LLR and Tidal Effects

Subgroup on LLR of the IAG/ETC Working Group 6

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1 Introduction

The analysis of Lunar Laser Ranging (LLR) data enables the determination of many parameters of the Earth-Moon system like lunar gravity coefficients, station and reflector coordinates, Earth Orientation Parameters (EOP) or quantities which parameterise relativistic effects in the solar system. The big advantage of LLR is the long time span of lunar observations (1970 - 2000). The accuracy of the normal points nowadays is about 1 cm.

At this time, year 2000, centres are analysing LLR data and contribute to the ILRS (International Laser Ranging Service). A fifth LLR centre, at the Shanghai observatory, submits EOP solutions to the IERS, but does not contribute to the ILRS and is not considered here in detail. A general problem of LLR is that funding is minimal and, if at all, thus covers only very small scientific investigations (e.g. relativity, lunar physics). Therefore many modelling activities or LLR analyses have to be performed beside the normal work. This is not a satisfactory situation, but LLR research has to be continued somehow.

In this paper, the tidal models used in the various LLR software packages of the four lunar analysis centres are listed/compared.

2 Tidal Effects in LLR

In principle, all tidal effects affecting the Earth-Moon distance at the mm-level should be considered, because the accuracy of the observations reaches the sub-cm level and an insufficient modelling causes systematic errors which affect the accuracy of other parameters to be determined, e.g. site coordinates or the EP parameter h. According to the IERS Conventions (1996), one should implement models of solid Earth tides, ocean loading, atmospheric loading, polar tides, diurnal and semi-diurnal tidal effects in UT1 (also those automatically contained in the nutation series) and polar motion.

Additionally one has to consider the secular tidal acceleration of the Moon (i.e. the Moon raises a tidal bulge on the non-ideal elastic Earth which in turn accelerates the Moon). Here the product k_2t can be estimated where k_2 is the Love number of the Earth and t is the lag angle (often expressed in time units). The secular tidal acceleration is responsible for the increase of the Earth-Moon distance of about 3.8 cm/year.

Like the Earth, the Moon behaves as an (an-)elastic body. That means one has to use an appropriate model with the lunar Love number k_m and the dissipation parameter t_m as typical model parameters. Both quantities can be determined in the global adjustment of the LLR data.

The four lunar analysis centres have implemented the various tidal effects with different accuracies, some effects have even been neglected. Those which have been dealt with in totally equivalent ways in each software are not mentioned here explicitly. For example, no one uses Eq. 17 of Chapter 7 of the IERS Conventions (1996) for the correction of the permanent tide; or considers short periodic tidal effects in polar motion. Also effects considered inherently e.g. by taking a nutation series, are not addressed here. The following tide-related models are considered in the respective software:

JPL (J. Williams)

Solid Earth tides are computed according to Eq. 8 of Chapter 7 of the IERS Conventions (1996), which models the degree 2 part. Additionally, a correction is applied to consider the different Love number for the K_1 tide with a maximum amplitude of about 1.2 cm (see e.g. IERS Standards (1992), p. 57). The pole tide effect and high-frequency variations in UT1 have also been implemented.

No attempt was made to determine Earth tidal parameters. Estimations had shown that the LLR observation time is correlated with the M_2 period which means this effect can hardly be determined by LLR. But solar tides or smaller lunar tides might be obtainable (Williams, 1999).

The (secular) tidal acceleration of the lunar orbit, k_2t, has been modelled and can be solved-for, including the diurnal (k_{20} = 0.34, k_{21} = 0.3, t_{21} = 0.012956 days) and semi-diurnal (k_{22} = 0.3, t_{22} = 0.006925 days) terms. The result for the value of t given here has been determined using the ephemeris DE330 (Williams, 1998).

The Moon is modelled as an elastic, dissipative body. The corresponding terms affect the librations of the Moon. The two parameters k_m (= 0.0287) and t_m (= 0.11523 days) have been determined during the global adjustment.

UTXMO (J. Györgyey Ries)

The effect of the solid Earth tides is computed following the work of Alsop and Kuo (1964), which was based on Bullen's model. The Love numbers of the tidal displacement are h_2 = 0.618 and l_2 = 0.088, which are hardcoded and cannot be estimated without substantial effort. Tides raised on the Moon by the Earth and the Sun are coded, but usually not used. Although the software is capable of estimating the lunar Love number, k_2, and k_2t, it has not been attempted. Additional tidal effects and ocean loading are not considered. At present, streamlining of the analysis process takes higher priority than estimation of tidal effects.

OBSPM (J. Chapront)

Solid Earth tides are computed according to Eq. 8 of Chapter 7 of the IERS Conventions (1996). The Love numbers of the tidal displacement are h_2 = 0.609 and l_2 = 0.0852.

