Report from
Subgroup 6 on Satellite Altimetry
of the IAG/ETC Working Group 6
'Solid Earth Tides in Space Geodetic Techniques'

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1 Introduction

The International Association of Geodesy Earth Tide Commission (IAG/ETC) Working Group 6, 'Solid Earth tides in space geodetic techniques', was established in July 1997. Initially, three terms of reference were established to define the goals of the working group. The first of these three terms of reference is aimed at extending the recommendations relating to tidal influences that are given in the IERS conventions [*McCarthy*, 1996]. This first term of reference been addressed by *Schuh* [1999]. The remaining two terms of reference are aimed at evaluating and comparing the potential of different space geodetic techniques to monitor tidal effects and to determine tidal parameters, and at determining parameters of the tidal models by space geodetic techniques. It was decided to separate working group 6 into subgroups, with each subgroup having the task of addressing the two remaining terms of reference as they pertain to a specific geodetic technique. This report summarizes the conclusions and recommendations of subgroup 6 on satellite altimetry with respect to these two terms of reference.

The report begins by describing the tidal effects that should be considered in satellite altimetry and the methods that are being used to treat these individual effects. Specific emphasis is then placed on describing the treatment of the permanent tide in satellite altimetry, particularly with regards to the definition of the mean sea surface, geoid, and geopotential. A summary of the capabilities and limitation of satellite altimetry to determine various tidal effects and parameters is then provided.

2 Tidal Effects That Should be Considered

Satellite altimeters measure the range between the satellite and the ocean surface that lies nadir to the satellite. The geocentric sea surface height, h, of the ocean surface observed by satellite altimeters is determined as the difference between the geocentric altitude of the host satellite, r, and the range measurement from the satellite altimeter, ρ.

$$h = r - \rho \tag{1}$$

The observed geocentric sea surface height can then be decomposed into a static surface, most often referred to as a mean sea surface height, h_{mss}, and the geocentric dynamic sea surface height, h_d.

$$h = h_{mss} + h_d \tag{2}$$

The geocentric dynamic sea surface height can then be decomposed into radial tidal displacements, h_t, and non-tidal oceanic variations that include the response to atmospheric forcing and the general circulation of the oceans, h_c.

$$h_d = h_t + h_c \tag{3}$$

Observing the non-tidal variations of the oceans is generally the primary goal of most satellite altimeter missions. However, it is the tidal displacements that provide the dominant signal in the geocentric dynamic sea surface heights observed by satellite altimeters. These tidal effects must first be considered before observing the non-tidal variations of the oceans. Those tidal displacements that must be considered include the body tides, h_{bt}, the ocean tides, h_{ot}, the associated load tides, h_{lt}, and the pole tide, h_{pt}.

$$h_t = h_{bt} + h_{ot} + h_{lt} + h_{pt} \tag{4}$$

Satellite altimeters only observe radial displacements of the ocean surface, so only radial tidal displacements need to be considered for satellite altimetry applications. Since satellite altimeters actually observe the total displacement at the sea surface that results from both the ocean tide and the associated load tide, many satellite altimeter applications and data products often combine the ocean tide and load tide displacements over the oceans into a total displacement that is sometimes referred to as the geocentric ocean tide. For clarity, here this total displacement is simply referred to as the (ocean+load) tide, h_{oplt}.

$$h_{oplt} = h_{ot} + h_{lt} \tag{5}$$

On average, the tidal displacements, excluding any contributions from the permanent tide, account for approximately 80% of the geocentric dynamic sea surface height, with the (ocean+load) tides having the largest contribution, followed by the body tides, and a very small (<0.1%) contribution from the pole tide.

3 Treatment of Tidal Effects

3.1 Satellite Altimetry Data Products

Data from satellite altimeter missions are accumulated by the individual institutions that manage the particular mission. These data are processed by the managing institutions and then distributed to the scientific community in data products referred to as Geophysical Data Records (GDRs). The GDRs provide the data that is necessary to derive the observed sea surface height that is inferred by each individual range measurement from the altimeter, and also provide quantities that predict certain geophysical contributions to the observed sea surface height. The GDRs usually provide the following quantities for each range measurement data point.

1. Time tag of the range measurement from the satellite altimeter.
2. Geolocation of the range measurement.
3. Range measurement from the satellite altimeter after applying various instrument corrections.
4. Altitude of the satellite above a reference ellipsoid.
5. Various instrument corrections that were applied to the raw range measurement.
6. Environmental corrections that should be applied to the observed range measurement to account for the effects of tropospheric and ionospheric delays, and the observed sea state roughness.
7. Geophysical quantities that contribute to the individual sea surface height measurement, such as the mean sea surface, the geoid, the ocean tide, the load tide, the body tide, the pole tide, and the inverse barometer response of the ocean surface to atmospheric pressure.

The geolocation and altitude associated with each range measurement are determined from the precise orbit determination (POD) of the satellite. Models are usually used for the environmental corrections. However, dual frequency altimeters are able to implicitly measure the ionospheric delay, and some missions such as the TOPEX/POSEIDON (T/P) mission also have a radiometer on the satellite that measures the wet tropospheric delay. Models are always used to generate each of the geophysical quantities that are associated with each individual sea surface height observation. It should be emphasized that only radial displacements of the observed sea surface are provided for all of the geophysical quantities.

3.2 Body Tides

Satellite altimetry data products most often, if not always, model the body tides by a purely radial elastic response of the solid Earth to the tidal potential. The elastic response is usually modeled using frequency independent Love numbers, h_n, for each degree n of the adopted tidal potential. The tidal potential is one that has been decomposed into its individual spectral components [e.g. *Cartwright and Edden*, 1973, *Tamura*, 1987]. The permanent tide (zero frequency) spectral component is specifically removed from the adopted tidal potential so that the model of the total body tide displacement that is provided on the satellite altimeter data products does not include the contribution from the permanent body tide displacement. As an example, the T/P GDRs model the body tides by applying Love numbers $h_2 = 0.609$, and $h_3 = 0.291$ to a *Cartwright and Taylor* [1971] tidal potential (excluding the permanent tide component) that has been extrapolated to the T/P era.

