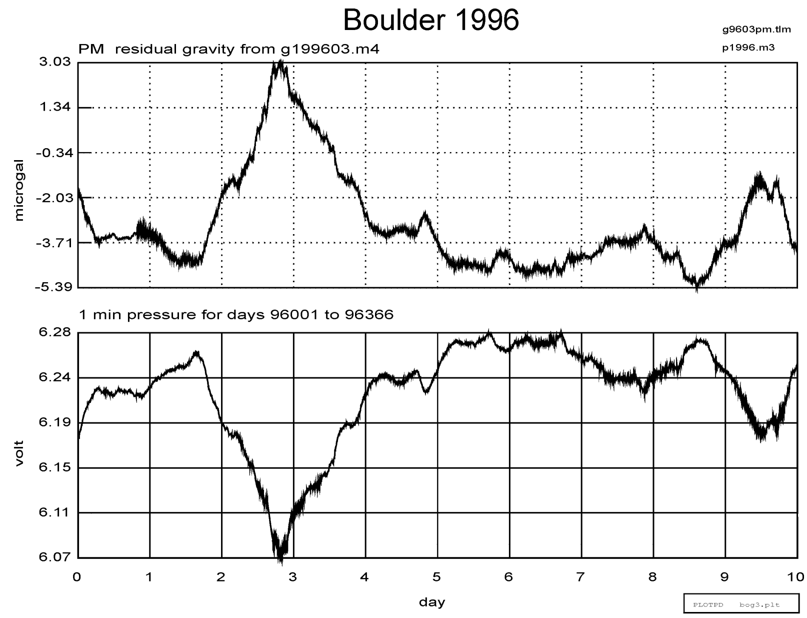

Figure 2. Residual gravity and pressure, 10 days.

Figure 3 shows identical treatment of data from the GGP station ST in Strasbourg (days 97113 - 97365). In this case both gravity and pressure are sampled every 2 sec, and the tidal parameters are of course different for this station.

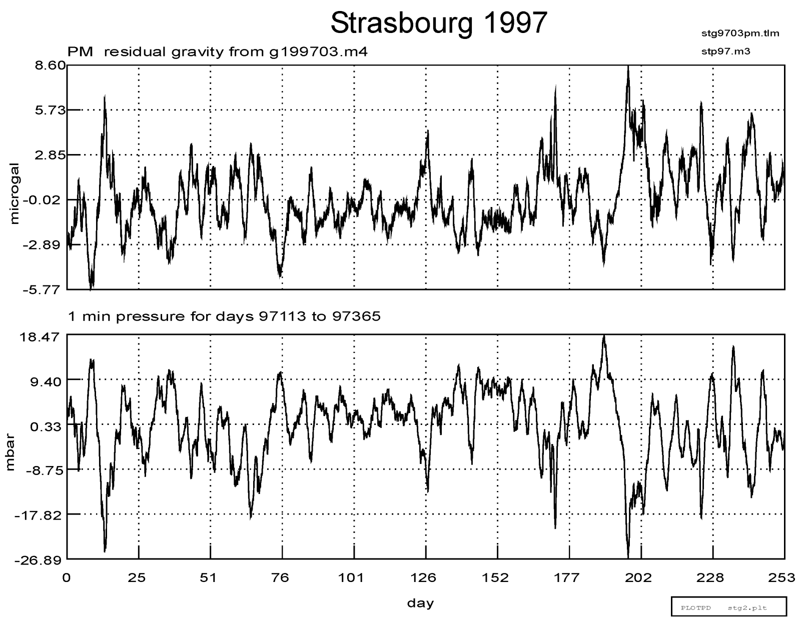

Figure 3. Residual gravity and pressure for Strasbourg

Despite the good correlation in these data sets, there are clearly times when the gravity does not respond directly to the pressure. An example can be seen in Figure 4, from Boulder in which the correlation is relatively poor, especially between minutes 2080 and 2140.