The (secular) tidal acceleration of the lunar orbit has been modelled, including the effect of both the diurnal (k_{20} = 0.34, k_{21} = 0.3, t_{21} = 0.0138569 days) and semi-diurnal (k_{22} = 0.3, t_{22} = 0.0068254 days) terms, where the numerical values have been adopted from the JPL ephemeris DE245. The secular lunar tidal acceleration, k_2t, is fitted. The effect of ocean loading at the sites has been considered, but without corrections due to the lunar node (IERS Conventions (1996), p. 53). Atmospheric loading is modelled using a simplified version of the formula given in the IERS Conventions (1996), p. 67: Dr_{al} = - 0.9 pr [mm], where pr = p_0-1013 mbar and p_0 is the local pressure reduced to sea level.

The Moon is modelled as an elastic, dissipative body where the two parameters k_m (= 0.0299) and t_m (= 0.16485 days) have been included in the computation of the lunar librations.

FSG (J. Müller)

Previous model:

Solid Earth tides were computed according to Eq. 8 and 9 (displacements due degree 3 tides) of Chapter 7 of the IERS Conventions (1996). The Love numbers of the tidal displacement are h_2 = 0.603, l_2 = 0.083, h_3= 0.292 and l_3 = 0.015. A correction was applied to consider the different Love number for the K_1 tide (see IERS Standards (1992), p. 57). Also the pole tide effect and high-frequency variations in UT1 have been implemented. The effect of ocean loading at the sites has been considered, following the recommendations of the IERS Conventions (1996), Chapter 7.

The secular tidal acceleration of the lunar orbit has been modelled, but only the semi-diurnal term (k_2 = 0.3, t_2 = 0.006939 days) which is also solved-for during each adjustment.

Again, the Moon is modelled as an elastic, dissipative body where the two parameters k_m (= 0.0267) and t_m (= 0.1709 days) are determined.

Tidal modelling since 2000:

The main parts of the model are the same as before. We detected and corrected a small error (wrong phase angle) in the term which models the K_1 tide effect. We implemented a new solid Earth tide model given by Mathews et al. (1997). This model considers further frequency- and latitude-dependent terms. The difference between the previous (but K_1 corrected) and the new models is less than 5 mm in radial direction and almost 0.1 mas (= 3 mm) transversal (Figure 1). Furthermore, we implemented a model to consider atmospheric loading which is similar to that of OBSPM.