Although frequency independent Love numbers are used to model the body tides, the effects of the free core nutation (FCN) resonance in the diurnal tidal band are sometimes accounted for by simply scaling the tide potential amplitude of the K_1 spectral line and some neighboring spectral lines by an appropriate scale factor. Multiplication of the scale factor by the adopted frequency independent Love number provides the Love number that accounts for the FCN resonance. Again using the T/P GDRs as an example, the effects of the FCN resonance are simply modeled by scaling the tide potential amplitude of the K_1 spectral line and the neighboring nodal lines by a factor $0.52/0.609 = 0.854$, since the appropriate Love number at the K_1 tidal frequency is approximately 0.52.

The use of frequency independent Love numbers, and especially the simplistic approach towards the FCN resonance in the diurnal band certainly introduces errors to the modeled body tides. However, these errors are considered to be much smaller than the instantaneous accuracies of the altimetric sea surface height measurements which are likely to be no better than approximately 4 cm. For example, the elastic semidiurnal Love number specified by *Mathews et al.* [1997], $h_2 = 0.6026$, implies a maximum error in the amplitude of the elastic M_2 body tide modeled on the T/P GDRs that ranges from 0.16 cm at the equator to 0.03 cm at $\pm 66°$ latitude. It should be realized that any errors in the body tide model are likely to be absorbed into any (ocean+load) tide models that are empirically determined from altimetric sea surface height data that remove body tide effects with the provided body tide model.

3.3 Ocean Tides and Load Tides

With the recent advances in modeling the ocean tides [e.g. *Shum et al.*, 1997], present altimetric missions sometimes provide for one empirically derived ocean tide model and one hydrodynamic ocean tide model on their data products. The benefit of providing both types of models on the data products is that the ocean tide effects can be removed from the sea surface height data with the use of either a data-dependent ocean tide model, or a data-independent ocean tide model. Furthermore, the hydrodynamic ocean tide models tend to provide better representations of the ocean tides in regions where the ocean tides have shorter wavelengths than can be observed by the spatial sampling characteristics of the altimeter ground track.

The models are usually provided on the data products as a field for the (ocean+load) tides, and a corresponding separate field for the associated radial load tide that was actually used to generate the (ocean+load) tide field. At least in the case of the T/P GDRs, the (ocean+load) tide fields provide radial (ocean+load) tide displacements using the diurnal and semidiurnal (ocean+load) tide heights as is provided by the chosen models, and adding to them a long-period pure ocean tide height that models the long-period ocean tides with a classical equilibrium response. Load tide displacements from the long-period ocean tides are ignored and not included in either of the (ocean+load) tide or load tide fields on the data products.

Here, the classical equilibrium long-period ocean tide response refers to a representation of the long-period ocean tide height as the product of the Love number factor $\gamma_n = (1 + k_n - h_n)$, and the long-period tidal potential. As with the body tide displacements provided on the data products, the permanent ocean tide is also excluded from the long-period ocean tide heights that are computed for the satellite altimetry data products by specifically removing the permanent tide (zero frequency) spectral component from the long-period tidal potential. The Love numbers used to compute the classical equilibrium long-period ocean tide are also assumed to be frequency independent. Again using the T/P GDRs as an example, the model for the long-period equilibrium ocean tides use the *Cartwright and Taylor* [1971] long-period tidal potential (excluding the permanent tide term), along with the Love numbers, $h_2 = 0.609$, $k_2 = 0.302$, $h_3 = 0.291$, and $k_3 = 0.093$.

The treatment of the long-period ocean tides is erroneous in many ways. Of most importance is the fact that the true long-period ocean tides probably do not have an equilibrium response to the tidal potential particularly at the periods of the two principal monthly and fortnightly tidal components. However, the equilibrium response for the long-period ocean tides is adopted because the long-period ocean tides are thought to have only small departures from an equilibrium response, and because the long-period ocean tides are an area of continuing investigation. Only very recently have models been developed that appear to model the long-period ocean tides better than the equilibrium response, and these models are still under evaluation. Indeed, data products for the upcoming Jason-1 altimetry mission will have fields for both the traditional classical equilibrium model of the long-period ocean tides, and a non-equilibrium model of the long-period ocean tides.

Nevertheless, even if the Earth's ocean tides did have an equilibrium response, the classical equilibrium model that has historically been adopted by the satellite altimetry data products ignores the effects of the loading and self-gravitation of the long-period ocean tides, and the requirement that the ocean tides conserve mass. The self-consistent equilibrium model of the long-period ocean tides, introduced by *Agnew and Farrell* [1978], would take these effects into account. Also, while the data products provide (ocean+load) tide heights from the diurnal and semidiurnal band, only pure ocean tide heights are being provided from the long-period band. It should however be conceded that those effects that are being ignored in the classical equilibrium representation of the long-period ocean tides are small enough ($<$ 1.0 cm) to ignore for most oceanographic applications of the altimetry data. More importantly, these effects are likely to be smaller than the actual departures of the true long-period ocean tide response from an equilibrium response.

