Calibration of pendulum by sinusoidal torque induced with the needles of a watch turning in the gravity field of the Earth

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Abstract

An astatized symmetrical vertical pendulum is monitoring torque Γ_M resulting of gravitational attractions exerted by two external masses M moving up and down. Local gravity field $\vec{g}$ produces the main pendulum restoring torque combined with a variable torque Γ_m of similar intensity induced by the rotation of the needles of a watch which is embedded on the pendulum. Transfer of fundamental units to calibrate the Γ_m torques is obtained by a reference torque Γ_μ resulting of precise displacements of a well known mass μ. We permanently monitored ratio between the gravitational effect Γ_M and calibrated Γ_m to determine G. The position of the pendulum is measured with a capacitive bridge. Bias voltages sent to two electrodes set-up at the bottom of the pendulum allows to feedback pendulum with a controlled electrostatic torque. We discuss potential interest of our prototype to design a multi pendulum system to check systematic effects for different geometries and various kinds of materials.

Keywords: tidal instrumentation, gravity, G gravitational constant, micro force, pendulum, calibration

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1. Introduction

Newtons constant of gravitation G, is a fundamental constant of nature that determines the gravitational force between two massive bodies. The value of G varies by over 400 parts per million, 20 times greater than the uncertainty on any kind of measurement. So the precise value of G remains an open question for the physicists. To better understand this variation, we prepare a series of experiments designed with the expertise gained at the Royal Observatory of Belgium in the tidal instrumentation. Indeed very large dynamics are necessary in this domain with instrumental long term stability and stable sensitivity. Surface laboratory measurements are affected by various effects induced by thermal stress, infrasonic waves, atmospheric pressure waves, human activities effects, etc. For example, tiltmeter recording in a surface laboratory could not detect the tides. In an underground laboratory at a depth of fifty meter, we have access to a more than sixty dB on the same signal.

Analysing the patterns of various tidal instrumentation, we select a vertical pendulum approach with the local gravity field like restoring force. Capacitive transducer positioning sensor which reach sufficiently the precision needed. We induce electrostatic feedback similar to system applied in the tidal gravimeters. Calibration process is based on stacking filtering developed in the tidal waves detection within long series of registration and on inertial calibration developed by a vertical motion platform. Our new experiment allows G to be measured in two independent ways which are from angular deflection and from the electrostatic feedback force needed to compensate effect of gravitational induction. The operating mode consist in recording simultaneously the gravitational induction and a well known calibrated torque. We meter the ratio between the two effects which are modulated with different periodicities allowing separation by stacking. So we could ignore the sensitivity of the pendulum itself. This paper introduces the

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different element of our research. Our G value obtained has less than one percent of discrepancy with CODATA value which could be attributed in the limitation of accuracy in the geometry and mass values determination. However the dynamics observed experimentally confirm potentiality of our method.

2. State of art

Physics of the gravitational forces continues to be mysterious. Since Galileo, this force become better and better described by theoretical models. But their origins stay an open question. Geophysicists in charge of determination of the Earth gravity field announce a very coherent pattern in relation with metrological standard units. Tidalists demodulate tidal signals induced by the permanent motions of Moon and Sun in the local coordinates. They reach dynamics better than 10^{10}. This reality continue to be based on the Newton principle of gravitation expressed by the short formula:

$$F = G\frac{M_1 M_2}{r^2} \qquad N \tag{1}$$

This simple representation of the gravitational interactions describes the astronomical observations and organises space navigation with enough accuracy.

Inversely the fixation of G value remains at the limit of instruments with systematic errors limiting the accuracy to 1/10000.

At the present time, the CODATA (Committee on Data for Science and Technology) recommended value of the Newton's constant, G, is announced with a relative uncertainty of 1.2×10^{-4} [Peter J. Mohr (2012)].

$$G = 6,67384(80)10^{-11} \qquad m^3 kg^{-1} s^{-2} \quad \epsilon_r = 120ppm \tag{2}$$

Noting that, on the one hand, the successively recommended values of G, and on the other hand the current measurement have no obvious overlap, the CODATA committee asked the scientific community to continue the experiments with various instruments and methods, to compare the systematic effects of each ones.

