The K1 triplet: Can Lunar nodal waves contribute to the study of the Free Core Nutation (FCN)?

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ABSTRACT

The longest series of superconducting gravimeters participating to the Global Geodynamics Project (GGP, Crossley et al., 1999) are now ranging between 10 and 18 years. It was possible to extract successfully the nodal waves for 12 series longer than 3,500 days using the VAV04 tidal analysis program (Venedikov and Vieira, 2004). In most of the cases the tidal parameters of the nodal waves agree with those of the main tidal constituent. The K1 triplet is especially interesting, being submitted to the resonance of the liquid core of the Earth. The amplitude factors of the three constituents should differ by 0.1% according to different Earth models. This effect is clearly seen in our results. We introduce a parameter $\rho\pm = |1- \delta_{K1}\pm/\delta_{K1})|$, free from calibration errors and ocean tides loading influence, to express the relative difference between K1 and its nodal companions $K1^-$ or $K1^+$. The $K1^-$ nodal wave has a too small amplitude to provide reliable results but the mean relative difference ρ^+ between K1 and $K1^+$ (0.113%±0.022%) is very close to the values 0.124% and 0.116% predicted respectively by the DDW99NH (Dehant et al., 1999) and the MAT01NH (Mathews, 2001) non hydrostatic models.

Keywords: superconducting gravimetry, Free Core Nutation, nodal waves

Foreword

The lunar nodal waves associated with the main tidal components have been fairly well separated from a 14 year long record of the superconducting gravimeter T003 (SG, Hinderer et al., 2007) of Brussels by Ducarme and Melchior (1998). The most interesting result concerned the K1 triplet associated with the 18.6124 year astronomical nutation. Fifteen year later most of the SGs operated since 1997 in the framework of the Global Geodynamics Program (GGP, Crossley et al., 1999) have records longer than 10 years that could be used for the same purpose.

1. Introduction

Let us consider the development of the tidal potential due to the Moon (Wenzel, 1997a)

$$W = \sum_2^\infty W_n = \frac{GM}{c}\sum_{n=2}^{\infty}(\frac{r}{c})^n \frac{1}{2n+1}\sum_{m=0}^{n} P_{nm}(\cos\theta).P_{nm}\cos(\pi/2-\delta).\cos(mH) \qquad (1)$$

with G gravitational constant, M mass of the Moon, r geocentric distance of the point of observation, c distance from the geocentre to the Moon, θ geocentric colatitude, δ declination of the Moon and H its hour angle. The P_{nm} are the fully normalized Legendre functions of degree n and order m. The order m is associated to the different tidal bands through the hour angle. The time variations of the potential are linked to r,δ and H. Expressing these quantities as a function of the astronomical arguments describing the motion of the celestial bodies inside the solar system, it is possible to develop the tidal potential in a sum of harmonic constituents, under the form

$$W = D\sum_{n=2}^{\max}\sum_{m=0}^{n}(\frac{r}{a})^n \Gamma_{nm}(\theta).P_{nm}(\cos\theta).\sum_i [C_i^{nm}\cos(\alpha_i t) + S_i^{nm}\sin(\alpha_i t)] \qquad (2)$$

with D [Newton.m], so called “Doodson constant”, $\bar{a}$ mean equatorial radius, Γ_{nm} normalisation coefficients and $P_{nm}(\cos\theta)$ geodetic coefficients. The arguments α_i are expressed in function of astronomical arguments. If we consider only Moon and Sun, neglecting the planets of the solar system, we can write

$$\alpha_i = a\tau + bs + ch + dp + eN' + fp_s$$

with τ mean local lunar time (H+180°), s mean tropic longitude of the Moon, h mean tropic longitude of the Sun, p mean tropic longitude of the lunar perigee, N’=-N mean tropic longitude of the ascending lunar node changed of sign and p_s mean tropic longitude of the solar perigee. The angular speed of a tidal wave is completely determined by its argument under the form (a,b,c,d,e,f). Among the different development of the tidal potential one generally use as standards the TAM1200 potential (Tamura, 1987) and the HW95 catalogue (Hartmann and Wenzel, 1995).