3.4 Pole Tide

Those satellite altimetry data products that provide a model for the pole tide do so by assuming that the period of the oscillating rotation axis is long enough for the pole tide displacement of the solid Earth and the oceans to be in equilibrium with the centrifugal potential that is generated by the oscillating rotation axis [e.g. *Wahr*, 1985]. The data products provide a single field for the radial geocentric pole tide displacement of the sea surface, and includes the radial pole tide displacement of both the solid Earth and the oceans. The radial geocentric pole tide at a point on the surface with latitude ϕ and longitude λ is computed as the elastic equilibrium response to the centrifugal potential, V_c.

$$h_{pt} = (1 + k_2)\frac{V_c(\phi, \lambda)}{g} \tag{6}$$

$$V_c = -\frac{a^2\Omega^2}{2}\sin 2\phi\,[(x - x_{av})\cos\lambda - (y - y_{av})\sin\lambda] \qquad (7)$$

The mean radius of the Earth, the mean rotation rate of the Earth, and the mean gravitational acceleration at the Earth's surface are denoted by a, Ω and g, respectively. The instantaneous location of the rotation axis, in units of radians, along the 0° meridian and 90°W meridian are denoted by x and y respectively, and x_{av} and y_{av} are the respective coordinates of the mean pole. Certainly, the Love number k_2 should be consistent with the long periods of 12 and 14 months that almost entirely describe the oscillation of the rotation axis, and IERS conventions [*McCarthy*, 1996] recommend using $k_2 = 0.2977$ for an elastic Earth. The pole location varies from its mean location by at most 0.8 arcseconds, which corresponds to a maximum geocentric pole tide displacement of approximately 5.5 cm. Any errors caused by ignoring anelastic effects are likely to be smaller than 1 mm and are therefore insignificant for altimetry applications. The use of equation (6) to model the pole tide is analogous to the use of the classical equilibrium response to model the long-period ocean tides. As such, it also ignores the self-gravitation and loading effects of the pole tide in the oceans.

Using T/P GDRs again as an example, the pole tide is computed using a value $k_2 = 0.302$ that is consistent with the Love number used to compute the long-period equilibrium ocean tides, and a mean pole location of $x_{av} = 0.042$ and $y_{av} = 0.293$ arcseconds. The difference between the Love number adopted for the T/P GDRs and that recommended in the IERS conventions implies a maximum error in the radial elastic geocentric pole displacement of less than 1 mm, which again is small enough to ignore.

4 Treatment of the Permanent Tide

4.1 Tidal Reference Frames

When describing the treatment of the permanent tide in geodetic quantities it become useful to define the various tidal reference frames to which the geodetic quantities refer. The following terminology which is most often used in discussions of the permanent tide [e.g. *Rapp et al.*, 1991; *Mathews*, 1999] is also used in this discussion of the permanent tide.

1. "Tide-free" values refer to values which exclude all tide effects.
2. "Mean" values refer to values which include both the direct and indirect permanent tide effects.
3. "Zero" values refer to values which include the indirect permanent tide effects only, and exclude the direct permanent tide effects.

Rapp et al. [1991] provide recommendations for the treatment of the permanent tide on the T/P GDRs and that discussion is extended here for general satellite altimetry applications.

4.2 The Permanent Tidal Potential and Deformations

In order to quantify the treatment of the permanent tide in satellite altimetry consider the direct permanent tide potential at the surface of the Earth, represented here by $\overline{V}$,

$$\overline{V} = \left(\frac{5}{4\pi}\right)^{1/2} H g P_{20}(\sin\phi) \qquad (8)$$

$$P_{20}(\sin\phi) = (1.5\sin^2\phi - 0.5) \qquad (9)$$

where P_{20} is the second degree unnormalized Legendre polynomial, and H is the permanent tide potential amplitude. The value of the permanent tide potential amplitude varies slightly in the literature where for example *Cartwright and Edden* [1973] have H = -0.31446 meters, or more recently *Tamura* [1987] has H = -0.314593 meters, while T/P algorithm specifications extrapolate the *Cartwright and Edden* [1973] tide potential to the T/P era and define H = -0.31458 meters.

The direct permanent tide potential causes the solid Earth to deform, with the radial component of this body tide deformation defined by $\overline{h}_b$.

$$\overline{h}_b = h_2(\omega_0)\frac{\overline{V}}{g} \tag{10}$$

The redistribution of mass associated with the permanent body tide results with an indirect permanent tide potential, $k_2(\omega_0)\overline{V}$. The total permanent tidal potential at the surface of the Earth and the geocentric equipotential surface of the permanent tide are then defined by $\overline{V}_t$ and $\overline{h}$, respectively.

$$\overline{V}_t = [1 + k_2(\omega_0)]\,\overline{V} \tag{11}$$

$$\overline{h} = \frac{\overline{V}_t}{g} = [1 + k_2(\omega_0)]\,\frac{\overline{V}}{g} \tag{12}$$

Here, $h_2(\omega_0)$ and $k_2(\omega_0)$ are used to explicitly indicate the requirement that second degree Love numbers at the zero frequency, ω_0, must be applied to the permanent tide.

The Love numbers at the zero frequency are presently unknown and are a subject for further investigation. However, present IERS conventions [*McCarthy*, 1996] use the fluid limit Love numbers, h_s and k_s, sometimes also referred to as the secular Love numbers, to define the tidal response at the zero frequency, $h_2(\omega_0) = h_s$ and $k_2(\omega_0) = k_s$. The fluid limit Love numbers are adopted at the zero frequency only because of a lack of observational evidence or any other theory that would indicate otherwise. The secular Love numbers are derived for a rotating Earth in hydrostatic equilibrium [*Lambeck*, 1980] and have the following important relationship.

$$h_s = (1 + k_s) \tag{13}$$

Lambeck [1980] defines k_s = 0.94, while *Mathews* [1999] defines k_s = 0.933 for the Preliminary Reference Earth Model (PREM). Use of the fluid limit Love numbers implies that the surface of the permanent body tide is coincident with the equipotential surface of the permanent tide, with $\overline{h}_b = \overline{h}$. This suggests that the solid Earth and the oceans react in tandem, as fluids, to the permanent tide potential.

