MODELLING OF GRAVITATIONAL EFFECTS DUE TO NONSTANDARD ATMOSPHERIC CONDITIONS

Martin LEDERER

Department of Levelling and Gravimetry
Land Survey Office, Czech Republic

Antonín ZEMAN

Department of Advanced Geodesy
Czech Technical University, Prague, Czech Republic

Summary : Models for estimating the gravitational effects of the Earth's atmosphere on gravity observations are defined. The models from the simplest Bouguer plate to the numerically more complicated models of the spherical cap, divided into the layers in which density is distributed according to height and, simultaneously, by a grid defined by geographical coordinates. Data from the European Center for Medium-Range Weather Forecasts (ECMRWF) was used. The values, computed from the investigated models, were compared with the residual values of gravity acceleration after elimination of Earth's tides, polar motion (free nutation) and other trends from the records of continual gravity data of the tidal gravimeter at the Geodetic Observatory Pecný (Ondřejov). The comparison is evidence that the Bouguer plate model with barometric pressure values, observed only at the places where gravity acceleration is observed, is sufficient. The correlations, however, indicate some advantages of the more sophisticated models.

1. INTRODUCTION

The atmosphere, surrounding the Earth, is a part there of from the point of view of gravitational effects. All geodetic observations, carried out within the atmosphere, are influenced both by direct gravitational effects of the atmosphere and by indirect (deformational) effects caused by variable loading of the Earth's surface by masses different from the masses of a normal atmosphere.

The atmosphere for which the values of the air temperature, pressure and density are expressed for individual heights above sea level by mean values, is referred to as the Standard Atmosphere. Static models of the density distribution of the atmosphere were proposed by different institutions, e.g., the C.I.R.A. model (Cospar International Reference Atmosphere) or U.S.S.A. (U.S. Standard Atmosphere) . Models are designed on the basis of observations and theoretical premises.

The effects of variable loading by the atmosphere were modelled by different authors using models ranging from extremely simple to considerably complicated, e.g., *Van Dam and Wahr (1987), Sun H.P. et al. (1995), Kostelecký and Zeman (1997)*.

The deformations of the Earth's surface and gravity variations due to the variable part of the atmosphere are mainly computed for the surface of a spherical elastic Earth by performing a convolution between the real local or regional barometric pressure and Green functions according to *Farrell (1972).*

The problem arises when we want to check the derived models. The best method is to compare model values with observations. For this purpose we can use the values of gravity acceleration from the tidal station at the Geodetic Observatory Pecný (Ondřejov) after eliminating tidal variations, the general drift of the observed values and variations caused by polar motion. Correlations between both values can then be used to determine the optimum variant of the model for computing the effects of anomalous atmospheric formations on the observed values of the gravity acceleration.

2. MODELS OF ATMOSPHERE

Bouguer plate model

To compute the direct gravitational effects of near atmospheric masses, one can use the model of an infinite cylindric plate with constant density σ_A and height b_A. Its gravitational effect is given as $\delta g = 2\pi G \sigma_A b_A$ (Bouguer plate), where G is the gravitational constant. If we also apply the relation for hydrostatic pressure of the atmosphere $p_N = \sigma_A g b_A$ ($p_N = 101\,325$ Pa), where g is the gravity acceleration, the local gravitational effect comes out as

$$\delta g_N^1 = 2\pi G p_N / g \,. \qquad (1)$$

For G = 6.672 x 10^{-11} kg^{-1} m^3 s^{-2} and g = 9.806 m s^{-2} the gravitational effect of pressure anomaly Dp = p – p_N = 100 Pa = 1 hPa (1mbar) is

$$\delta g_N^1 = -0.43 \times 10^{-8}\, m\, s^{-2} / 1hPa \,. \qquad (2)$$

The minus sign expresses the decrease of the observed value of gravity acceleration for positive value of the atmospheric masses above the point where the gravity acceleration is observed.