The main problem in G determination is that the system to measured is influenced by multiple various effects. The value of certain ones are several orders of size greater. Remembering that the gravitational force of the Earth is the main one, we have also to take into account forces induced by electrostatic and magnetic processes. In addition, the geometry is influenced by temperature gradients, and we have also to consider mechanical effects of displacements due to micro seismic events, industrial noises or airflows, thermal drift and so on. Those forces can totally overwhelm the force to measure, or combine with it in a non-linear fashion, preventing its evaluation.

In general, the experimental devices try to eliminate the Earth gravitation, and correct all other perturbations separately, e.g. by working in the vacuum.
Cavendish tried to get rid of the Earth gravitation by using a torsion balance whose movements are orthogonal to the Earth force. In fact, the Earth gravitation was present anyway through the weight of the balance applied to the rod that affects its torsional force. This effect occurs at the molecular level, along the complete rod.
Despite those difficulties, the resulting error obtained in 1798 was less that 1%, which is really outstanding.

In 2010, measurement techniques improved this result up to 4 significant digits. The internal coherency of the values issued from the *CODATA* referenced experiments, shows smaller value than the dispersion of the absolute values of the G constant. So, it seems that results are influenced systematically by undetermined parameters. Problems could be induced on the instrumentation or on gravity interaction into processes. Our goal will experience following earth tides instrumentation expertise : the simplest as possible techniques with a maximum of control for each kind of physical interactions. First assessment is that we could compare the modulated gravity action obtained by moving mass with the gravity action of the mass of the Earth.

Absolute gravimeters gives the g value with $0.01ppm$ of accuracy by direct reference to the standard units. Objective of our method consists to use the pendulum principle to measure G referring to g with gravitational modulation function of mass expressed in *kilogram*, geometry and displacements in *meter*.

Let us recall that g is equal to G multiplied by the mass of the Earth divided by the square Earth mean radius. Our system based on the ratio between two actions becomes unable to detect a change of G.

3. Functional analysis of the G balance

In this context, we propose to exploit the earth local gravity field value g, to rely a gravitational force between two masses to the fundamental references.

From our expertise gained with experiences with tidal instrumentation, a list of specifications has been established for the design of an experimental device able to measure Newtons constant.

1. Our system uses a pendulum approach [Naslin (2009)] (see Fig. 1) referred to g with actions expressed in term of torques induced by various effects. The sum $\sum \Gamma_i = 0$ corresponds to the equilibrium point.
2. Astatization obtained with an horizontal axis of rotation just over centre of mass of pendulum allowing comparing very weak horizontal forces with larger vertical ones. Central positioning rejects largely translation mode of the micro seismic noise acting on the support.
3. Geometry defining the gravitational torque for different positions of the moving parallelepiped masses. Design decreases the effect of lateral discrepancy by larger attractive masses than the pendulum ones.
4. Axis of rotation experimental selection of cutter knives with their edges set perpendicular on a drill rig cylinders (see Fig. 3 and 4). This point determines the instrumentation approach because the position of the center of rotation directly involves the period and thus the sensitivity of the instrument.
5. Minimisation of thermal, acoustic and magnetic influences induced by the motion of the twin attractive masses. A constant loading effect on the ground rejecting risk of pendulum tilt.
6. Use of electronics based on symmetric phase detector connected to capacitive transducer [van Ruymbeke (1980)] with a limited gap between plates large enough for pendulum operating quite at critical damping just by air friction. It allows very high precision in the positioning without hysteresis.
7. Application of bias signal using the bottom capacitance plate to induce electrostatic torques for determination of step response or to apply electrostatic feedback to keep constant the position of the pendulum mass allowing neglecting the inelasticity of the suspension. Pulse Width Modulated (PWM) signals on both sides decrease non linear coupling between electrostatic induction and instantaneous position of the mass vibrating under environmental influences.
8. Filtering of random noise by stacking approach allows separating different signals just by linear averaging.

For this case of functional analysis, our approach is similar to a method of fundamental physics, where the simplification of our instruments is necessary to observe nature (and avoid to mask any phenomenon).

Therefore, the issues discussed above lead us to develop flexible simple instruments (with interchangeable pieces), ergonomic (for facilitated setting).