From the tidal potential it is possible to compute the different tidal components. In this study we focus on the vertical component of the tidal force i.e. the variation of gravity. The Earth body submitted to the tidal forces is deformed and this deformation produces an additional change of potential. The global effect on the tidal gravity changes is characterized by the so called “amplitude factor”. For a given tidal wave, the amplitude factor δ is defined as the ratio A/A_a (Melchior, 1983) of the effective amplitude A with respect to the astronomical tide of amplitude A_a. Several theoretical models of the Earth response to the tidal forces have been developed in the last decades: Wahr-Dehant-Zschau (Dehant, 1987), DDW99 (Dehant et al., 1999), MATH01 (Mathews, 2001). Their results provide the so called body tides with amplitude A_{th} and amplitude factor values $\delta_{th}= A_{th}/A_a$. It is thus possible to define the different body tides models by a vector $\mathbf{R}(\delta_{th}.A_a, 0)$, expressing the fact that the body tide is in phase with the astronomical one. The analysis of the observations will provide an observed tidal vector $\mathbf{A_o}(\delta A_a,\alpha)$, where α is the difference between the observed and the astronomical local phases with lag counted as negative. Unhappily it is generally not possible to compare directly the observed and body tides vectors as the ocean tides effect is still mixed up in the observations. The tidal loading vector **L**, which takes into account the direct attraction of the water masses, the flexion of the ground and the associated change of potential, is generally evaluated by performing a convolution integral between the ocean tide models and the load Green’s function computed by Farrell (Farrell, 1972). We subtract the tidal loading effects $\mathbf{L}(L,\lambda)$ to get the so called “corrected” tidal parameters: amplitude factor δ_c and phase difference α_c.

$$\mathbf{A_c}(\delta_c A_a, \alpha_c) = \mathbf{A_o} - \mathbf{L} \qquad (3)$$

which can be directly compared with the body tides models **R**.

The Earth response is different for the different degrees of the potential. For W_2 the recent body tides models agree at the level of a few tenth of percent and these different models have been evaluated using tidal gravity observations, mainly superconducting gravimeters data provided by the GGP consortium. The DDW99 and MATH01 models agree with the observations corrected for the ocean tides loading at the level of 10^{-3} (Baker and Bos, 2003; Ducarme et al., 2001, 2002, 2007, 2009).

2. Constrains on the tidal analysis procedure

The analysis of earth tide observations is usually carried out by least squares adjustment. A general description of the procedure and of its advantages can be found for example in Wenzel 1997b. The goal of the tidal analysis is to determine the so called tidal parameters i.e. amplitude factors (ratio between the observed amplitude A_o and the theoretical one A_{th}) and phase differences (difference between the observed phase α_o and the theoretical one α_a), for different tidal "wavegroups". The wavegroup concept was proposed by Venedikov (1961). Due to the limited resolution of any analysis technique, the frequency resolution is limited by the recording length T. According to the Rayleigh criterion the separation of the waves is generally restricted to $\Delta f \geq 1/T$. However the Rayleigh criterion should be used as a rule of thumb only. For the least squares adjustment method, where the frequencies are known beforehand, the separation depends on the recording length T and on the signal-to-noise ratio. For high signal to noise ratios, as it is the case with SGs, waves with frequency differences $\Delta f < 1/T$ can be sometimes separated. In any case it is impossible to determine individual tidal parameters for all the tidal waves contained in any tidal potential catalogue. Instead, average tidal parameters are determined for "wavegroups" containing neighbouring waves. The Rayleigh criterion applies in this case on the frequency difference between the main wave of two neighbouring wavegroups. It is supposed that the tidal parameters are identical for all the waves inside a wavegroup. This assumption is generally not verified as different degrees of the potential are mixed inside of the same group. To cope with this problem the usual practice is to multiply the theoretical amplitude of the waves which are not belonging to the same degree as the main wave of the group by the ratio of the theoretical amplitude factors. For example, if the tidal gravity factors for (2,2) and (3,2) terms in (2,2) group are δ_2 and δ_3 (Melchior, 1983), the theoretical amplitude of any (3,2) term will be multiplied by δ_3/δ_2. If the observed tidal factor of the group is δ, the contribution of a (3,2) term is in fact $\delta.\delta_3/\delta_2 \approx \delta_3$ if $\delta_2 \approx \delta$. This approximation is generally valid as the observed and theoretical tidal factors agree generally within a few per cent while the discrepancy between the theoretical factors of different degrees of the potential are of the order of 10%. Moreover the contribution of the components deriving from W_2 are much larger than the signal coming from the higher degrees of the potential, so that the residual effect becomes generally negligible. This procedure should be applied also to the terms generated by W_4.

3. First approach of the nodal waves

As a matter of fact the argument of the nodal waves differ only from the argument of their closest neighbour by the variable N' associated to the Lunar node, which has an angular speed of 0°.00220641 per hour. According to the Rayleigh criterion, the period required to separate such waves is thus 18.6124 years. In section 4 we discuss how it is possible to relax considerably this condition.

Let us consider first the data of the superconducting gravimeter CD021 at station Membach (BE). It is one of the longest and most precise series observed with a superconducting gravimeter (Hinderer et al., 2007) in the framework of the Global Geodynamics Project (GGP, Crossley et al., 1999). The Tables 1 and 2 present the characteristics of the principal nodal waves and the tidal factors computed with the ETERNA (Wenzel, 1996) software. It is noticed at the first glance that there do not generally exist a pair of nodal waves symmetrical with respect to the main tidal constituent. The exceptions are M1, K1 and NO1. NO1- (1,0,0,1,-1,0) with an amplitude of $0.7 nms^{-2}$ is not negligible, but it is located very close to M1+ (1,0,0,0,1,0), which has a similar amplitude (Table 2). The difference in angular speed is only p-2N' i.e. $2.29\ 10^{-4}$ deg/hour. The period of commensurability becomes then 179 years! We cannot separate both components simultaneously The separation of $M1^+$ becomes possible if we keep NO1 and NO1- in one and the same group. Inversely results for $NO1^-$ are obtained by grouping M1 and $M1^+$. However the precision is low.