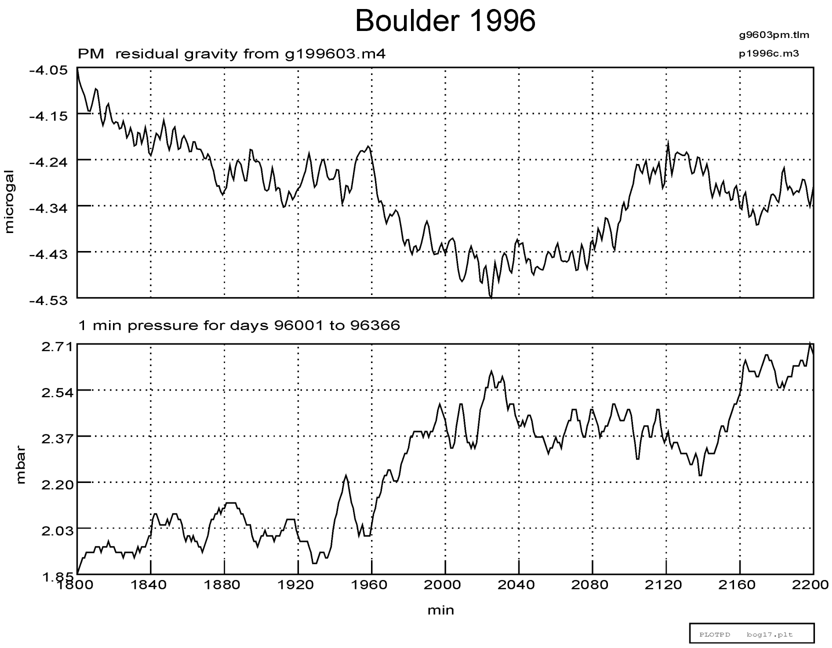

Figure 4. Residual gravity and pressure, detail showing poor correlation.

The question therefore arises - will the simple application of (1) sometimes inject an artificial and erroneous pressure signal into the gravity, instead of correcting for it?

We show the residual gravity (1) for BO for two standard calculations, one assuming that α is - 0.3 µgal $mbar^{-1}$ and the second for a best fit of the whole of the series (Figure 5). In the latter case α = - 0.239 and the overall r_{pg} = - 0.666, which is only a modest correlation.

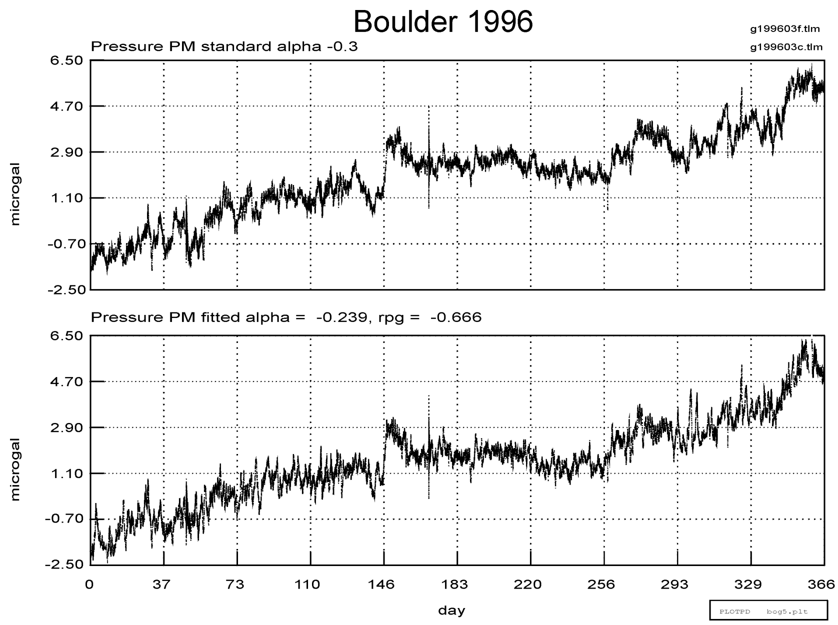

Figure 5. Corrected gravity for BO: nominal α (upper), fitted α (lower).

We repeat this for the ST series and obtain for the fitted case α = - 0.270 μgal mbar^{-1} and the overall correlation is very high, r_{pg} = - 0.910 (Figure 6). Note the both the admittance and the cross correlation are higher for ST than for BO.