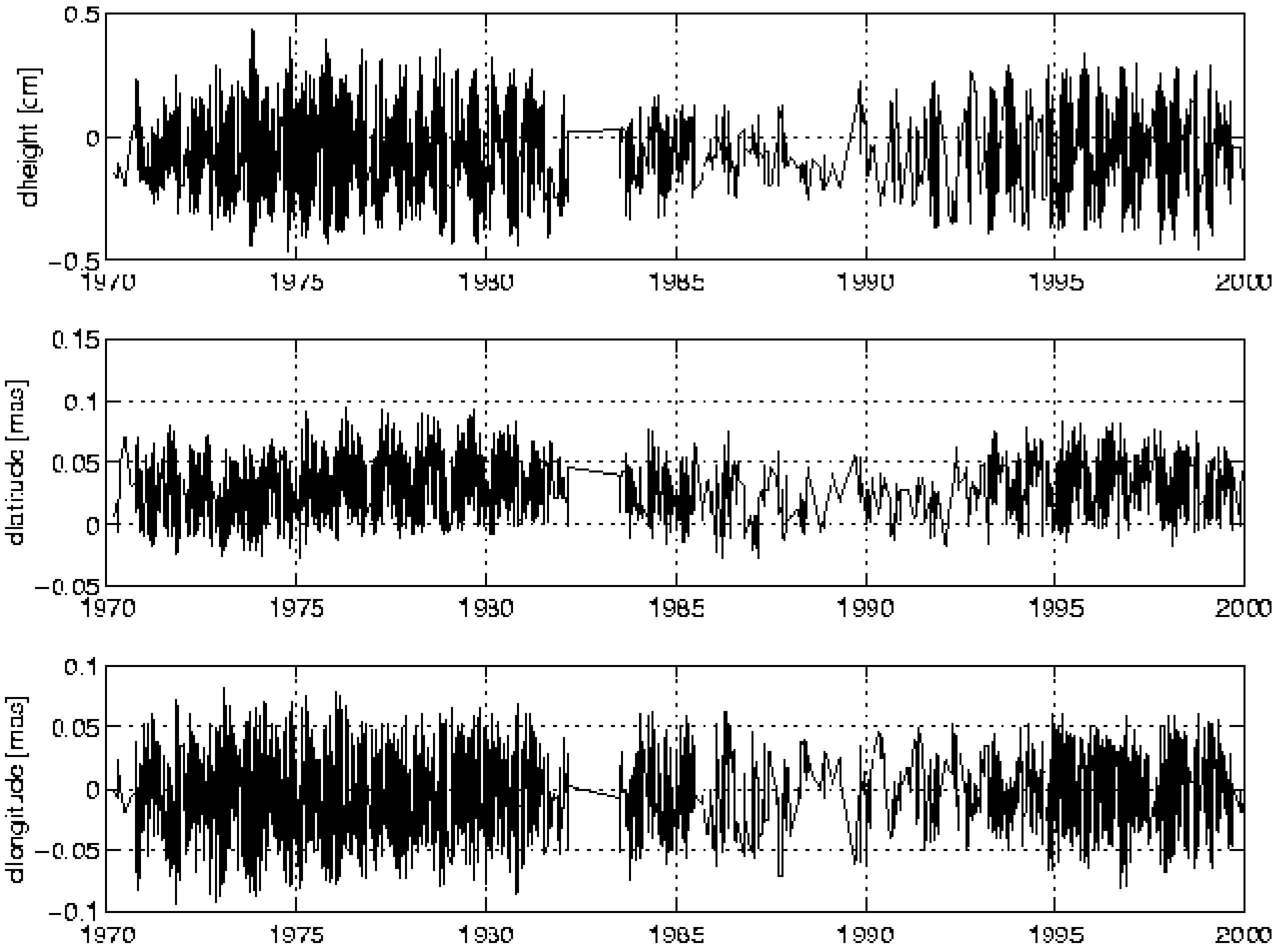

Figure 1: The difference between the previous and new solid Earth tide models at FSG.

The software implementations of the various tidal effects at the four lunar analysis centres are summarised in Tables 1a and 1b. There we have also indicated whether tidal parameters are determined or not.

	JPL (J. Williams)	**UTXMO** (J. Györgyey Ries)	**OBSPM** (J. Chapront)	**FSG** (J. Müller)	**FSG 2000**
Earth tides (no corr, of the permanent tide, Eq. 17)	IERS Conv. 1996, Eq. 8 (degree 2) + corr. of K_1	Alsop and Kuo (1964)	IERS Conv. 1996, Eq. 8 (degree 2)	IERS Conv. 1996, Eq. 8, 9 (degree 3) + corr. of K_1	Matthews et al. (1997)

Ocean loading	?	-	IERS Conv. 1996 (without corr. due to lunar node)	IERS Conv. 1996	
Atmospheric loading	?	-	IERS Conv. 1996 (p_{t0} vs p_0, simplified)	-	IERS Conv. 1996 (p_t vs p_{avg}, simplified)
Pole tide	yes	-	?	yes	
(Sub-)diurnal UT1 variations	yes	-	?	yes	

Table 1a: Tidal effects implemented in the various lunar analysis softwares (recommended by the IERS Conventions 1996).

	JPL (J. Williams)	**UTXMO** (J. Györgyey Ries)	**OBSPM** (J. Chapront)	**FSG** (J. Müller)
Secular tidal ecceleration of the moon (+ 3.8 cm/y) potential Love$_{earth}$ k, lag angle	diurnal (k_{20}, k_{21}, $_{21}$), subdiurnal (k_{22}, $_{22}$) (det. with eph. DE330)	-	diurnal (k_{20}, k_{21}, $_{21}$), subdiurnal (k_{22}, $_{22}$) (det. with eph. DE245)	diurnal (k_2, $_2$)
estimation	$k_{2121;}$ k_{2222}	-	k_2	k_2
Moon as an elastic, dissipative body potential Love$_{moon}$ k_m, dissipation parameter $_m$	yes	-	yes	yes
estimation	yes	-	?	yes

Table 1b: Tidal effects implemented in the various lunar analysis softwares (relevant for LLR).

3 Conclusion

The comparison of the software packages shows that the various tidal effects are handled very differently at the lunar analysis centres. The (solid) tidal effects not yet considered may still reach the mm-level and should be implemented. As a first step, the tidal modelling could be improved and homogenised. However, the general funding of geophysical research is a big problem.

All analysis centres estimate the secular tidal acceleration and the lunar tidal parameters. The results differ slightly depending on the parameterisation, the ephemeris used or other modelling properties. The comparison of the differences and the possible effect for solid Earth physics and/or lunar physics should be further investigated. Some aspects of this are discussed in Dickey et al. (1994). Also, the capability of LLR to determine additional tidal parameters has to be investigated separately. A first step in this direction is carried out by Müller and Tesmer (2000).

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