In most of the cases the tidal parameters of the nodal waves agree with those of the main tidal constituent within one or two σ (RMS error). The main exceptions are P1 and K1 in the diurnal band, M2 in the semi-diurnal band and perhaps M3 in the ter-diurnal one. In the diurnal band the amplitude factors are frequency dependent due to the FCN resonance

(Ducarme et al., 2007). The slope of the resonance being steeper close to K1 and the nodal waves larger we can perhaps get some useful information on the FCN from the K1 triplet. Concerning M2 and M3 one can suspect a different resonance of the nodal waves with respect to the main tidal constituent in the ocean tides loading. However it is not confirmed by the analysis of the ocean tides records at Oostende (BE) between 1945 and 2006 as shown in Table 3.

4. K1 (1, 1, 0, 0, 0, 0) and its nodal waves $K1^-$ (1, 1, 0, 0,-1, 0) and $K1^+$ (1, 1, 0, 0, 1, 0)

As seen in the previous section, the K1 triplet (Table 1) is especially interesting, being submitted to the resonance of the liquid core of the Earth. The amplitude factors of the three constituents should differ by 0.1% according to different Earth models (Table 4). The GGP data base is incorporating the observations of 26 tidal gravity stations between 1997 and 2010. From the point of view of the Rayleigh criterion no series already reaches the 18.6124 year data length required for the separation of the nodal waves. Including data prior to GGP the series of Brussels (more than 18 years), Cantley (16.5 years) and Membach (14.5 years) hardly reach the required time span. Most of the stations however reach a data span larger than 10 year.

To save a maximum of series, we can use the advantages of the VAV04 tidal analysis program (Venedikov and Vieira, 2004). The main difference with respect to the more popular ETERNA software (Wenzel, 1996) resides in the filtering technique used to separate the tidal signal in the spectrum. ETERNA is applying overlapping high pass filters on the original data to produce filtered series still including all the complete tidal signal, while VAV04 is applying different odd and even filters to separate the tidal bands at different angular speed Ω: D (Ω=15°/h), SD (Ω=30°/h), TD (Ω=45°/h), QD (Ω=60°/h) and so on…. Moreover the filter length is generally limited to 48h and always applied without overlapping. The least square adjustment is applied on these discrete series of filtered data. The main advantage of VAV04 for the determination of the small nodal waves is the automatic elimination of noisy data (Venedikov and Ducarme, 2000) based on a statistical study of the residues of the filtered data in the four frequency bands: D (Ω=15°/h), SD (Ω=30°/h), TD (Ω=45°/h) and QD (Ω=60°/h). The m.s.d. $\sigma(\Omega)$ is used to define a threshold level $t_S\sigma(\Omega)$ where t_S is supposed to be a Student coefficient. Venedikov used the classical value $t_S = 3$ (the 3 sigma rule). VAV04 provides also a tool to relax the Rayleigh criterion for the separation of the nodal waves by numerical experimentation. To decide if a finer separation is justified we can use the Akaike information criterion (AIC, Sakamoto et al., 1986). For a given data set the optimal separation corresponds to a minimal value of AIC. After a systematic experimentation we were able to separate the nodal waves without degrading the AIC value for series close to 3,500 days or 9.5 years as a minimum (Table 4). It is only half of the length based on the Rayleigh criterion.

The separation of the nodal waves is not valid for Bad Homburg and Sutherland as the error on K1 is increased by a factor of two after the separation of $K1^-$ and $K1^+$. We present here the results of 12 GGP stations.

As seen from Table 1, the nodal wave $K1^-$ (1, 1, 0, 0,-1, 0) has a much smaller amplitude than the symmetrical wave $K1^+$ (1, 1, 0, 0, 1, 0) and is thus determined with a much lower precision. The associated RMS errors on the amplitude factors are of the order of respectively 0.15% and 0.02%, corresponding to the inverse of the amplitude ratio. K1 and its nodal companions correspond to the annual modulation of the meteorological wave S1. the tidal factors of $K1^-$ is thus much more affected by environmental conditions. It is clearly seen in the Brussels results, which is not providing a reliable amplitude factor for $K1^-$, although it is the only series longer than 18 years.