An additional change of gravity acceleration is caused by elastic deformation of the Earth's surface due to its loading by atmospheric masses. If the masses are directly above the point at which the effects are being computed, it is necessary to reduce the Green functions of Farrell's solution to Lamé's parameters for the very surface layers of the given Earth model. This yields approximate value $\delta g_E^1 = 0.04 \times 10^{-8}\, m\, s^{-2} / 1hPa$. The total change of the gravity acceleration from the modelling with the Bouguer plate is then

$$\delta g^1 = -0.39 \times 10^{-8}\, m\, s^{-2} / 1hPa \,. \qquad (3)$$

Spherical cap model

This model takes into consideration, in contrast to the Bouguer plate model, the curvature of the Earth surface (the atmosphere is then modelled by a spherical layer).

To find the direct effect, we used the elementary relation for the intensity of the gravitational field but modified to read

$$\delta g_N^2 = -G \int_{m'} \frac{dm_i'}{D_i^2} \cos \alpha_i \,, \qquad (4)$$

where quantities D_i (distances between the elements of mass dm_i and point P, where the effects are being computed) and $\mathbf{a_i}$ (cos a_i are the projections of the intensity vectors on to the geocentric radiusvector of point P) are shown in Fig.1. and are defined by the following two relations using geocentric spherical coordinates ρ, Φ, Λ.

$$D_i = \left(R^2 + \rho_i^2 - 2R\rho_i \cos \vartheta_i\right)^{1/2}, \qquad (5)$$

$$\cos \alpha_i = \frac{\rho_i^2 - D_i^2 - R^2}{2D_i R} \,. \qquad (6)$$

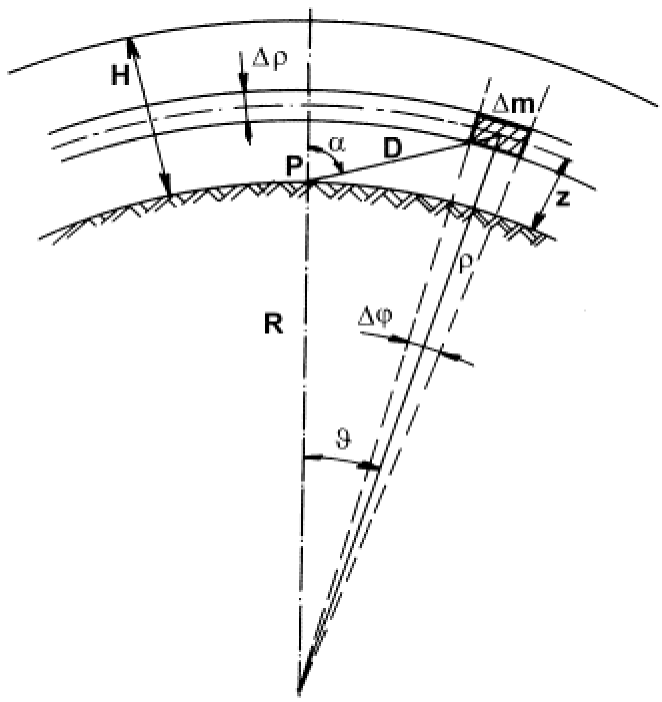

Fig.1.

In Eqs. (5) and (6) $\rho_i = R + z_i$ and $\vartheta_i = \sin\,\Phi \sin\,\Phi_i + \cos\Phi\cos\Phi_i\cos(\Lambda - \Lambda_i)$. The computation will be carried out as the sum of the effects of the finite elements of mass $\Delta m_i = \sigma_i \Delta\tau_i$, where $\Delta\tau_i$ are the finite elements of the volume which must be defined individually for a particular configuration. The density of the atmosphere can be modelled in two ways:

a) by a constant for the whole layer using the relation

$$\sigma_i = \Delta p_i / (gH), \qquad (7)$$

b) by a relation which takes into account that the barometric pressure decreases with increasing height. This relation can be expressed as an exponential function of the of density varying with height

$$\sigma_i = \sigma_0 e^{-z_i/H}, \qquad (8)$$

where σ_0 is the density reduced to the Earth's surface. H and z_i are symbols for the height of the whole layer and the heights of the individual layers, respectively – see Fig. 1.