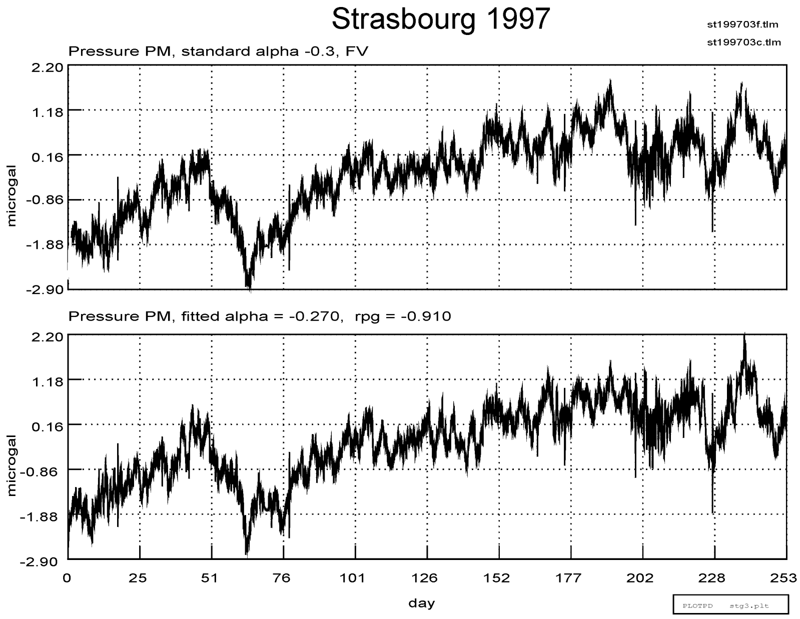

Figure 6. Corrected gravity for ST: nominal alpha (upper), fitted alpha (lower).

Relation Between Cross Correlation and Admittance

We now compute (1) and (2) for various subsets of the data:

- using the raw 5 or 2 sec data, with an averaging window of 1 hour (non-overlapping)
- the same for 6 hour and 1 day averages
- using the 1 minute data for each station, with an averaging window of 1 hour (non-overlapping)
- the same for 6 hour and 1 day averages.

Any non-pressure related signals in the gravity residuals obviously will corrupt the determination of correlation and admittance, especially for short averaging windows. Data spikes and instrumental disturbances should have no correlation with pressure. The period during the large surface waves of large earthquakes, for example, is one in which the correlation will be particularly bad. This is why a thorough cleaning of the data necessary.

For each experiment we produce a scatter plot of the cross correlation and admittance. Figure 7 shows the result for BO with a 1 hour averaging of the raw 5 sec data. Although the correlation coefficient must lie between -1 and 1 by definition, the values for the admittance of each block can be large; we show only those admittances between -2 and 2. There are some extreme values lying outside this plot.