If no specific relationship between the Love numbers $h_2(\omega_0)$ and $k_2(\omega_0)$ is assumed, then the classical equilibrium theory that is often applied to the long-period ocean tides might also be extended to the permanent ocean tide. Classical equilibrium theory derives the ocean tide displacement with respect to the ocean bottom, $\overline{h}_o$, by subtracting the body tide from the surface of the tide-generating potential.

$$\overline{h}_o = \overline{h} - \overline{h}_b = \mathcal{O}\,[1 + k_2(\omega_0) - h_2(\omega_0)]\,\frac{\overline{V}}{g} \tag{14}$$

Here, $\mathcal{O}$ defines the ocean function [e.g. *Munk and Macdonald*, 1960] which has a value of 1 over the oceans and 0 over land. Equation (14) further emphasizes that there is a zero net displacement of the ocean surface with respect to the ocean bottom if the fluid limit Love numbers are applied at the zero frequency, since $(1 + k_s - h_s) = 0$.

4.3 The Mean Sea Surface

From equations (2) and (3), and using $< X >$ to denote the mean value of a time dependent parameter X, the mean of the geocentric sea surface heights observed by satellite altimeters, $<h>$, is:

$$<h>= <h_{mss}> + <h_t> \tag{15}$$

For the purposes of this discussion it is assumed that $<h_c> = 0$. If h_{bt} and h_{ot} are defined to include the respective indirect permanent tide deformations, then the mean deformation of all tidal quantities $<h_t> = \overline{h}$, since over the oceans $<h_{bt}> + <h_{ot}> = \overline{h}_b + \overline{h}_o = \overline{h}$, and $<h_{lt}> = <h_{pt}> = 0$. The quantity h_{mss} would be considered to be the "tide-free" mean sea surface and $<h>$ would be the "zero" or "mean" mean sea surface since $<h>$ would include the indirect permanent tide deformation $\overline{h}$. In this case the "zero" and "mean" values are equivalent since there are no direct permanent tidal deformations. However, if h_{bt} is defined to exclude the indirect permanent tide deformation, then $<h_t> = 0$, and $<h> = h_{mss}$ and the mean sea surface h_{mss} would be the "zero" or "mean" value, defined to include the indirect permanent tide deformation, $\overline{h}$.

Rapp et al. [1991] recommend that reported sea surface heights "should have permanent tide effects included when the values are reported". In accordance with this recommendation, the zero frequency spectral line (permanent tide) component of the tidal potential is ignored when generating the radial body tide height and the classical equilibrium representation of the long-period ocean tide height for tidal fields on the T/P GDRs [*Benada*, 1997], such that $<h_t> = 0$. The philosophy behind this recommendation appears to be based on the assumption that most users of satellite altimetry data subtract tidal effects from the observed geocentric sea surface heights. Since the subtracted tidal effects do not include the permanent tide then the residual geocentric sea surface heights are considered to be "zero" (or "mean") values since they include the indirect permanent tide deformation, $\overline{h}$. As a consequence of these recommendations, a mean sea surface generated from the "zero" valued geocentric sea surface heights would therefore be considered to be a "mean", or equivalently "zero", mean sea surface, and the observed geocentric dynamic sea surface heights with respect to that mean surface would be "tide-free" values.

4.4 The Geopotential and Geoid

The treatment of the permanent tide with respect to the POD of the satellite, and the definition of a geoid height are also of particular importance to satellite altimetry. The POD of the satellite is necessary to derive the geocentric satellite altitude, and to subsequently derive the geocentric sea surface height that is observed by the altimeter. The geoid height is sometimes used as an alternative to the mean sea surface as the reference for the geocentric dynamic sea surface heights. Both of these applications require knowledge of the Earth's geopotential and the treatment of the permanent tide with respect to the adopted geopotential is necessary to ensure that the complete altimetric system is in a consistent tidal reference frame.

The Earth's geopotential, W, is represented here by the usual spherical harmonic expansion,

$$W = \frac{GM}{a} \sum_n \sum_m \left(\frac{a}{r}\right)^{n+1} P_{nm}(\sin\phi)\left[C_{nm}\cos(m\lambda) + S_{nm}\sin(m\lambda)\right] \tag{16}$$

where GM is the product of the gravitational constant and the mass of the Earth, and r, λ, and ϕ are the spherical coordinates of a point external to the surface of the Earth. For simplicity, here unnormalized spherical harmonic coefficients C_{nm} and S_{nm} are used. Also pertinent to this discussion is the oblateness term $J_2 = -C_{20}$. The direct permanent tide potential in equation (8) can be rewritten in a form that is easily compared to the spherical harmonic coefficients in equation (16) by considering the potential W at the surface of the Earth ($r = a$).

$$\overline{V} = \frac{GM}{a} C_{20}^{pt} P_{20}(\sin\phi) \tag{17}$$

$$C_{20}^{pt} = \left(\frac{5}{4\pi}\right)^{1/2} Hg \frac{a}{GM} \tag{18}$$

Using constants from the IERS conventions [*McCarthy*, 1996], and the permanent tide potential amplitude from *Tamura* [1987], then C_{20}^{pt} = -3.10555 e-8. For the following discussion a corresponding oblateness coefficient for the the permanent tide, $J_2^{pt} = -C_{20}^{pt}$, is also defined.

The question arises as to which tidal reference frame is being referred to by the C_{20} geopotential coefficient. In order to distinguish between the three tidal reference frames, three different values of J_2 are explicitly defined here, with J_2^{tf}, J_2^m, and J_2^z being the “tide-free”, “mean”, and “zero” values respectively.

$$J_2^m = J_2^{tf} + [1 + k_2(\omega_0)]\, J_2^{pt} \tag{19}$$

$$J_2^z = J_2^{tf} + k_2(\omega_0) J_2^{pt} \tag{20}$$

Of course, without any better knowledge the fluid limit Love number k_s should also be applicable for the geopotential and in fact section 6 of the IERS conventions [*McCarthy*, 1996, p. 47] states that “...to obtain the effect of the permanent tide on the geopotential, one can use the same formula as equation (6) using the fluid limit Love number which is $k = 0.94$”.