A direct comparison of the tidal amplitude factors of $K1^-$, K1 and $K1^+$ given in Table 4 with the theoretical values is not possible as we did not apply any ocean load correction. As a matter of fact we do not have ocean tides models for these nodal waves. We can indeed suppose that, inside the K1 group, the ocean load correction is directly proportional to the amplitudes of the different waves due to the very close frequencies. This hypothesis is not in contradiction with the results of the Oostende tide gauge, given the associated RMS errors (Table 3). We decided thus, as a first approximation of the slope of the resonance, to use the normalized differences

$\rho^- = (\delta_{K1}^- - \delta_{K1})/\delta_{K1} = \delta_{K1}^-/\delta_{K1} - 1$

and (4)

$\rho^+ = (\delta_{K1} - \delta_{K1}^+)/\delta_{K1} = 1 - \delta_{K1}^+/\delta_{K1}$.

It has the advantage to suppress the calibration errors and to reduce drastically the ocean load contribution from the result if the load vector **L** is proportional to the amplitude of the different waves.

Neglecting other perturbation sources than ocean tides we can write

$$\mathbf{A_o} = \mathbf{R} + \mathbf{L} \qquad (5)$$

and derive the two components of $K1^-$ and K1

$\mathbf{A^-_o}(\delta^-_{th}.A^-_a + L^-\cos\lambda^-, L^-\sin\lambda^-)$ and $\mathbf{A_o}(\delta_{th}.A_a + L\cos\lambda, L\sin\lambda)$

If $A^-_a = x\,A_a$ we state $L^- = xL$, $\lambda^- = \lambda$ to get for K1- and K1

$$A_o^- = \sqrt{(\delta_{th}^- x A_a + xL\cos\lambda)^2 + xL\sin^2\lambda} = \delta_{th}^- x A_a \sqrt{1 + 2L\cos\lambda/\delta_{th}^- A_a + L^2/(\delta_{th}^- A_a)^2}$$

$$A_o = \sqrt{(\delta_{th} A_a + L\cos\lambda)^2 + L\sin^2\lambda} = \delta_{th} A_a \sqrt{1 + 2L\cos\lambda/\delta A_a + L^2/(\delta_{th} A_a)^2}$$

$$\delta^- = A_o^-/xA_a = \delta_{th}^- \sqrt{1 + 2L\cos\lambda/\delta_{th}^- A_a + L^2/(\delta_{th}^- A_a)^2}$$

$$\delta = A_o/A_a = \delta_{th}\sqrt{1 + 2L/\delta_{th} A_a + L^2/\delta_{th}^2 A_a^2}$$

so that we get

$\delta^-/\delta \approx \delta_{th}^-/\delta_{th}$ considering $\delta_{th}^- \cong \delta_{th}$ under the square root

The ocean load contribution is thus largely eliminated from the ratio of the observed amplitude factors, which is then close to the ratio of the body tides amplitude factors.

A similar demonstration is valid for $K1^+$.

5. Discussion of the results

Table 5 presents the relative variations of the amplitude factors inside the K1 triplet using the ρ parameter and the corresponding values for different body tides models. We note that the non hydrostatic models provide lower values of ρ^- and ρ^+ than the hydrostatic ones. It is due to the shift of the resonance toward longer periods. The same results are graphically displayed in Figure 1.

As expected the standard deviation is much larger on ρ^- (0.31%) than on ρ^+ (0.08%). The mean value $\bar{\rho}^- = 0.262\% \pm 0.088\%$ is not really compatible with any of the models. On the contrary the mean value $\bar{\rho}^+ = 1.113\% \pm 0.022\%$ is close to the non hydrostatic models. It confirms the results presented in Ducarme et al., 2009 for the corrected amplitude factor δ_c of

the wave O1 and the ratio $\delta_c(O1)/\delta_c(K1)$, using the data of the West European Network (WEN). The hydrostatic models are offset by a bit more than the associated RMS error.
Looking at Figure 1 there is an obvious correlation (r=0.7) between the observed values of ρ^- and ρ^+. Larger or smaller values of ρ^- are preferentially associated with similar values of ρ^+, the slope of the regression line being close to 3, i.e. the perturbations are three times larger for ρ^- than for ρ^+. It should be noted that correlated extreme values are found also among the WEN stations for which the tidal loading is weak in the diurnal band (Ducarme et al., 2009), while stations with a large loading, such as Matsushiro and Wuhan, do not show any correlation. The perturbations are not due to ocean tides loading but their origin is more likely to be found in the environmental noise concentrated on S1, as K1 corresponds to the annual modulation of S1. The noise propagation around S1 was already pointed out in Ducarme and Melchior, 1998.