The elastic effect δg_E^2 of mass Dm_i at angular distance J_i from point P is given by *the Farrell (1972)* solution. In the computations we interpolate the tabulated values of the Green functions $G(J_i)$ and values Dm_i, now located at discrete points on the Earth's surface ($z_i = 0$)

$$\delta g_E^2 = \frac{\Delta m_i G(\vartheta_i)}{10^{18} R \vartheta_i}. \qquad (9)$$

Expression (9) is normalized by the numerical value of the term $10^{18} R \vartheta_i$ (R = 6.371 x 10^6m, J_i in radians), hence the result is in units of m s^{-2}.

Models of the spherical cap in which the barometric pressure depends on height its surface is divided into a grid by

geographical coordinates

To come even closer to reality we have used the air pressure data given in the $2.5^O \times 2.5^O$ grid of geographical coordinates (for practical computations we shall use data in the intervals from 10^O to 80^O in latitude and from -20^O to 60^O in longitude and with 12-hour time interval). The barometric pressure data for a grid of this extent are from the ECMWF (European Center for Medium-Range Weather Forecast). For the reason of data division we have used for the computation a modified spherical cap model which is not circularly symmetric but is represented by the area limited by geographical coordinates $\left(\varphi_{\min}, \varphi_{\max}; \lambda_{\min}, \lambda_{\max}\right)$.

The formulas will be the same as for the previous spherical cap model, only the following formula is used to compute the mass elements – see Fig. 2.

$$\Delta m_i = \sigma_i \rho_i^2 \cos \varphi_i \Delta \rho_i \Delta \varphi_i \Delta \lambda_i \quad , \Delta \rho_i = \Delta h_i \, . \qquad (10)$$

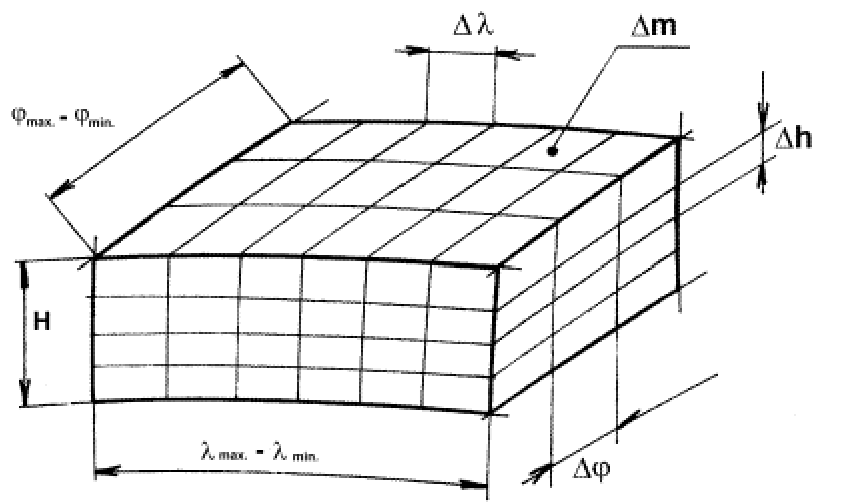

Fig. 2.

Densities σ_i are now computed for each mass element separately. Using the relation between density and barometric pressure, we can compute the densities from the interpolated values of the observed barometric pressure given by the ECMWF data. The resulting direct gravitational effect is again given by the formula

$$\delta g_N^3 = -G \frac{\Delta m_i}{D_i^2} \cos \alpha_i \, . \qquad (11)$$

The indirect effect is computed in the same way as for the previous spherical cap model.

3. RESIDUAL GRAVITY ACCELERATION AFTER ANALYSES OF TIDAL GRAVIMETER RECORDS

The tidal gravimeter Askania Gs 15 No. 228 (Geodetic Observatory Pecný) provides digital records of gravity acceleration observations at 1-hour intervals. The observations are, however, affected by different influences which have to be removed to be able to compare them with the computed model values of the gravity acceleration variations caused by variable barometric pressure.