The POD of satellites requires use of both a geopotential and a background body tide model to determine the total potential acting on the satellite. It should be emphasized that very often the direct and indirect potential arising from the body tides are not derived from spectrally decomposed tide potentials, but rather from lunar and solar ephemerides, as is the case in the IERS conventions [Section 6, *McCarthy*, 1996]. In doing so, a constant Love number k_{nm} is usually assumed across each degree (n) and order (m) of the tide potential. For the second degree zonal term ($n = 2, m = 0$), a Love number k_{20} of approximately 0.3 is usually used. In such cases the indirect permanent tide contribution is then restored to the geopotential either beforehand by removing a term $k_{20}J_2^{pt}$ from the complete indirect tide potential that is computed from the luni-solar ephemerides to ensure use of only the time varying part of the indirect tide potential, or after the fact by adding a term $k_{20}J_2^{pt}$ back into the applied or estimated geopotential. In doing so, the geopotential is always defined in the “zero” tidal reference frame.

The potential for confusion, particularly with regards to the defined terminology, when restoring the geopotential to its “zero” value should be quite evident. For example, *Lemoine et al.* [1998, pp. 3-14, eq. 3.3.1-5], have adopted what they refer to be a “zero” J_2 value for that project that is defined as follows:

$$J_2(\text{zero}) = J_2(\text{tide-free}) + (0.3 \times 3.11080e - 8) \tag{21}$$

Evidently, the terms “tide-free” and “zero” are inconsistent with each other in this equation since the Love number applied to the permanent tide is a value of 0.3. A similar conflict with terminology appears to be evident in *Rapp et al.* [1991], where the J_2 value from the GEM-T2 model is referred to be the “tide-free” value, and the “zero” value is restored with the use of a Love number of 0.3. It is likely that the defined “zero” value is consistent with the defined terminology, but what has been referred to as the “tide-free” value is probably not consistent and is instead some “pseudo-tide-free” value that is really $J_2^{tf} + [k_2(\omega_0) - k_{20}]J_2^{pt}$. Namely, the “zero” value is simply restored by adding the adopted background indirect permanent tide potential, $k_{20}J_2^{pt}$ to the “pseudo-tide-free” value, as is described above.

The geoid is an equipotential surface that very closely represents the mean sea surface. The geoid is another commonly used reference surface that is provided in the satellite altimeter data products. Denoting W (see equation (16)) as the Earth's geopotential, and U as the normal potential defined by an equipotential reference ellipsoid, then a disturbing potential T is determined as:

$$T = W - U \tag{22}$$

The geoid height, N can be computed from Bruns' formula [*Heiskanen and Moritz*, 1993]:

$$N = \frac{T}{\gamma} \tag{23}$$

where γ is the normal value of gravity. For clarity, "tide-free", "mean", and "zero" geopotentials, W^{tf}, W^{m}, and W^{z}, and normal potentials, U^{tf}, U^{m}, and U^{z}, are explicitly defined here, where:

$$W^m = W^{tf} + [1 + k_2(\omega_0)]\,\overline{V} \tag{24}$$
$$W^z = W^{tf} + k_2(\omega_0)\overline{V} \tag{25}$$
$$U^m = U^{tf} + [1 + k_2(\omega_0)]\,\overline{V} \tag{26}$$
$$U^z = U^{tf} + k_2(\omega_0)\overline{V} \tag{27}$$

Similarly, the corresponding "tide-free", "mean", and "zero" valued geoid heights with respect to the respective reference ellipsoid are defined as follows:

$$N^{tf} = \frac{(W^{tf} - U^{tf})}{\gamma} \tag{28}$$
$$N^{m} = \frac{(W^{m} - U^{m})}{\gamma} \tag{29}$$
$$N^{z} = \frac{(W^{z} - U^{z})}{\gamma} \tag{30}$$

As long as the geopotential and the normal potential are defined in identical tidal reference frames, the geoid height with respect to the equipotential reference ellipsoid would itself be tide-free in the strictest sense, with $N^{tf} = N^z = N^m$. Meanwhile the geocentric geoid surface, $N_g = W/\gamma$, would be in a tidal reference frame that is identical to that used for the two potentials U and W. As such, it is really the tidal reference frame of the equipotential reference ellipsoid that is important.

With the nomenclature defined above, the "tide-free", "mean", and "zero" geocentric geoid surfaces would then be defined by N_g^{tf}, N_g^{m}, and N_g^{z}.

$$N_g^{tf} = N^{tf} + \frac{U^{tf}}{\gamma} \tag{31}$$
$$N_g^{m} = N^{m} + \frac{U^{m}}{\gamma} = N_g^{tf} + [1 + k_2(\omega_0)]\,\frac{\overline{V}}{\gamma} \approx N_g^{tf} + \overline{h} \tag{32}$$
$$N_g^{z} = N^{z} + \frac{U^{z}}{\gamma} = N_g^{tf} + k_2(\omega_0)\frac{\overline{V}}{\gamma} \tag{33}$$

By making the approximation $\gamma \approx g$, and inserting equation (12) into equation (32) it should be realized that the "mean" geocentric geoid surface is consistent with the "mean" mean sea surface in the sense that both surfaces include the permanent body and ocean tide displacement. Therefore, for satellite altimetry applications, which usually define the mean sea surface in the "mean" tidal reference frame, the geocentric geoid surface should also be provided in the “mean” tidal reference frame. In this way, geocentric dynamic sea surface heights that are referenced to the “mean” geoid would be “tide-free” values, just as they would be if referenced to the “mean” mean sea surface. Given a “zero” geocentric geoid surface N_g^z, the “mean” geocentric geoid surface is simply derived as follows:

$$\begin{aligned} N_g^m &= N_g^z + \frac{\overline{V}}{g} \\ &= N_g^z + \left(\frac{5}{4\pi}\right)^{1/2} H P_{20}(\sin\phi) \end{aligned} \tag{34}$$

where the approximation $\gamma \approx g$ is made. Note that equation (19) of *Rapp et al.* [1991] is an equivalent expression to equation (34).