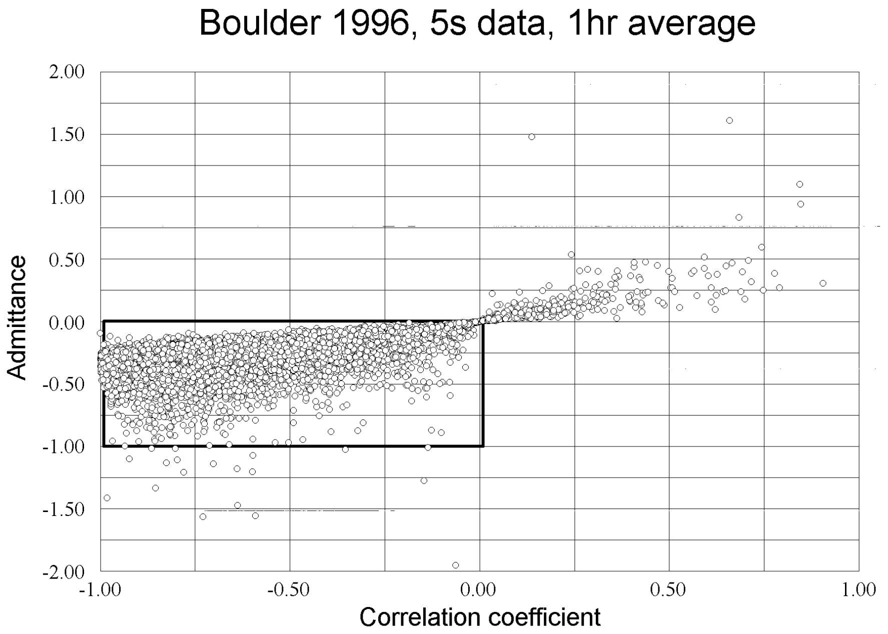

Figure 7. Scatter plot of Admittance vs Cross Correlation

There is clearly a well-defined pattern to the scatter plot, with a strong suggestion of a linear relation between α and r_{pg} extending from negative to positive correlations. Also obvious is the concentration of values around (0,0), indicating that where there is no correlation the admittance is also very small. For the rest of the study we concentrate on the portion of the plot between (-1,0) for both the correlation and also for α (in the heavy box).

We now quantify the above assumption and fit a straight line to the plot (Figure 8); in fact we show two fits, one for the L_1 norm (black) and the other for the L_2 norm (white). The slope of the two lines is 0.400 and 0.429 respectively. Of the 8784 hourly values for this year, 397 lie outside the acceptable limits for α .

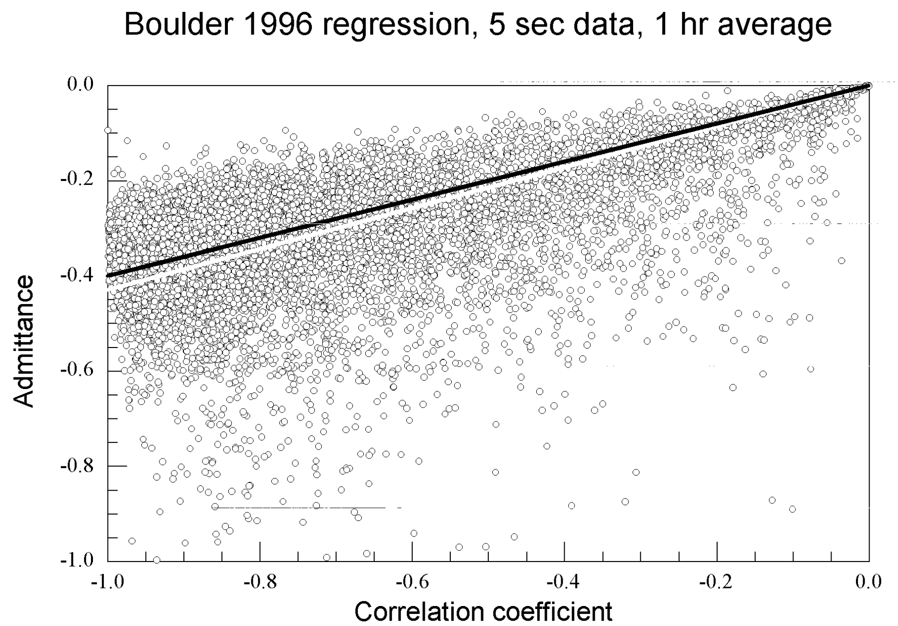

Figure 8. Admittance - correlation regression, 1 hour averaging.

Bearing in mind that the L_1 norm is more robust to outliers, we take its slope of 0.400 as indicative of the (α, r_{pg}) relationship. Note this value is also the intercept on the admittance axis when the pressure is perfectly anti-correlated with the gravity. Let us call this intercept the admittance factor α_0 for this data set.

As a contrast, we show in Figure 9 the same scatter plot, with fitted lines (almost coincident), for a 1 day averaging window of the same data. This time all 366 points lie within the acceptable limits for α.