The main effects are due to the Earth's tides and gravimeter drift. We can also judge the significance of eliminating other influences such as ocean tides and polar motion (free nutation).

After introducing all the corrections, we can assume that the most significant residual effect is (except for errors of observations and other negligible effects) the effect of the varying distribution of atmospheric masses.

n <u>Effect of the Earth’s tides</u>

The effect of tidal waves of synthetic tides was removed by theoretical reductions. Computations were made using the special software of the GO Pecný.

n <u>Gravimeter drift</u>

The gravimeter drift was approximated by a polynomial of optimum degree for a given time interval of observations and then removed by reductions from the observed values. Fig. 3. shows the analyzed time interval (1.7.1994 – 27.11.1994) where the values were cleared of tidal effects and approximated by a 3rd degree polynomial.

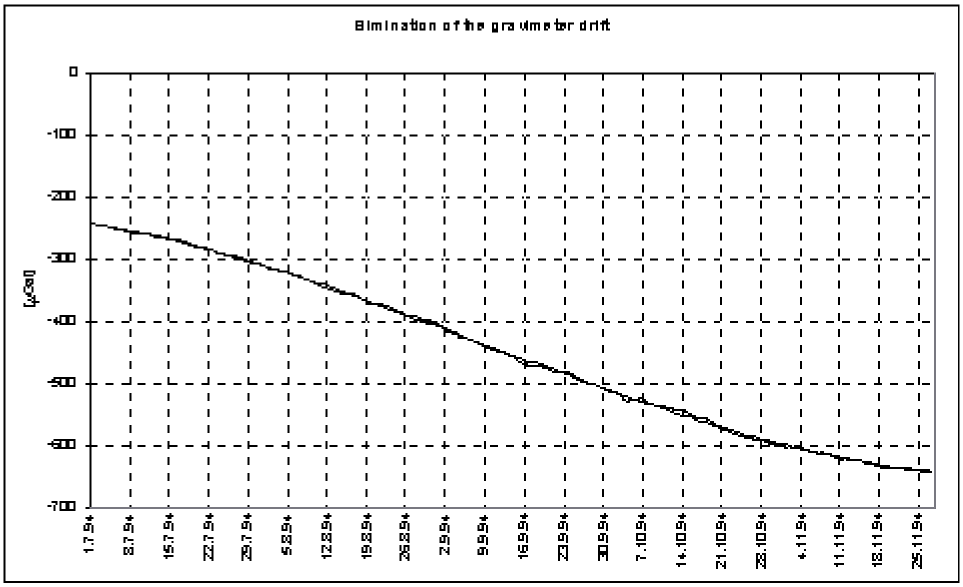

Fig. 3.

n <u>Polar motion (free nutation)</u>

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The effect was removed with the help of the following formula

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$$\delta g_P = -18.9 \sin 2\vartheta \left(x \cos \lambda - y \sin \lambda \right) , \qquad (12)$$

where x, y are the pole coordinates in arc seconds , j, l are geographic coordinates of the point where the effect is being computed, J = 90-j. The coordinates of the CIO pole were taken from the IERS Bulletin. The resulting values are in $\mu Gal \left[10^{-8} m\, s^{-2} \right]$.