Rapp et al. [1991] recommend adopting the “mean” geoid on the T/P GDRs in order to have the provided geoid surface and mean sea surface both defined in the identical “mean” tidal reference frame. They recommend generating this geoid height with respect to the reference ellipsoid by adding the term $[(5/4\pi)^{\frac{1}{2}} H P_{20}(\sin\phi)]$ to the “zero” geoid height N^z. Of course, this adjustment to the “zero” geoid height is only appropriate if the reference ellipsoid is defined to be in the “zero” tidal frame. However, it is unclear from T/P documentation whether or not the adopted reference ellipsoid is in the “zero” tidal reference frame, and the reference ellipsoid is simply defined by an Earth radius of 6378.1363 km and a flattening of 1/298.257. Nevertheless, this adjustment to the geoid height has been adopted in the T/P GDRs.

5 Capability of Determining Tidal Effects and Parameters

Recent satellite altimeter missions, and in particular the Geosat Exact Repeat Mission and the T/P mission, have provided significant improvements to our knowledge of the Earth's ocean tides, and particularly our knowledge of the diurnal and semidiurnal ocean tides. This is due in large part to the fact that the ocean tides are the dominant signal in the geocentric dynamic sea surface heights observed by satellite altimeters. Our knowledge of the Earth's long-period ocean tides has also been somewhat improved from satellite altimetry. However, investigation of the Earth's long-period ocean tides is still very much a subject of continuing investigations that are likely to continue to improve our knowledge of these ocean tides as longer durations of high accuracy altimetric data become available.

The first three years after the T/P mission saw significant interest in modeling the ocean tides with more than 20 new models becoming available during that period [e.g. *Andersen et al.*, 1995; *Shum et al.*, 1997]. These models ranged from purely empirical models that were derived from the new generation of altimetric data, to purely hydrodynamic models whose goal was to model the ocean tides for oceanographic use of the altimetric data, and subsequently to hybrid models that either assimilated empirical models into hydrodynamic equations of motion or estimated empirical adjustments to apriori hydrodynamic models. The empirical and hybrid ocean tide models agree with each other to within 2-3 cm [*Shum et al.*, 1997], and are likely to have similar accuracies at least in the deep oceans [e.g. *Desai et al.*, 1997]. This should be of no surprise as all of the models are somewhat dependent on the same T/P sea surface height observations.

Some of the most recent global ocean tide models, among the many that are available, include those from *Eanes* [1999], *Ray* [1999], and *Le Provost et al.* [1998]. Each of these three models might be considered to be hybrid models that in some fashion take advantage of the higher spatial resolution that is available from hydrodynamic models while using the satellite altimetry data to constrain the long wavelength response of the ocean tides. The high accuracy and long duration of the T/P altimetry data has also sparked interest in the so called overtides which principally occur in shallow water regions. They are caused by the nonlinear interaction between the principal ocean tides. *Andersen* [1999] has used T/P altimetry data to observe the M_4 and M_6 ocean tides in the northwest European shelf region, where in some cases the amplitude of the M_4 constituent can exceed 50 cm. Certainly, such large ocean tides should have associated load tide effects on neighboring regions and future analyses should probably be extended to include investigations of load tide effects from the overtides.

The new generation of ocean tide models that have been developed from the T/P sea surface height data have been used to predict the effects of the ocean tides on the Earth's polar motion and rotation rate. For example, a comparison by *Chao et al.* [1995] of VLBI observations of the diurnal and semidiurnal tidal variations of the Earth's rotation rate to respective variations that are predicted by T/P-derived ocean tide models shows good agreement to within 2-3 microseconds. A similar comparison by *Chao et al.* [1996] for diurnal and semidiurnal tidal variations in polar motion also shows good agreement to within 10-30 microarcseconds for the largest tides. *Desai and Wahr* [1999] used a T/P-derived model of the monthly and fortnightly ocean tides to predict the effects of these long-period ocean tides on the Earth's rotation rate. This analysis indicates that the predicted fortnightly variations of the Earth's rotation rate are fairly well determined from the T/P ocean tide model, but that there is potential for improvement of the the predicted monthly variations of the Earth's rotation rate, particularly in the component that results from tidal currents.

The high accuracy of the ocean tide models that have recently been determined from satellite altimetry also allows use of these models to place observational constraints on the anelasticity of the Earth at the tidal periods. This is particularly useful since observations of the anelasticity of the Earth have usually been limited to the seismic periods of approximately 1 hour and the Chandler wobble period of 14 months, while the tidal periods principally range from 12 hours to 28 days. Ocean tide models that are derived from satellite altimetry, and the spherical harmonic coefficients of the ocean tides that are determined from satellite tracking data, are both usually based on elastic Earth tide models. Therefore, the ocean tide coefficients that are determined from each of the two geodetic techniques actually include the effects from the anelasticity of the Earth, although the coefficients derived from satellite altimetry data are far less sensitive to the Earth's anelasticity than those derived from satellite tracking data. Reconciling the coefficients from the two geodetic techniques provides an estimate of the anelasticity in the solid Earth. *Ray et al.* [1996] have performed such a comparison using a T/P-derived model of the M_2 ocean tide, which has a period of 12.4 hours. Their estimates of the lag in the body tide due to anelasticity agrees well with theoretical estimates. In a similar fashion, observational constraints on the Earth's anelasticity at periods of 5-30 days can also be determined by reconciling observed long-period tidal variations of the Earth's rotation rate with the predicted contributions to these variations from the respective elastic body tides and ocean tides. The residual in the observed variations after removing the respective contributions from the elastic body tide and the ocean tide is explained by mantle anelasticity. A preliminary analysis by *Desai and Wahr* [1998] that determines this residual at the fortnightly period by using a T/P-derived ocean tide model to predict the contribution from the ocean tides, also shows good agreement between this residual and theoretical estimates of the contribution from mantle anelasticity.