4. G Symmetrical Vertical Pendulum

A pendulum can be displaced from its rest position by an horizontal force that is significantly lower than the weight of the pendulum (remind that the torques Γ_i are the orthogonal projections of the forces F_i on the axis of rotation of the pendulum instrument). The ratio is equal to be displacement divided by the length of the pendulum. In such conditions, it is quite possible to measure horizontal force that is 10^{-8} times smaller than the weight of the pendulum. Since in our case, the gravitational forces F_M are quiet orthogonal to $\vec{g}$, we have to take in account for the torque determination its orthogonal component if the pendulum is vertical.

The weakness of the gravitational attraction obliges to use an astatic pendulum which could have restoring force divided by more than 1000 compared to vertical pendulum.

The natural period of an astatic pendulum (between 15 and 20 seconds) minimizes external perturbations. A classical pendulum would need a length of more than 50 m to reach same sensitivity (see right side of Fig. 1).

For a simple pendulum, the restoring force is $-mgsin(\theta)$. For the center of mass at a distance L of the axis of rotation, the restoring torque is $\Gamma = -mgLsin(\theta)$. For a Symmetrical Vertical Pendulum (SVP) (see Fig.1, left), the restoring force depends of θ proportionally to the addition of the lower torque $\Gamma_l = -mg(L + \Delta L)sin(\theta)$ and the upper torque $\Gamma_u = -m(-g)(L - \Delta L)sin(\theta)$. If $\frac{\Delta L}{L}$ decreases, sensitivity to external force increases until to reach an infinite value at the neutral equilibrium for $\Delta L = 0$.

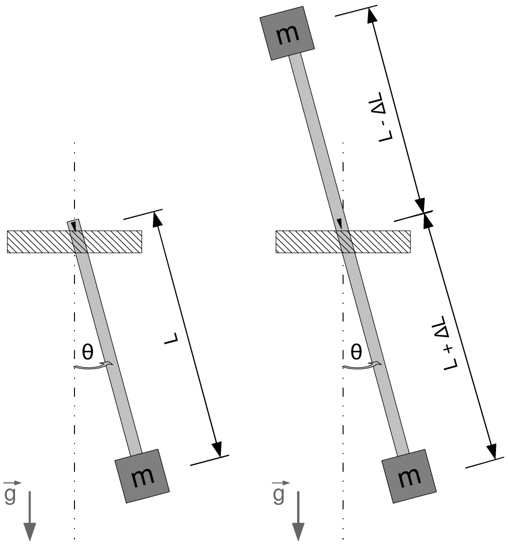

Figure 1: Principle of the simple pendulum (left) and of the astatized SVP pendulum (right).

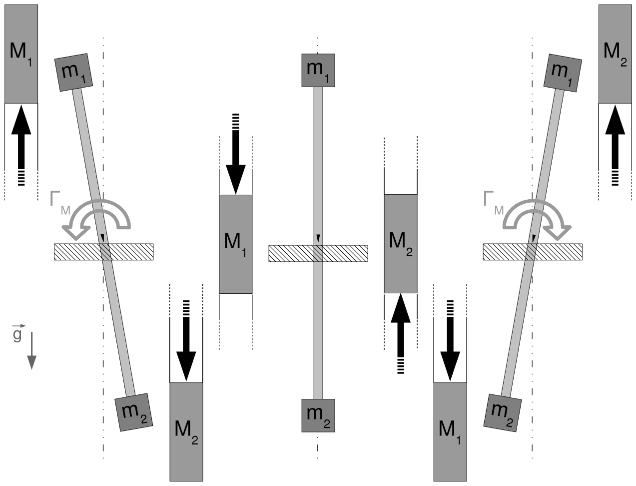

Figure 2: Principle for G measurement.

A belt rolling on a cylinder with horizontal axis of rotation supports two attractive masses which have inverse vertical motion. We control the mass displacement with a stepper motor and a gearbox driving the rotation of this cylinder. The loading of that system on the ground supporting the pendulums become constant (see Fig.2). The equilibrium position of the angular position θ of the pendulum figures that the combination of all the torques, equals 0. An additional modulated electrostatic feedback loop could be adjusted to keep θ constant.