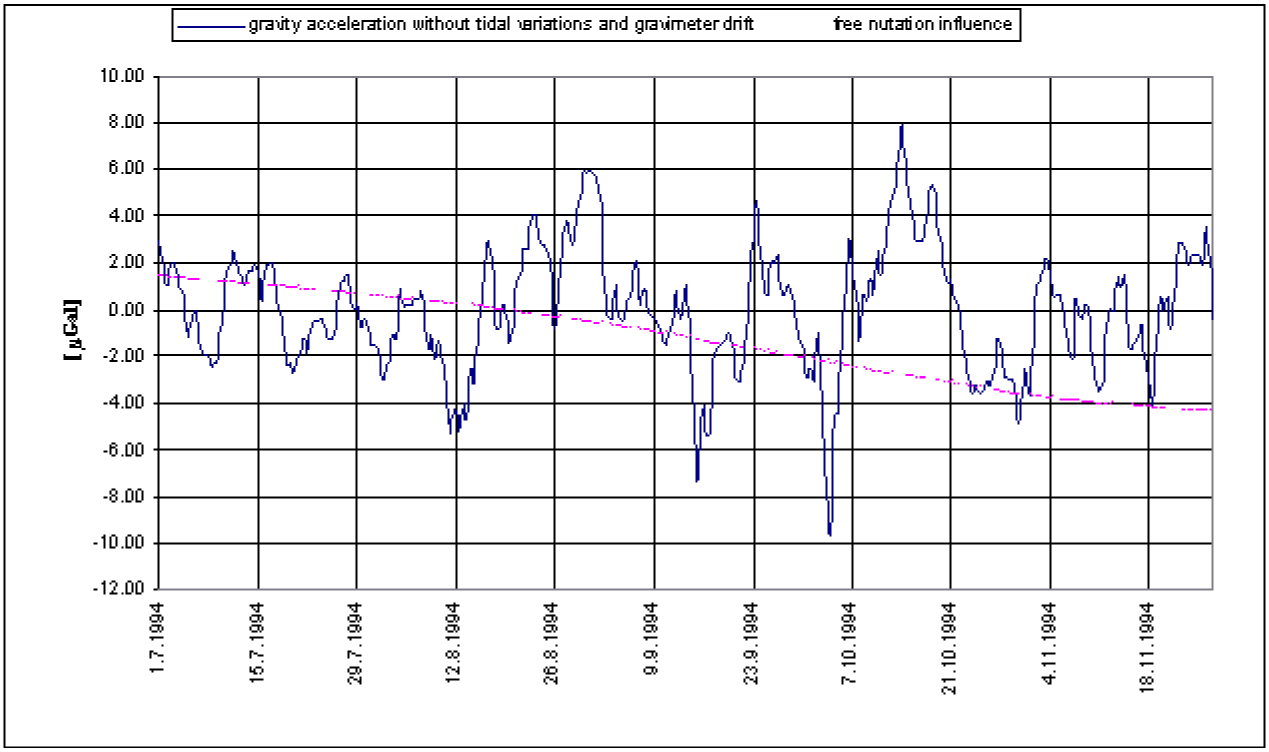

Fig. 4.

4. CORRELATIONS BETWEEN THE RESIDUAL VALUES OF GRAVITY ACCELERATION AND THE VALUES COMPUTED FROM DIFFERENT MODELS OF THE ATMOSPHERE

Correlation coefficients were computed for comparing the individual models with the observed (and reduced) values. The slopes of the regression lines were then computed. These values express the dependence of the variations of the gravity acceleration on the variations of barometric pressure. We used 146 days of observation i.e. 292 data altogether. The models are compared with the “observed” values in Table 1 and Fig. 5.

Table 1.

Model	Correlation r	Regression $\mu Gal / hPa$
Bouguer plate	0.45	0.39
Spherical cap – pressure varying with height	0.45	0.24
Spherical cap – pressure varying with height and position	0.62	0.23