Satellite altimetry also has the capacity to place constraints on the Love number h_2. A preliminary analysis by *Ray et al.* [1995] provides estimates of h_2 at the M_2, N_2, O_1, and K_1 frequencies by combining tidal estimates from satellite altimetry with in situ measurements of the ocean tides. They use the satellite altimeter data to determine the total tidal displacement of the sea surface.

A model for the load tides is used to determine the load tide contribution, and the ocean tide contribution is determined from in situ tide gauge measurements. The remaining tidal displacements at the location of the tide gauge sites results from the body tides as observed by the satellite altimeter. The results from *Ray et al.* [1995] clearly indicate the effect of the FCN resonance at the K_1 frequency, and also provide good agreement with VLBI observations and theoretical estimates of h_2 at the M_2 frequency.

6 Limitations

The finite sampling of the ocean surface by satellite altimeters limits observations of the Earth's ocean tides to only the response which is of longer wavelength than the spatial sampling interval of the neighboring ground tracks. This means that it becomes difficult to observe and model the ocean tides from altimetric data in those areas where the ocean tide response has relatively large gradients. Fortunately, areas with large gradients usually occur in shallow water and coastal areas, which only account for a minor part of the Earth's oceans. As a consequence, the altimetric ocean tide models have already provided significant advances to global geodetic applications that only require knowledge of the long-wavelength response of the ocean tides. However, more localized geodetic applications still have the potential for significant future improvements both from longer durations of altimetric data and from the combination of data from multiple altimetry missions. For example, limitations in the accuracy of load tide models that are derived from the altimetric ocean tide models should be evident in regions both over land and the oceans that lie near areas with large gradient ocean tides, such as near coastlines.

Indeed, most of the recent ocean tide models show good agreement (within 2-3 cm) in the majority of the oceans, but can differ from each other by more than 10 cm in some shallow water areas. This limitation is especially significant in purely empirical ocean tide models that are derived exclusively from the altimetric data without any apriori assumptions about bathymetry or hydrodynamics. On the opposite end of the spectrum, purely hydrodynamic models are limited by poor knowledge of bathymetry and friction. It is the hybrid models that generally provide a good compromise by allowing the altimetric data or models to constrain the hydrodynamic equations of motion with the observed long wavelength response of the oceans, while applying the hydrodynamic model at a higher spatial resolution than is available from the sampling characteristics of the satellite ground track.

Satellite altimetry also has the fundamental limitation that results from the fact that radar altimeters cannot be used over ice sheets. As such, most oceanographic satellite altimeters have orbits that are configured to limit sampling over ice sheets while maximizing sampling of areas not covered by ice. This limits the use of satellite altimetry to observing the Earth's tides only in those areas within latitudes of approximately ± 70 degrees. Hydrodynamic ocean tide models still provide the best method of modeling the ocean tides in polar regions that are usually covered by ice sheets. This limitation is particularly important in tidal components such as the long-period ocean tides whose principal spherical harmonic component, the second degree zonal component, is particularly sensitive to the response in the extreme polar latitudes.

7 Future Perspectives

The current treatment of tidal effects in satellite altimetry data products appears to be sufficient for the accuracies that are presently available from satellite altimetry sea surface height measurements. However, as the accuracy of future satellite altimeters improves to the level of 1 cm (or better) it is likely that closer attention will need to be given to some of the finer details that are involved with modeling the tidal effects. In particular, the treatment of the body tides would probably need to be brought closer in tune with the IERS conventions, particularly with regards to the use of frequency dependent Love numbers. The treatment of the long-period ocean tides also has the potential for improvement, by using non-equilibrium models for the shorter period (< 30 days) constituents, by adopting a self-consistent equilibrium representation of the longer period constituents that are more likely to have an equilibrium response, and by accounting for the long-period load tides.

Certainly, data from multiple altimeter missions and longer durations of altimetry data are likely to continue to provide improvements to our knowledge of the Earth's ocean tides and consequently the load tides. In particular, there is potential for improvement in the spatial resolution of the altimetric ocean tide models particularly in high gradient areas such as shallow water and coastal regions. Indeed there are plans to have the soon to be launched T/P follow on satellite, Jason-1, and the T/P satellite fly on interleaving ground tracks. Meanwhile the accuracies of the smaller amplitude tidal components are likely to improve as longer durations of data become available. Improved accuracies in the long-period monthly and fortnightly ocean tide models are especially anticipated, while expanded interest in observing and modeling the overtides should also be expected. The improvements to the spatial resolution and accuracies of the altimetric tidal observations are likely to be accompanied by revised analyses of other geophysical applications of the altimetric tidal observations, many of which have so far only been preliminary analyses.

From the discussion on the treatment of the permanent tide it becomes evident that the actual values of the Love numbers at the zero frequency are neither important or necessary in satellite altimetry. The geopotential can always be defined in the "zero" tidal reference frame with knowledge of the Love number that has been adopted in the background body tide model that is used when generating the geopotential, and without explicit knowledge of the Love number $k_2(\omega_0)$. It is the tidal reference frame of the adopted reference ellipsoid that is of most importance to satellite altimetry. This is particularly true since all parameters including the altitude, sea surface height, mean sea surface, and geoid are usually provided with respect to the reference ellipsoid. Perhaps the least confusing method of dealing with the permanent tide would be to define the reference ellipsoid in the "mean" tidal reference frame. This only requires knowledge of the "zero" geopotential and the permanent tidal potential. In this way, the physical representation of the reference ellipsoid would include the deformation that results from the permanent body and ocean tides. The altimetric sea surface height, the mean sea surface height, and the geoid height with respect to this ellipsoid would then all be "tide-free", while the respective geocentric values would all be in the "mean" tidal frame. No adjustment of the geoid height as is defined by equation (34) would be necessary, and any potential for confusion that might arise from such adjustments would be removed, particularly with regards to the consistency of this adjustment with the defined reference ellipsoid.