6. Conclusions

A strict application of the Rayleigh criterion should limit the separation of the nodal waves to series of 18 years minimum. The longest series of superconducting gravimeters participating to the GGP consortium are now ranging between 10 and 18 years. It was possible to extract successfully the nodal waves for 12 series longer than 3,500 days using the advantages of the VAV04 tidal analysis program. Most of the nodal waves do not provide a new insight into tidal theory with the notable exception of the K1 triplet. The slope of the FCN resonance curve is producing differences in the amplitude factors inside the triplet at the level of 0.1%. This effect is clearly seen in our results. We introduce a parameter $\rho^{\pm} = \left|1- \delta_{K1}{}^{\pm}/\delta_{K1})\right|$, free from calibration errors and ocean tides loading influence, to express the relative difference between K1 and its nodal companions $K1^-$ or $K1^+$. The $K1^-$ nodal wave has a too small amplitude to provide reliable results but the mean relative difference ρ^+ between K1 and $K1^+$ (0.113%±0.022%) is very close to the values 0.124% and 0.116% predicted respectively by the DDW99NH (Dehant et al., 1999) and the MAT01NH (Mathews, 2001) models. It should be worth to introduce the nodal wave $K1^+$ in the determination of the FCN parameters, besides O1, P1, K1, PSI1 and PHI1.

Acknowledgements

The data used in this study were extracted from the Global Geodynamics Project (GGP) data base and preprocessed at the International Centre for Earth tides (ICET).

Bibliography

Baker, T. F., Bos, M. S., 2003. Validating Earth and ocean models using tidal gravity measurements. *Geophys J. Int., 152, 468-485*
Crossley, D., Hinderer, J., Casula, G., Francis, O., Hsu, H. T., Imanishi, Y., Jentzsch, G., Kääriaïnen, J., Merriam, J., Meurers, B., Neumeyer, J., Richter, B., Shibuya, K., Sato, T., Van Dam, T., 1999. Network of superconducting gravimeters benefits a number of disciplines. *EOS, 80, 11, 121/125-126.*
Dehant, V., 1987. Tidal parameters for an inelastic Earth. *Physics of the Earth and Planetary Interiors*, 49, 97-116, 1987.
Dehant V., Defraigne P., Wahr J., 1999. Tides for a convective Earth. *J. Geophys. Res., 104,B1, 1035-1058.*
Ducarme B., Melchior P., 1998. Eight lunar nodal waves and third degree waves derived from the 14 year series of observations with the superconducting gravimeter GWR/T3 in Brussels

Proc. 13th Int. Symp. on Earth Tides, Brussels July 22-25, 1997. Observatoire Royal de Belgique, Série Géophysique, Brussels, 347-356
Ducarme B., Sun H.P., 2001. Tidal gravity results from GGP network in connection with tidal loading and Earth response. Proc. 14th Int. Symp. On Earth Tides. *Journal of Geodetic Society of Japan*, 47, 1, 308-315.
Ducarme B., Sun H.P., Xu J.Q., 2002. New investigation of tidal gravity results from GGP network. *Bull .Inf. Marées Terrestres*, 136, 10761-10776
Ducarme B., Sun H. P., Xu J. Q., 2007. Determination of the free core nutation period from tidal gravity observations of the GGP superconducting gravimeter network. *Journal of Geodesy*, 81, 179-187 *(DOI:* 10.1007/s00190-006-0098-9)
Ducarme B., Rosat S., Vandercoilden L., Xu J.Q., Sun H.P., 2009. European tidal gravity observations: Comparison with Earth Tides models and estimation of the Free Core Nutation (FCN) parameters. Proceedings of the 2007 IAG General Assembly, Perugia, Italy, July 2 - 13, 2007, Observing our Changing Earth, M.G. Sideris (ed.), Springer Verlag, *International Association of Geodesy Symposia*, 133, 523-532*(DOI10.1007/978-3-540-85426-5)*.
Farrell, W. E., 1972. Deformation of the Earth by surface load. *Rev. Geoph.*, 10,761-779.
Hartmann T., Wenzel H. G., 1995. Catalogue HW95 of the tide generating potential. *Bull. Inf. Marées Terrestres*, 123, 9278-9301.
Hinderer, J., D. Crossley, and R. Warburton, 2007, Superconducting Gravimetry, in: Treatise on Geophysics, Vol 3., eds. T. Herring and G. Schubert, Elsevier
Mathews, P. M., 2001. Love numbers and gravimetric factor for diurnal tides. Proc. 14thInt. Symp. Earth Tides. *J. Geod. Soc. Jpn.,* 47 (1), 231-236.
Melchior P. 1983. The tides of the Planet Earth, Pergamon Press, 641 pp.
Sakamoto Y., Ishiguro M., Kitagawa G. 1986. Akaike information criterion statistics, *D. Reidel Publishing Company, Tokyo,* 290 pp.
Tamura, Y., 1987. A harmonic development of the tide-generating potential, *Bull. d'Inform. Marées Terrestres*, 99, 6813-6855.
Venedikov, A.P., Vieira, R., 2004: Guidebook for the practical use of the computer program VAV – version 2003. Bull. Inf. Marées Terrestres, 139, 11037-11102.
Wenzel, H. G., 1996. The nanogal software: earth tide data preprocessing package. *Bull. Inf. Marées Terrestres, 124, 9425-9439*
Wenzel H.G., 1997a. Tide Generating Potential for the Earth. *Tidal Phenomena, Lecture Notes in Earth Sciences,* Helmut Wilhelm, Walter Zürn and Hans-Georg Wenzel editors, 66, 9-26.
Wenzel H.G., 1997b. Analysis of Earth Tide Observations. *Tidal Phenomena, Lecture Notes in Earth Sciences,* Helmut Wilhelm, Walter Zürn and Hans-Georg Wenzel editors, 66, 59-76