Principle of SVP obliges to have a very high stability in the positioning of the axis of rotation. To select an appropriate system, a $10kg$ mass is fixed on a vertical beam supporting the axis of rotation prototype to evaluate (see Fig.3). An optical detector triggers a magnetic pulser keeping in oscillation the pendulum with a minimum of interaction. Periods are recorded by counting a quartz oscillator.

Series of tests confirms high quality of a support consisting in a Stanley $^{\mathrm{TM}}$cutter blade $N^{\circ}1992$ pushing perpendicularly on a $3mm$ in diameter drill rig cylinder used for PCB boring. The prototype pendulum oscillating on 1 second period with such heavy mass did not show abnormal features. The presented system referring to the randomness of

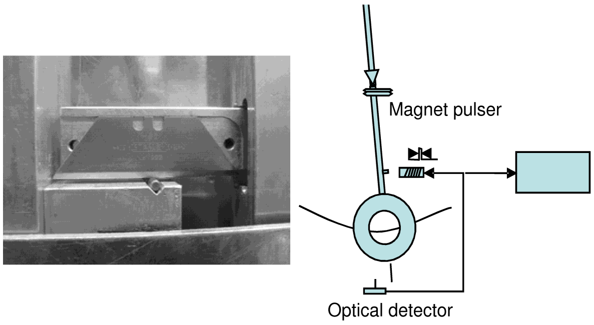

Figure 3: Detail of a blade swivel and the test pendulum principle.

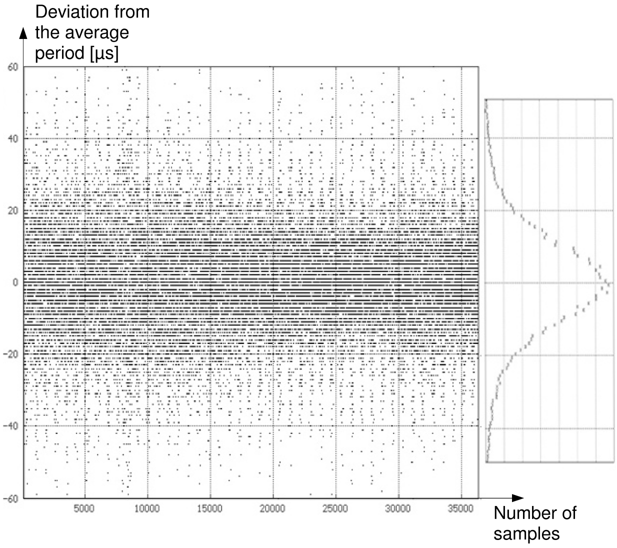

Figure 4: The figure shows an histogram of a series of 45000 periods measurements.

the histogram on 45000 periods (see Fig.4) is well adapted to the requirements of our heavy SVP device.

5. SVP scale factor determination

A calibration system is required to refer to the fundamental standards of mass and length with like a secondary standard the well known intensity of the local gravity field.

Figure 5 shows property of torque induction by mass motion. If we move for a pendulum of axis of rotation Y, a mass m from point P to P' in the earth gravity field parallel to Z axis, we only have to consider the motion x to x' which is the projection of $P - P'$ on horizontal axis, perpendicular to the axis of rotation.

The low intensity of the measured gravitational forces compared to various random noise sources could reduce the signal / noise ratio. So, we choose filtering by averaging based on stacking method with well known frequencies and modulation on induced periodic phenomena. It improves dramatically the signal / noise ratio by effective rejection of all random noise [Zhu et al. (2009)] [Van Ruymbeke et al. (2003)]. To meet requirement of stacking, we place on the pendulum an analogue watch with tiny needles moved by Quartz mechanism (see Fig. 6). The rotation of two needles produce two periodic sine torque with periods of 1 minute and 1 hour and stable amplitude.