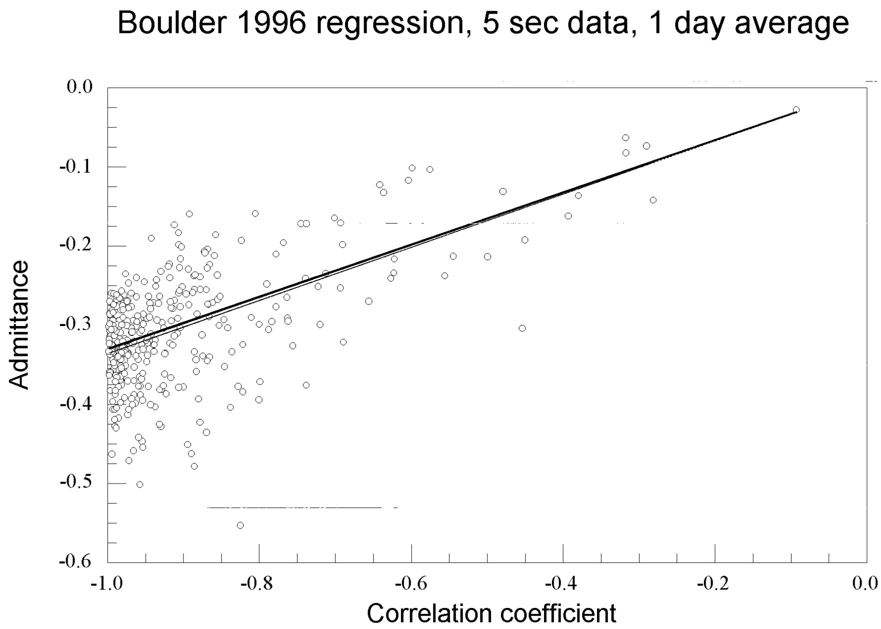

Figure 9. Admittance - correlation regression, 1 day averaging.

The admittance factors α_0 for the L_1 and L_2 lines are 0.330 and 0.336 respectively. Obviously, as the averaging window lengthens, the value of α_0 decreases because the lower frequencies have more influence in the data.

We performed a variety of similar experiments for averaging windows of 1 hour, 6 hours and 1 day on the Boulder data decimated to 1 min, and also on the ST data at 2 sec and 1 minute. Space does not permit all the plots to be shown, but they show the same characteristics as Figures 8 and 9, i.e. a linear trend with a well defined slope. These slopes, the admittance factors, are shown in summary form in Figure 10.

The symbols in Figure 10 refer to the station and year of the data (e.g. BO96, ST97) and the averaging window (e.g. -1hr), plotted against the data sampling. Note that we have added results from another year of BO data – 1997 - to compare the stability of the admittance with 1996; this data set is not otherwise discussed here.

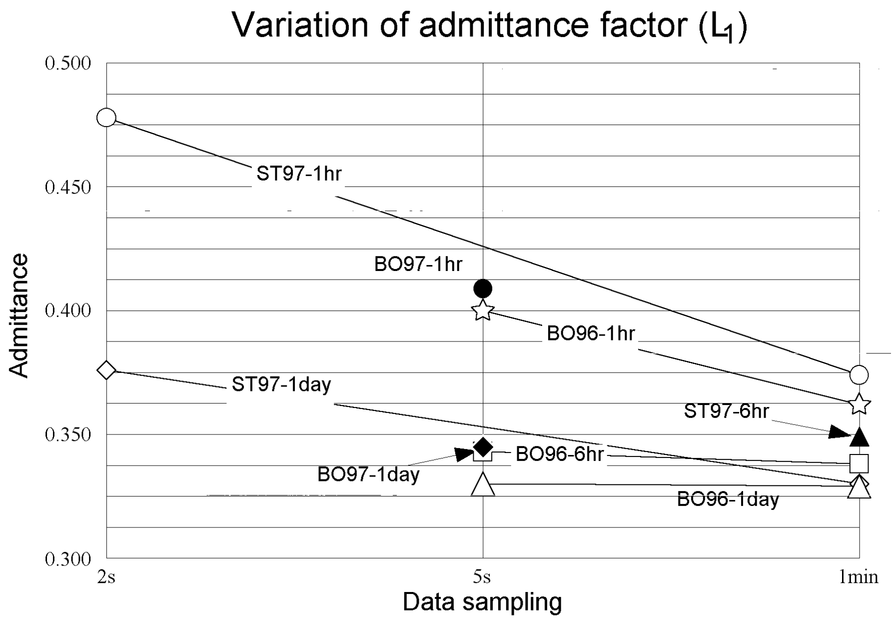

Figure 10. Admittance factors for a variety of data sets and averaging windows.