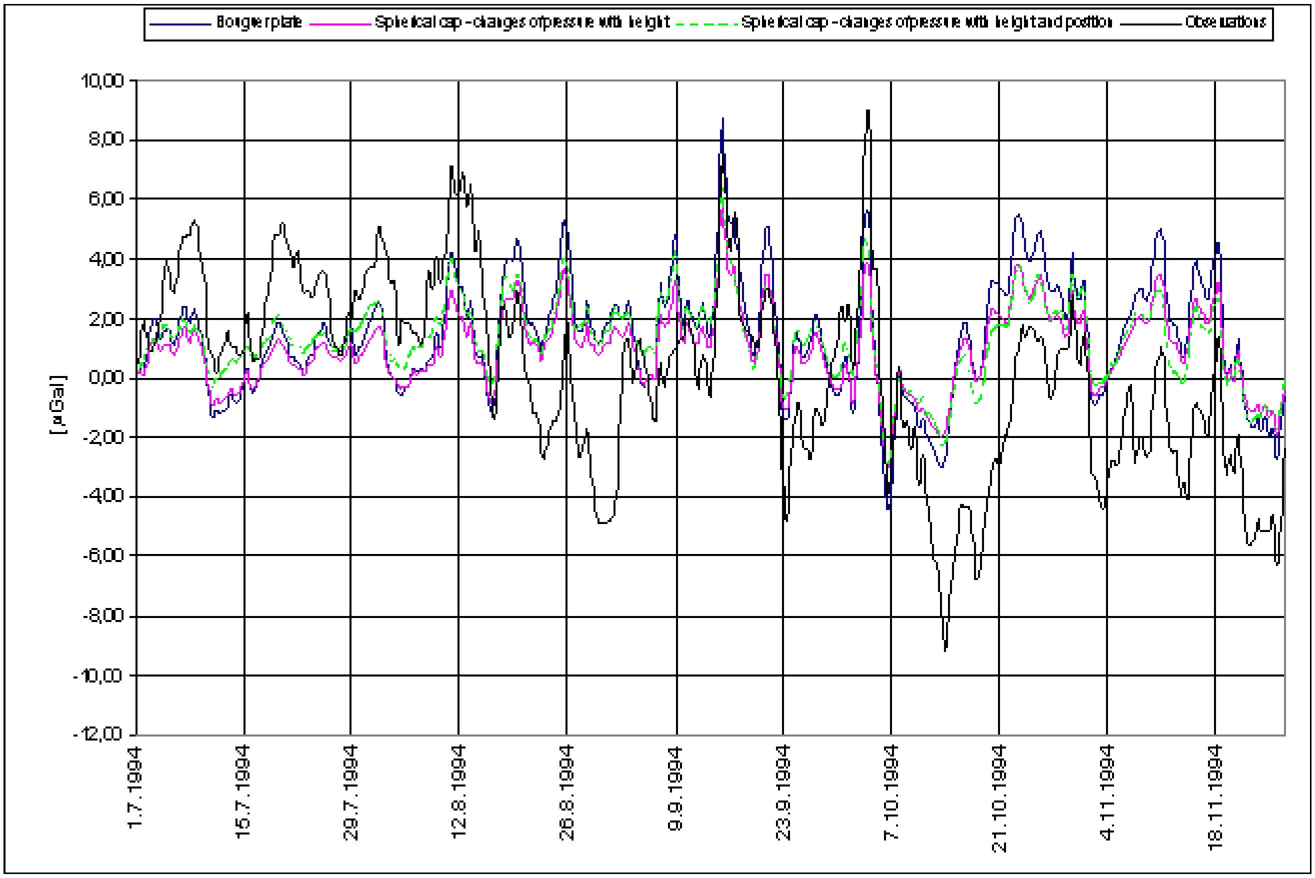

Fig. 5.: Comparisons of the models with “observed” values

The same comparisons for shorter time interval are in Fig. 6.