An analogue watch is a simple and suitable solution. Indeed, using current technologies, the electrical power supply and the long term stability of the movement are ensured. The sine torques generated by the rotation of the needles

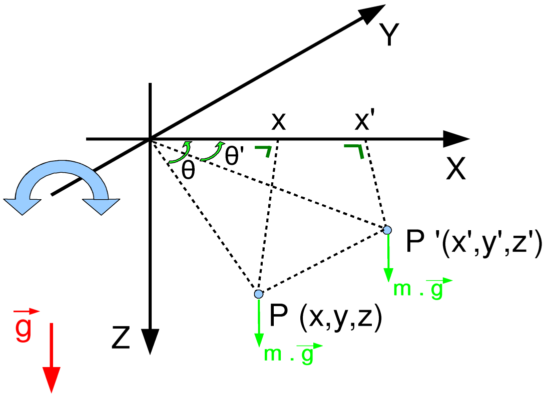

Figure 5: The torque produced by the motion of the mass m from P to P' is equal to $d\Gamma = |mg(x - x')|$.

Figure 6: Example of a solar watch mounted on a pendulum pivot for testing

are easy to model. Moreover, a waterproof watch allows to avoid the Archimedes effects due to the changes of atmospheric density. However, the watch manufacturer are unable to achieve needles systems with a repeatability needed for this metrological use. Moreover, the metallic needles are sensitive to thermal gradient (but can be modelled).

Before setting on a pendulum for the $\Gamma_M \leftrightarrow \Gamma_m$ comparison, the watch has to be calibrated in absolute referring to the fundamental standards of *mass*, *length* and using the value of the Earth gravity field as a secondary standard (known with enough accuracy).

For this calibration of each watch, a specific lightweight optimized pendulum instrument (see Fig. 7) was designed. This instrument embeds the watch and an miniaturized electromechanical system in charge to produce a standard torque Γ_μ.

Such a system is equipped with a micro brushless DC motor and an epicyclic reducer. The accuracy is limited by the errors of determination of the mass and the amplitude of displacement (see Fig. 8).

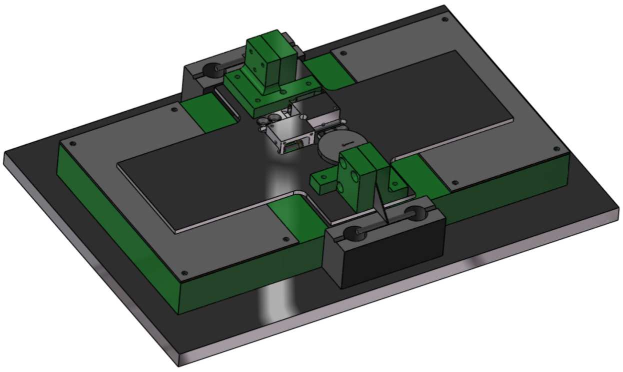

Figure 7: CAD view of the pendulum designed for watch calibration. The DC motor system is set up on the balance in parallel with the watch. The goal of this small lightweight pendulum becomes to conduct the calibration of the watches by direct comparison with two torques related to fundamental units.

Figure 8: Example of a micro motor used for calibration

Given its use of standard, this system has been designed to be embedded on a lightweight pendulum and to enable a relative uncertainty on the torque less than $100ppm$. For this specific problem, it is easy to separate the sine wave induced to the watch torque and to the slope generated by the linear motion of the small weight at very slow and constant speed (see Fig. 9).

The advantage of this calibration method is that each watch is individually calibrated. This allows the followings properties:

- As mentioned above, the position of the watch is not critical because, only the projection on the X axis is inducing a periodic torque (see Fig. 5).
- The individual calibration of each watch enables to mount a series of pendulum instruments, reacting to the movement of a couple of attractive masses. By extension, this allows gravitational optics experiments.
- We can validate the quality of our SVP instrument by monitoring the ratio between the amplitudes of the torques introduced by the rotation of the two needles which must be ideally constant.

The angular deviation of our pendulum instruments are measured through capacitive sensors. They are ideally suited for measuring displacements of such magnitudes. They also allow exerting reaction forces, to fully compensate the displacements. This allows controlling potential causes of errors:

- The mechanical properties of the pivots could induce perturbations like hysteresis.
- The gravitation forces are, of course, a non-linear function of the displacement; this aspect is made more complex due to the calibration features causing additional displacements.