Our conclusions from Figure 10 are consistent with the variation of admittance with period (or frequency). We know that at high frequencies (periods of hours – minutes) the admittance approach a constant value of approximately –0.4 (e.g. Merriam, 1992; Crossley et al., 1995). At low frequencies (periods of months) the large scale atmospheric variations become important and the admittance decreases to values between -0.2 and -0.3. The data for Strasbourg in Figure 10 shows a high value of α (-0.478) probably because of the very high sampling periods and short averaging window.

Correcting Residual Gravity

One possible way to use the variables is to use the time dependent admittances (Figures 8 and 9) directly. The high scatter, however, includes a substantial number of values greater than 0.4, and these are not physically realistic. No doubt they would serve to reduce the residual gravity, but this would be an artificial reduction, much like that found in the frequency dependent treatment when the averaging window is reduced (Crossley et al., 1995).

Instead we make an assumption that the cross correlation can be used to derive an admittance from the straight line L_1 fits in the above figures. Thus, for each averaging window, we compute the admittance

$$\alpha = \alpha_0 \, r_{pg}, \qquad (3)$$

assuming a linear relationship. The constant α_0 is the admittance factor which is just the slope (intercept) of the L_1 line. With this approach the admittance α is reasonably constrained in value. Even so, we also decide that for values of the cross correlation outside (-1,0) we will take the admittance to be zero.

We now apply a version of (1) to correct the gravity at each sample point (2s, 5s, 1 min ...).

$$g_{ci} = g_i - \alpha_i \, (\mathrm{p} - \mathrm{p}_{\mathrm{ref}}), \qquad (4)$$

where p_{ref} is a reference pressure and α_i is the admittance computed from (3). We can take p_{ref} typically either as a fixed nominal value for the station (as in absolute gravity pressure corrections), the first value (FV) of the series, or the series mean value. Figure 11 shows the results of taking the reference pressure to be the FV and computing (4) for the BO 1 minute data using a 1 hour averaging window.

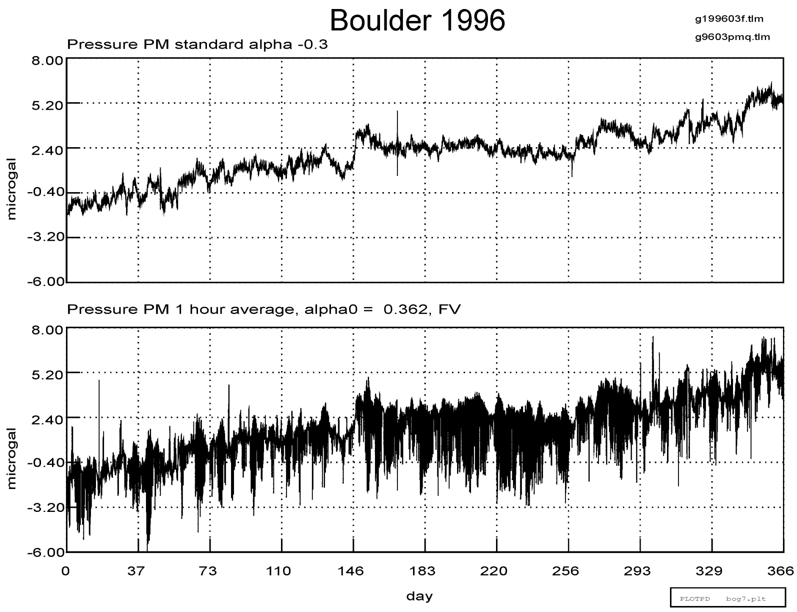

Figure 11. Gravity correction: fixed admittance (upper), variable alpha (lower). Note both plots are shown at the same scale.