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8 References

1. Agnew, D. C., and W. E. Farrell, Self-consistent equilibrium ocean tides, *Geophys. J. R. Astro. Soc.*, 55, 171-181, 1978.

2. Andersen, O. B., P. L. Woodworth, and R. A. Flather, Intercomparisons of recent ocean tide models, *J. Geophys. Res.*, 100 (C12), 25261-25282, 1995.

3. Andersen, O. B., Shallow water tides in the northwestern European shelf region from TOPEX/POSEIDON altimetry, *J. Geophys. Res.*, 104 (C4), 7729-7741, 1999.

4. Benada, J. R., Physical Oceanography Distributed Active Archive Center, PO.DAAC, Merged GDR (TOPEX/POSEIDON) Generation B, MGDR-B, Version 2.0, June 23, 1997.

5. Cartwright, D. E., and A. C. Edden, Corrected Tables of Tidal Harmonics, *Geophys. J. R. Astro. Soc.*, 33, 253-264, 1973.

6. Cartwright, D. E., and R. J. Taylor, New computations of the tide-generating potential, *Geophys. J. R. Astro. Soc.*, 23, 45-74, 1971.

7. Chao, B. F., R. D. Ray, and G. D. Egbert, Diurnal/semidiurnal oceanic tidal angular momentum: Topex/Poseidon models in comparison with Earth's rotation rate, *Geophys. Res. Lett.*, 22, 1993-1996, 1995.

8. Chao, B. F., R. D. Ray, J. M. Gipson, G. D. Egbert, and C. Ma, Diurnal/semidiurnal polar motion excited by oceanic tidal angular momentum, *J. Geophys. Res.*, 101, (B9), 20,151-20,163, 1996.

9. Desai, S. D., J. M. Wahr, Y. Chao, Error analysis of empirical ocean tide models estimated from TOPEX/POSEIDON altimetry, *J. Geophys. Res.*, 102 (C11), 25157-25172, 1997.

10. Desai, S. D., and J. M. Wahr, Monthly and fortnightly ocean tide models empirically estimated exclusively from TOPEX/POSEIDON altimetry with implications for mantle anelasticity, *EOS Trans. AGU*, 79, 1998.

11. Desai, S. D., and J. M. Wahr, Monthly and fortnightly tidal variations of the Earth's rotation rate predicted by a TOPEX/POSEIDON empirical ocean tide model, *Geophys. Res. Lett.*, 26 (8), 1035-1038, 1999.

12. Eanes, R. J., An improved ocean tide model from satellite altimetry, *EOS Trans. AGU*, 80(46), 1999.

13. Heiskanen, W. A., and H. Moritz, Physical Geodesy, reprint, Institute of Physical Geodesy, Technical University, Graz, Austria, 1993.

14. Lambeck, K., The Earth's variable rotation: Geophysical causes and consequences, Cambridge University Press, 1980.

15. Lemoine, F. G. et al., The development of the joint NASA GSFC and the national imagery and mapping agency (NIMA) geopotential model EGM96, NASA Tech. Memo. 206861, 1998.

16. Le Provost, C., M. L. Genco, F. Lyard, P. Vincent, and P. Canceil, Tidal spectroscopy of the world ocean tides from a finite element hydrodynamic model, *J. Geophys. Res.*, 99 (C12), 24777-24798, 1994.

17. Le Provost, C., F. Lyard, J. M. Molines, M. L. Genco, and F. Rabilloud, A hydrodynamic ocean tide model improved by assimilating a satellite altimeter-derived data set, *J. Geophys. Res.*, 103 (C3), 5513-5529, 1998.

18. Mathews, P. M., V. Dehant, J. M. Gipson, Tidal station displacements, *J. Geophys. Res.*, 102 (B9), 20,469-20,477, 1997.

19. Mathews, P. M., Explanatory Supplement to the Section "Treatment of the Permanent Tide" of the IERS Conventions (1996), Deutsches Geodatisches Forschungsinstitut, Report 71, editor H. Schuh, pp 9-12, 1999.

20. McCarthy, D. D., IERS Conventions (1996), IERS Technical Note 21, July 1996.

21. Munk, W. H. and G. J. F. Macdonald, The Rotation of the Earth, a Geophysical Discussion, Cambridge at the University Press, 1960.

22. Rapp, R. H., R. S. Nerem, C. K. Shum, S. M. Klosko, R. G. Williamson, Consideration of permanent tidal deformation in the orbit determination and data analysis for the TOPEX/POSEIDON mission, NASA Tech. Memo. 100775, January, 1991.

23. Ray, R. D., S. Bettadpur, R. J. Eanes, E. J. O. Schrama, Geometrical determination of the Love number h_2 at four tidal frequencies, *Geophys. Res. Lett.*, 22 (16), 2175-2178, 1995.

24. Ray, R. D., R. J. Eanes, and B. F. Chao, Detection of tidal dissipation in the solid Earth by satellite tracking and altimetry, *Nature*, 381, 595-597, 1996.

25. Ray, R. D., A global ocean tide model from TOPEX/POSEIDON altimetry: GOT99.2, NASA Tech. Memo. 209478, 1999.

26. Schuh, H., Explanatory Supplement to the IERS Conventions (1996), Chapters 6 and 7, An Overview, Deutsches Geodatisches Forschungsinstitut, Report 71, editor H. Schuh, pp 5-8, 1999.

27. Shum et al., Accuracy assessment of recent ocean tide models, *J. Geophys. Res.*, 102 (C11), 25173-25194, 1997.

28. Tamura, Y., A harmonic development of the tide-generating potential, *Bulletin d'Informations Marees Terrestres*, 99, 6813-6855, 1987.

29. Wahr, J. M., Deformation Induced by Polar Motion, *J. Geophys. Res.*, 90 (B11), 9363-9368, 1985.