Table 1: Principal nodal waves derived from the potential of degree 2 (W2). Amplitudes are given at 45° latitude

a) diurnal waves

Wave	τ	s	h	p	N'	Angular speed °/hour	Ampl. nm/s^2	δ σ	α° σ°	origin
2Q1-	1	-3	0	2	-1	12.85207978	1.48	1.1554 ±.0077	-0.556 ±.383	nodal
2Q1	1	-3	0	2	0	12.85428619	7.87	1.1518 ±.0015	-0.664 ±.074	Ellipt. Q1
σ1-	1	-3	2	0	-1	12.92493343	1.79	1.1481 ±.0065	-1.320 ±.322	nodal
σ1	1	-3	2	0	0	12.92713984	9.49	1.1480 ±.0012	-0.761 ±.060	variation O1
Q1-	1	-2	0	1	-1	13.39645449	11.22	1.1453 ±.0010	-0.182 ±.050	nodal
Q1	1	-2	0	1	0	13.39866089	59.49	1.1469 ±.0002	-0.212 ±.009	Ellipt.O1
O1-	1	-1	0	0	-1	13.94082919	58.62	1.1490 ±.0002	0.120 ±.001	nodal
O1	1	-1	0	0	0	13.94303560	310.73	1.14935 ±.00004	0.1072 ±.0018	L declin.
LK1-	1	0	0	-1	-1	14.48520390	1.63	1.1518 ±.0078	0.660 ±.386	nodal
LK1	1	0	0	-1	0	14.48741031	8.78	1.1523 ±.0015	0.212 ±.074	Ellipt. O1
(NO1-)	1	0	0	-1	1	14.49448753	0.69	1.1700 ±.0148	1.256 ±.726	nodal
NO1	1	0	0	1	0	14.49669393	24.43	1.1526 ±.0006	0.189 ±.027	Ellipt. $K1^m$
NO1+	1	0	0	1	1	14.49890034	4.90	1.1548 ±.0026	0.354 ±.128	nodal
P1-	1	1	-2	0	-1	14.95672495	1.63	1.1598 ±.0067	0.821 ±.329	nodal.
P1	1	1	-2	0	0	14.95893136	144.55	1.1496 ±.0001	0.228 .004	S declin.
K1-	1	1	0	0	-1	15.03886223	8.65	1.1435 ±.0013	0.394 ±.065	nodal
K1	1	1	0	0	0	15.04106864	436.80	1.13715 ±.00003	0.2813 ±.0013	LS declin.
K1+	1	1	0	0	1	15.04327505	59.28	1.1360 ±.0002	0.310 ±.010	nodal
J1	1	2	0	-1	0	15.58544335	24.44	1.1585 ±.0005	0.151 ±.022	Ellipt. $K1^m$
J1+	1	2	0	-1	1	15.59008516	4.85	1.1544 ±.0023	0.283 ±.112	nodal.
OO1	1	3	0	0	0	16.13910168	13.36	1.1563 ±.0008	0.088 ±.041	3L declin.
OO1+	1	3	0	0	1	16.14130809	8.56	1.1558 ±.0012	0.099 ±.061	nodal
NU1	1	4	0	-1	0	16.68347639	2.56	1.1556 ±.0042	0.377 ±.208	Ellipt. OO1
NU1+	1	4	0	-1	1	16.68568279	1.64	1.1557 ±.0062	0.206 ±.309	nodal

b)semi-diurnal waves

Wave	τ	s	h	p	N’	Angular speed °/hour	Ampl. nm/s²	δ σ	α° σ°	origin
N2-	2	-1	0	1	-1	28.43752313	2.69	1.1739 ±.0029	3.061 ±.141	nodal
N2	2	-1	0	1	1	28.43972953	71.96	1.1723 ±.0001	3.111 ±.005	Ellipt. M2
M2-	2	0	0	0	-1	28.98189783	14.02	1.1915 ±..0005	2.436 ±.025	nodal.
M2	2	0	0	0	0	28.98410424	375.80	1.18731 ±.00002	2.4446 ±.0009	L princ.
K2	2	2	0	0	0	30.08213728	47.51	1.1939 ±.0002	1.027 ±.007	LS decl.
K2+	2	2	0	0	1	30.08434369	14.16	1.1950 ±.0005	1.178 ±.024	nodal
η2	2	3	0	-1	0	30.62651199	2.66	1.1954 ±.0028	0.359 ±.136	Ellipt. K2m
η2+	2	3	0	0	1	30.62871839	1.16	1.1926 ±.0065	-0.193 ±.310	nodal