The result is not encouraging. The lower curve shows large spikes that on closer inspection are coincident with places where the cross correlation falls to low values for some of the reasons given above. The spikes occur because a fixed reference is being used; identical results are found when using a mean pressure value as reference. If we extend the averaging window to 1 day the situation improves noticeably (Figure 12), basically because both the cross correlation and the admittance are much more stable in time.

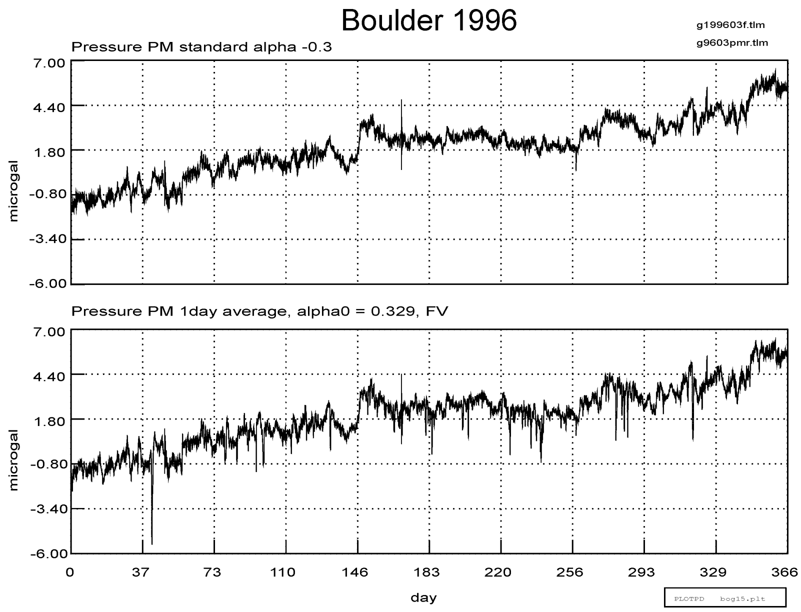

Figure 12. As Figure 11 but for 1 day averaging window.

We admit, however, that the gravity residuals for variable α, even for 1 day averaging, are still noisier than the standard correction. We can suggest two remedies, either restrict the value of α to a narrower range than permitted through (3), or change the pressure reference. In the latter case we need to ensure that when α is small (or even zero if r_{pg} is out of bounds) the correction is referred to the previous gravity and pressure values rather than a fixed reference. This latter idea can be implemented by using a moving pressure reference in the gravity correction:

$$\begin{aligned} \Delta g_i &= 0 && ,i=1 \\ \Delta g_i &= \Delta g_{i-1} - \alpha_i\,(p_i - p_{i-1}) && ,i>1 \\ g_{ci} &= g_i - \Delta g_i \end{aligned} \qquad (5)$$

This is slightly more complicated than (4), but ensures that the accumulated gravity corrected always refers only to the previous value. It can easily be shown that (5) reduces to (4) when α is constant, so that the choice of pressure reference is not critical in this latter case.

In Figure (13) we show the effect of implementing (5) on the Boulder data set. It is evident that the spikiness in the residual has gone (one might even say it is a little smoother), but instead we now see a slow drift of the signal away from the level when the admittance is constant.