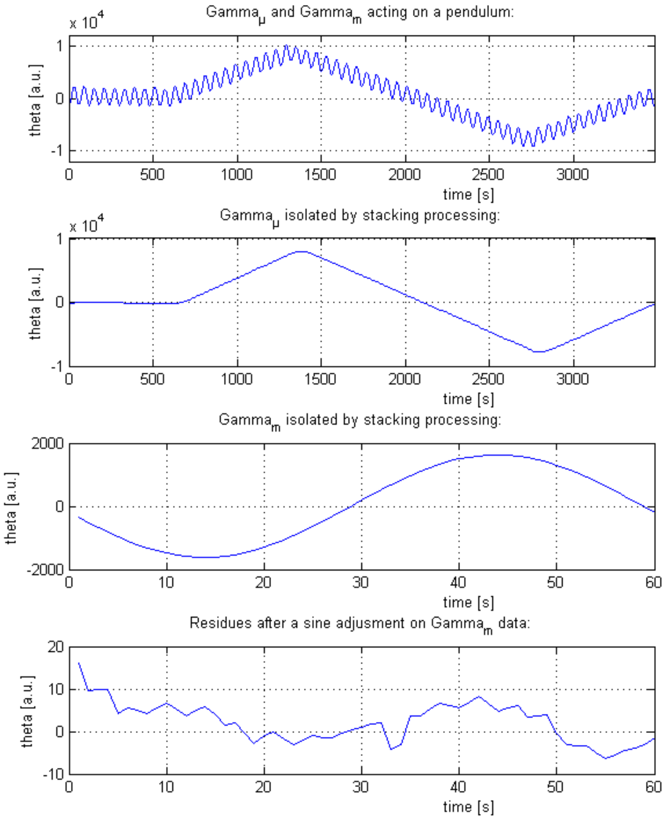

Figure 9: Exeprimental data obtained with a prototype pendulum in 2011 at ROB. The scale factor between the $Gamma_\mu$ torque (second graph from the top) and the $Gamma_m$ torque induced by the seconds needle of a watch (third graph from the top) can easily be compared after stacking processing. The amplitude of the residues proves the reality of the technique.

Mastery of these techniques of design, calibration and measurement enables to set up a system composed by three independent SVP, to conduct a new campaign of multi-dimensional gravitational interaction experiments (see Fig. 10). We can fix accurately the relative sensitivity between the three pendulums by the use of three calibrated watches.

The value of G is evaluated by comparing the measurement of the attractive forces on the 3 pendulum, with their theoretical value obtained by computation. This computation needs an integration by finite elements. We selected the parallelepiped for all elements influencing the result, that is in fact, all moving parts (including translating and rotating ones). Manufacturing parallelepipeds is obviously much easier than spheres

Let us remark that the distance between an attracting and an attracted object is conditioned by the protection features avoiding the pendulum being influenced by drafts, gradients of temperatures, and so on. This results in a minimal distance of 2 or 3 cm.

6. Conclusion

This paper describes how to generate calibrated torque on gravitational balance. Calibration of this torque is referenced to a known mass and a fixed motion with its speed expressed in meter and second. The well determined Earth gravity field g which is proportional to the product $G \times M$ with M the Earth mass, induces the modulation of forces which produces the torque acting in the pendulum. So the procedure of calibration only consists in comparison of parameters with same physical dimensions.

Our prototype simply made with aluminium has promising characteristics. It operates without vacuum and shows a high rejection of ground vibration. A large enough support with metal strips fixing axis of rotation could work for

Figure 10: Picture of the set of 3 pendulums (each equipped with a calibrated watch), with a couple of attractive masses, moving vertically up and down. The gravitational torque on each pendulum is directly measured by comparison with two torques of the watches needles.

heavy masses without inelastic problem. The pendulum is calibrated by a fully independent action carried out simultaneously with gravitational measurements. Geometrical discrepancies are well rejected by a symmetrical approach. Improving our methods by a more careful setting of our balance could be justified.

Installing the pendulum in an underground laboratory at constant temperature and pressure could potentially increase the significance of results. We evaluate for the G SVP balance that the signal-to-noise ratio of 10000 is accessible. A new set of 3 SVP meters the attraction of the two attractive masses moving between them. So we have 3 signatures which allows to trace origin of systematic effects. By comparison of individual output signals it could be possible to detect eventualities of various kinds of effects susceptible to influence G constant value. By the simplicity of our prototype, this could be a good experimental system to study gravitational physics.

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