Table 2: Principal nodal waves derived from the potential of degree 3 (W_3)
The amplitude is given at 45° latitude

Wave	τ	s	h	p	N’	Angular speed °/hour	Ampl. nm/s²	δ *σ*	α° *σ°*	origin
M1-	1	0	0	0	-1	14.48984571	0.93	1.0866 ±*0123*	1.691 ±*649*	nodal
M1	1	0	0	0	0	14.49205212	6.28	1.0795 ±*0019*	0.922 ±*010*	L Princ.
M1+	1	0	0	0	1	14.49425853	0.81	1.0777 ±*0097*	0.761 ±*517*	nodal
3MK2-	2	-1	0	0	-1	28.43288131	1.10	1.0704 ±.0064	0.410 ±.342	nodal
3MK2	2	-1	0	0	0	28.43508772	6.47	1.0675 ±.0011	0.093 ±.059	L decl.
3MO2	2	1	0	0	0	29.53312076	5.97	1.0658 ±.0012	-0.408 ±.062	L decl.
3MO2+	2	1	0	0	1	29.53532717	1.12	1.0658 ±.0061	-0.065 ±.329	nodal
M3-	3	0	0	0	-1	43.47394995	0.29	1.0383 ±.0137	0.307 ±.758	nodal
M3	3	0	0	0	0	43.47615636	5.23	1.0615 ±.0008	0.461 ±.042	L Princ.

Table 3: Some nodal waves observed by the Oostende tide gage (1945-2006)

wave	Doodson argument	Amplitude (cm)	Amplitude factor
K1-	165.545	0.19±0.06	0.68±0.21
K1	165.555	5.662±0.060	0.409±.006
K1+	165.565	0.64±0.05	0.34±0.03
M2-	255.545	6.03±0.04	16.83±0.10
M2	255.555	181.23±0.04	18.885±0.004
M3-	355.545	0.045±0.025	10.1±5.9
M3	355.555	0.921±.0026	11.69±0.33

Table 4 : K1 and its two nodal waves as observed by the GGP network, **N** number of days
Δ(AIC) : relative diminution of the Akaike Information Criterion after **n** iterations

Station	**N**	**n**	**Δ(AIC)**	**K_1^-**		**K_1**		**K_1^+**	
			%	δ σ	α° σ°	δ σ	α° σ°	δ σ	α° σ°
Brussels	6699	5	-0.16	(1.1338)	0.583	1.13712	0.248	1.1363	0.333
				±.0021	±.108	±.00004	±.002	±.0003	±.016
Cantley	5881	3	-0.16	1.1480	0.612	1.14725	0.586	1.1462	0.648
				±.0015	±.074	±.00003	±.002	±.0002	±.011
Membach	5938	3	-0.45	1.1408	0.337	1.13716	0.280	1.1358	0.308
				±.0011	±.057	±.00002	±.001	±.0002	±.008
Canberra	4450	0	-0,14	1.1299	-0.747	1.12965	-0.831	1.1295	-0.866
				±.0018	±.091	±.00004	±.002	±.0003	±.0013
Metsahovi	4905	3	-0.38	1.1485	0.199	1.13998	0.083	1.1374	0.144
				±.0019	±.093	±.00004	±.002	±.0003	±.0014
Strasbourg	5024	0	-0.29	1.1387	0.379	1.13695	0.269	1.1355	0.276
				±.0014	±.070	±.00003	±.001	±.0002	±.010
Wettzell	4500	0	-1.31	1.1442	0.277	1.13673	0.204	1.1334	0.230
				±.0014	±.072	±.00003	±.002	±.0002	±.011
Medicina	5069	3	-0.94	1.1369	0.859	1.13484	0.355	1.1341	0.405
				±.0014	±.070	±.00003	±.001	±.0002	±.0010
Matsushiro	4008	3	-0.36	1.1928	0.031	1.18466	-0.068	1.1836	-0.127
				±.0021	±.099	±.00005	±.002	±.0003	±.0015
Moxa	3657	3	-0.51	1.1400	0.357	1.13631	0.227	1.1350	0.224
				±.0012	±.058	±.00003	±.001	±.0002	±.009
Vienna	3425	0	-0.72	1.1358	0.216	1.13392	0.204	1.1330	0.246
				±.0021	±.106	±.00005	±.003	±.0003	±.0016
Wuhan	3319	0	-0.60	1.1548	-0.634	1.15350	-0.464	1.1528	-0.570
				±.0032	±.160	±.00006	±.003	±.0005	±.024
theory	Wahr-Dehant-Zschau			1.13326		1.13189		1.13032	
	DDW99 H			1.13330		1.13197		1.13043	
	DDW99 NH			1.13530		1.13405		1.13264	
	Mathews NH			1.13610		1.13494		1.13361	