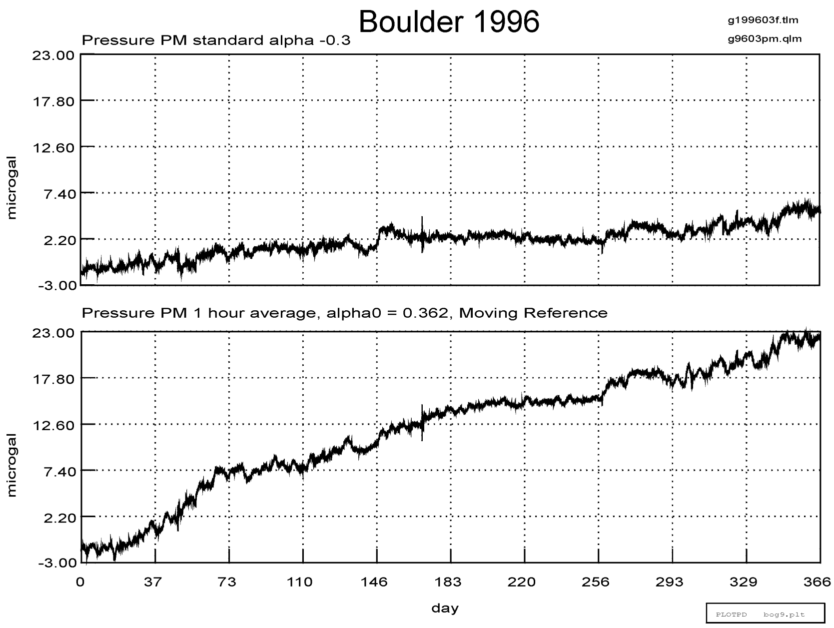

Figure 13. Corrected gravity using a variable admittance and moving average for the pressure reference.

The reason for this behavior is not hard to discover, as Figure 14 makes clear. We show a short section of the data, plotting the admittance and cross correlation, for a 1 hour average, showing the gravity corrections evolving according to (5). It can be seen that when the cross correlation becomes zero (or positive), then the admittance is zero and Δg does not evolve (it has a flat spot) and the gravity is not corrected at all for these points. The cumulative aspect of this method of correction then ensures that the gravity wanders away from its expected values.

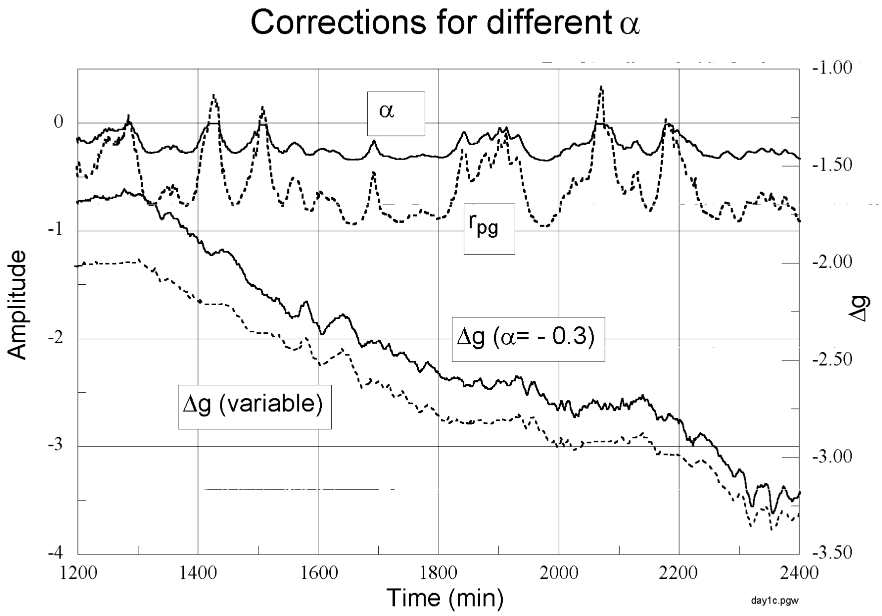

Figure 14. Detail on gravity correction using variable admittance.

Discussion and Conclusions

We have applied the corrections (4) and (5) to numerous other combinations of averaging windows and data sets, but were unable to find a case where there was a noticeable improvement over the use of a fixed α. We did however find that:

(a) the longer the averaging window, the closer we approached the solution for a constant α
(b) for both the FV and moving reference pressure models, the residuals improved when the averaging lengths were made longer
(c) the Strasbourg data set showed similar behavior to the examples shown above, even though the overall correlation was significantly higher than for BO.

We have also recently experimented on synthetic data sets and are able to shed some further insights on the pressure - gravity correlation and admittance. These and other findings will be discussed elsewhere.

Our principle conclusion is simply that the short term cross correlation appears too unreliable to serve as a basis for computing a time domain admittance. Long averaging windows are clearly more stable, but do not lead to superior results over a fixed admittance. We are thus reminded that it is fortunate in gravity studies that the simple local fixed admittance works so well for 90% of the atmospheric loading.

Acknowledgments

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