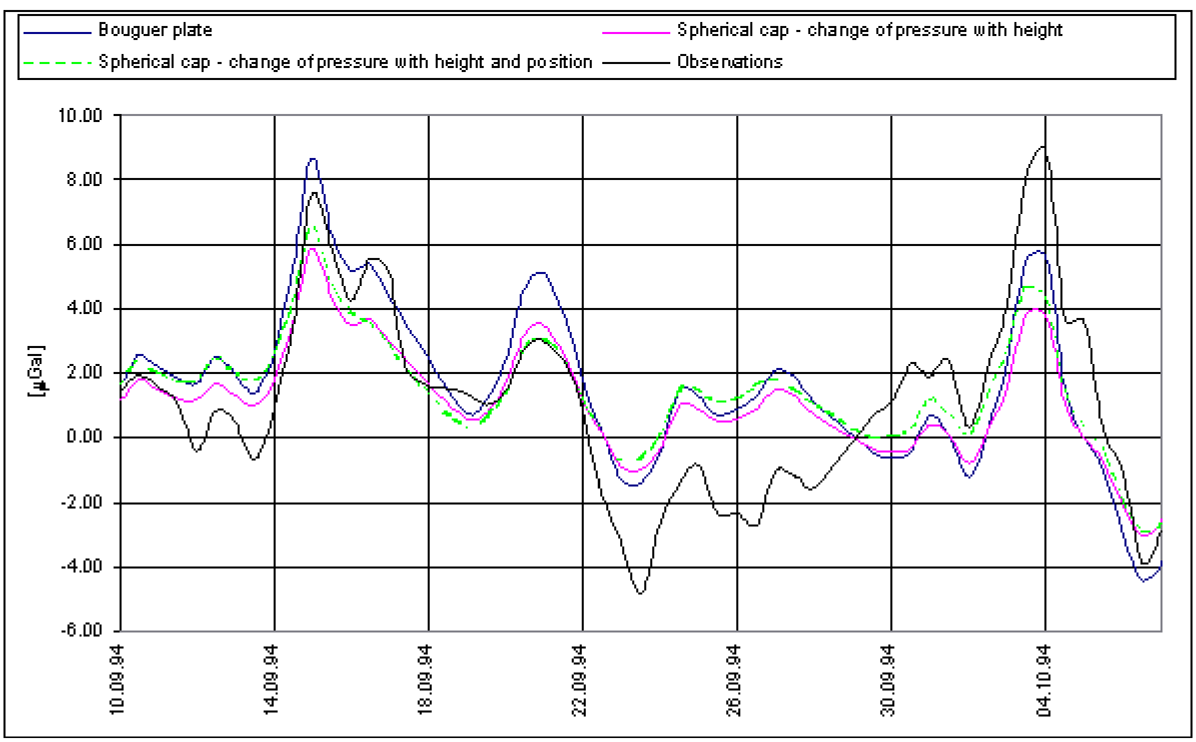

Fig. 6.: Comparisons of the models with “observed” values – detail

5. CONCLUSION

We have derived some mathematical models which can serve to compute the gravitational effects of nonstandard atmospheric conditions. The Bouguer plate model is very simple from the mathematical point of view. Other models come closer to reality. This means they take into consideration the real "shape" of the atmosphere, the variations of pressure with height and, in the last model, we are not restricted only to local barometric pressure but we have used data from a grid of larger extent. The computed results can then be compared with the real "observed" values.

Having compared the resulting values and correlation coefficients of all solutions, we can conclude that the models are not significantly different. Not even the model using interpolation in the grid of pressure data of larger extent represents a considerable improvement although the correlation has increased slightly. This supports the conclusion that the local anomalous barometric pressure has the greatest influence, and that, in most cases, the information on barometric pressure variations in a wider vicinity of the point where the effects are to be computed, does not need to be used.

For practical purposes, the Bouguer plate is the best model which is, moreover, very simple for computations. It is clear from this paper that the results from this model are very near to the real values of the effect.

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References

[1] Farrell W.E.,1972: Deformation of the Earth by Surface Loads. Rev. of Geoph. and Space Physics,10, 761 – 797.

[2] Van Dam T.M., Wahr J. M., 1987: Displacements of the Earth's Surface Due to Atmospheric Loading: Effects on Gravity and Baseline Measurements. J. Geophys. Res.,92,1281 – 1286.

[3] Sun H.P., Ducarme B. and Dehant V.,1993:Preliminary Investigations of the Atmospheric Pressure Effect on Vertical Displacement and Gravity Observations. B.I.M. (Marees Terrestres, Bulletin D'Informations), No. 114, 8334 – 8362.

[4] Sun H.P., Ducarme B. and Dehant V.,1995: Effects of the Atmospheric Pressure on Surface Displacements. J. Geodesy, 70, 131 – 139.

[5] Zeman A., 1978: Estimate of the Effect of Changes of Atmospheric Pressure on the Results of Levelling. Studia geoph. et geod.,22, 241 – 247.

[6] Kostelecký J., Zeman A., 1997: Study of the Influence of the Indirect Effect of Ocean Tides and of the Direct Deformation Effect of the Atmosphere on the Results of GPS Observations. Studia geoph. et. geod.,41, 93 – 102.