Table 5: normalised variations of the amplitude factors around K1

Station	**Number of days**	$\rho^- = (\delta_{K1}^- - \delta_{K1})/\delta_{K1}$ %	$\rho^+ = (\delta_{K1} - \delta_{K1}^+)/\delta_{K1}$ %
Brussels*	6699	-0.292	0.072
Cantley	5881	0.065	0.092
Membach*	5938	0.320	0.120
Canberra	4450	0.022	0.013
Metsahovi	4905	0.747	0.226
Strasbourg*	5024	0.154	0.128
Wettzell*	4500	0.657	0.293
Medicina*	5069	0.181	0.065
Matsushiro	4008	0.687	0.089
Moxa*	3657	0.325	0.115
Vienna*	3425	0.166	0.081
Wuhan	3319	0.113	0.061
	mean	**0.262±.088**	**0.113±.022**
	Standard deviation	0.306	0.078
Theory	Wahr-Dehant-Zschau	0.121	0.139
	DDW99 H	0.117	0,136
	DDW99 NH	0.110	0,124
	Mathews NH	0.102	0.116

* stations belonging to the West European Network (Ducarme et al., 2009)

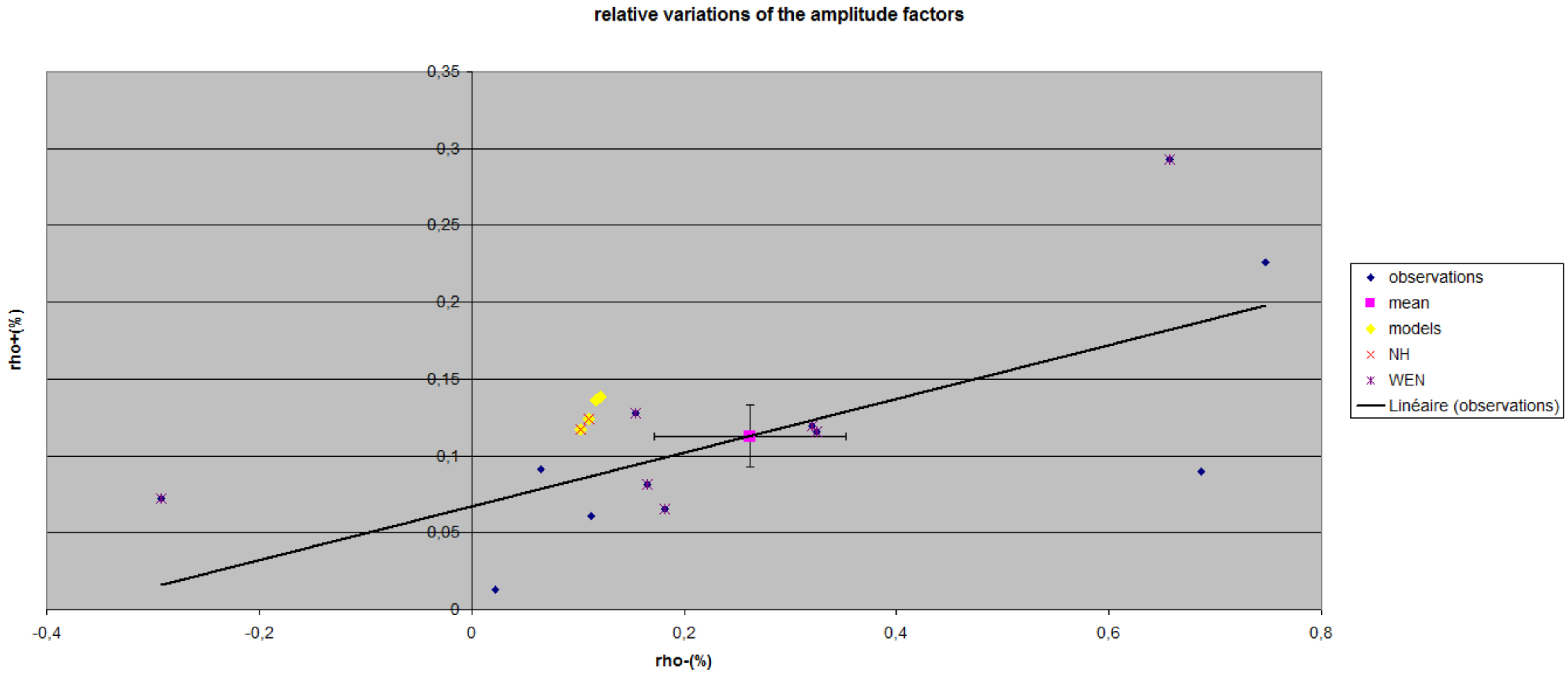

Figure 1: Relative variations of the amplitude factors
$rho^{-} = (\delta_{K1}^{-} - \delta_{K1})/\delta_{K1}$, $rho^{+} = (\delta_{K1} - \delta_{K1}^{+})/\delta_{K1}$
NH: Non Hydrostatic Earth models
WEN: stations belonging to the West European Network (Ducarme et al., 2009)